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SIMULATION OF FRESHWATER-SEAWATER INTERFACE BY EMPLOYING CARTESIAN MESH ON THE FEM MODEL

(Case Study : Semarang Aquifer – INDONESIA)

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ABSTRACT: The increasing concentration of human settlements, agricultural development, and economics activities in coastal zone will impose the shortage of fresh groundwater. This may trigger a number of environmental problems, such as; groundwater decline, seawater intrusion, and land subsidence. In order to solve the negative impacts, the understanding of freshwater-seawater interface dynamics due to the human intervention become importance. Analysis by using numerical model needs a good representation of geological system leading to huge number of elements requirement, particularly for three-dimensional model. Now days, computer hardware allows to support this requirement. However, mesh generation for data input of the Finite Element Method (FEM) model is a time consuming task, and the implementation of the Mesh Free Method (MFM) requires special technique for impositing the essential boundary conditions. Therefore, this paper is addressed to develop a simple modeling technique by employing Cartesian mesh system on the FEM model. Verification of the model showed a good agreement with some other results for benchmark problem that is mainly used in seawater intrusion simulation. In addition, the model has been implemented to simulate the interface behavior at Semarang coastal aquifer. Simulation was performed by using the 2D and the 3D model with multi-layer aquifer approach. Comparing with field observation data, the simulation showed accurate and reasonable results.

Keywords: interface, coastal aquifer, multi layer, Cartesian mesh.

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INTRODUCTION

Numerical modeling of Finite Element Method (FEM) is realized as the best method to analyze seawater intrusion phenomena. However, mesh generation for input data is well known as a formidable and time consuming task, therefore most of researchers simplify the complex of hydrogeological condition into a single layer of aquifer. This kind of simplification may give lack of accuracy such as shown by work of Frind (1980) and Rastogi et al (2002). Frind described the important influence of aquitard within a continuous system of aquifer-aquitard in controlling dynamics of the entire system, and Rastogi et al found that except dispersivity all other hydrogeological parameters produce significant effect to the seawater intrusion on multilayered-aquifers system.

Semarang aquifer is an extensive groundwater abstraction area in Indonesia, and it has been identified suffer from seawater intrusion. In order to understand the interface phenomena in this area, a new FEM model employing Cartesian Mesh is developed to overcome mesh generation problem in simulating the impact of groundwater extraction from multilayered aquifers.

GOVERNING EQUATION

The interface of freshwater-seawater is analyzed by using variable density approach in which requires solution of two partial differential equations representing fluid and solute mass balances. The fluid mass balance describes the groundwater flow at saturated-unsaturated medium, and the solute mass balance represents a mixing process on the freshwater-seawater interface.

Fluid Mass Balance

By assuming the contribution of solute dispersion to the mass average flux is negligible, the mass balance of fluid per unit aquifer volume at a point in the aquifer may be written as (Bear in Voss, 2002)

$$\left(\rho S_{op}\right)\frac{\partial p}{\partial t} + \varepsilon\frac{\partial \rho}{\partial C}\frac{\partial C}{\partial t} + \nabla(\varepsilon\rho v) = Q_p \qquad 1)$$

where ρ is fluid density (ML^{-3}), S_{op} is specific pressure storativity ($ML^{-1}T^{-2}$) , p is the pressure($ML^{-1}T^{2}$), C the solute concentration of fluid as a mass fraction (M_sM^{-1}), ε aquifer volumetric porosity (-), v fluid velocity (LT^{-1}), and Q_p a fluid mass source ($ML^{-3}T^{-1}$). In addition, the fluid velocity at any point in aquifer may be calculated through the application of Darcy's law;

$$v = -\left(\frac{k_{ij}}{\varepsilon\mu}\right).(\nabla p - \rho g) \qquad (2)$$

k_{ij} the permeability tensor (L^2), μ the fluid viscosity($ML^{-1}T^{-1}$), ε the porosity(-), and g the gravitational acceleration(LT^{-2}).

Solute Mass Balance

The solute mass balance equation including advective and dispersive spreading mechanisms is described mathematically by Bear (Voss, 2002) as;

$$\varepsilon\rho\frac{\partial C}{\partial t} + \varepsilon\rho v.\nabla C - \nabla\left[\varepsilon\rho\left(D_m I + D\right).\nabla C\right] = Q_p(C^* - C) \qquad (3)$$

where C is solute concentration as a mass fraction (M_sM^{-1}), D_m is apparent molecular diffusivity of solute in solution in a porous medium including tortuosity effects (L^2T^{-1}), I identity tensor (ones on diagonal, zero elsewhere), D dispersion tensor (L^2T^{-1}), and C* solute concentration of fluid sources (M_sM^{-1}).

Relation between density and concentration is given as a linear function;

$$\rho = \rho(c) \cong \rho_0 + \frac{\partial \rho}{\partial c}(c - c_0) \qquad 4)$$

where ρ_0 = fluid density at base solute concentration (ML^{-3}), and ∂ρ/∂c = constant of proportionality for each solute.

NUMERICAL MODEL

Mesh Generation

Mesh generation code for 2D and 3D numerical model by employing the Cartesian grid system developed in this paper are integrated with numerical model by a loose coupling system. As a loose coupling technique, the codes organize output files according to structure of data required by the numerical code. The 2D and 3D mesh are visualized with GL-View software, and structural contour maps of geological material layers in 3D-model with Model Viewer software.

By using the Cartesian system, the grid can be generated in a very short time and has simple arithmetic operation. Yet, its disadvantage is incorrect treatment of the geometry shape in a computational domain (Ono K et al, 1999). The Cartesian grid locates node points on a uniform grid, and the grid elements are rectangles in two dimensions and blocks in three dimensions. The Cartesian mesh is not body fitted. There is no embedding meshes and it is frequently used a "stair-cased" description of any geometry.

This mesh generation codes were designed with some operation steps:

a. *Defining of background mesh* to cover all modeling area and to define size of mesh, element connectivity, satellite nodes around central node, and nodes within an element.
b. *Surface map generation* through interpolating borehole data by using surface fitting employing the Moving Least Square Method.
c. *Assignment of hydrogeological parameters* as data input into space discretization with three different approaches; node-wise, cell-wise, and element-wise.

d. *Eliminating of in-active elements* (mesh) to reduce memory demand in simulation.
e. *Fixing of satellite nodes, element connectivity, and nodes information* by re-numbering nodes and elements, element connectivity, and satellite nodes of elements after elimination in-active mesh.
f. *Preparing of information of boundary nodes* to avoid difficulties in identifying boundary nodes to impose the essential boundary condition.

Numerical Code

Numerical code used SUTRA2D3D software as a basis code development. The code employed finite element and finite difference method to approximate the governing equations. The hybridization of both methods gain advantages in which it is naturally mass lumping, in contrast to common formulation of the finite element method, which requires mass lumping to avoid oscillatory solutions. In addition, the hybrid finite element and finite difference methods has all the ability of finite element method to handle irregular geometries and complex boundary conditions (Voss, 2003).

In this paper, the original code of SUTRA2D3D has been modified with four considerations;

(1) Problem to be solved has been simplified for groundwater flow and solute transport only.
(2) Ability to perform simulation with the Cartesian grid system or the FEM mesh.
(3) Efficiency of memory demand by recognizing parameter values for elements based on material type identity.
(4) Numerical solver uses Pre-Conditioned Bi-Conjugate gradient method.

Model Verifications

The 2D-Model Verification

The Henry problem is always used as benchmark problem of variable density models. Verification is mainly done to against other numerical results. The problem was formulated as a homogeneous, isotropic, confined, and rectangular aquifer with height is 1 m and length is 2 m. The top and bottom boundaries are impermeable, with a constant freshwater flux is $6.6.10^{-2}$ kg/sec entering the aquifer along the vertical at x = 0 m, and the seaside boundary at x = 2 m is a constant saltwater head. Hydrogeological parameters are that hydraulic conductivity is 1 cm/sec, effective porosity is 0.35, coefficient of molecular diffusion is $6.6.10^{-6}$ m^2s^{-1}, and specific storage is set to zero. Reference density is 1,000 kgm^{-3}, and brine density is 1,025 kgm^{-3}.

Simulation was performed in a regularly aquifer discretization with 231 nodes with time step increased 60 seconds. The problem, initially, was composed of freshwater, and then saltwater begins to intrude the freshwater from the sea boundary. Comparison with numerical result produced with Cartesian Mesh shows a good agreement with some other published results. Furthermore, to investigate the effect of jagged form of mesh to the numerical results, the vertical sea side of original geometry of Henry's problem is modified by prolong bottom boundary 1 m to create coastal slope. A simulation was run by discretizing the problem domain with the FEM mesh and the Cartesian mesh of 300 elements. Numerical results in figure (1) show a good agreement for both types of mesh.

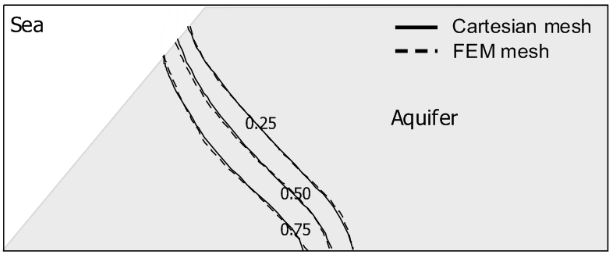

Fig. 1. Simulation result for modified slope at seawater boundary of the Henry's problem.

The 3D-Model Verification

Verification of the 3D-Model used circular island problem that concerns to post-drought recharge and restoration of groundwater lens on coastal unconfined aquifer. The problem poses flow in the sea undergoes a prolonged drought, and all ground water beneath the island becomes saline. Then, fresh rainwater recharge to the island begins and continues at a constant rate, raising the water table on the island, flushing out seawater, and eventually establishing a stable freshwater lens and a diffuse saltwater-freshwater interface.

Voss (2003) simulated the problem by representing only one fourth of the entire island. The 3D FEM mesh consists of 43,706 nodes and 40,000 elements. The horizontal direction is discretized with 20 m spaces, vertically 5 m except 5 meter from top surface with 1 m. By using the Cartesian mesh, it was created 33,218 nodes and 29,800 elements. Boundary condition of the hydrogeological system is set to be no flow crosses the inner and the bottom boundary. The vertical outer boundary is specified as hydrostatic seawater pressure. Along the top boundary, nodes above sea level receive freshwater recharge totaling 18.6658 kg/s of recharge for the entire circular island. At nodes below sea level, the pressure is specified to be hydrostatic seawater pressure and concentration of seawater.

Initial condition of salt concentration is set as seawater, and initial pressures are obtained through an extra simulation. The extra simulation was carried out by setting boundary condition that no recharge at the surface and specified pressure head along the sea bottom and the outer boundary. The model was run under transient condition to approximate both pressure and concentration using the time step of 0.2 year. The modeling achieves a new steady state after 100 time steps (20 years). The numerical results of both mesh systems are compared in figure (2) with small bias around the top of surface boundary. However, the simulations provide consistent variable-density fluid flow and solute transport results by exhibiting a good agreement to produce concentration distribution.

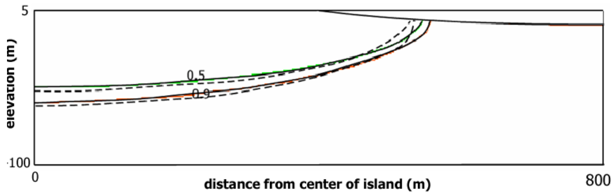

Fig. 2. Salinity concentration resulted with FEM-mesh and Cartesian mesh (dash line) simulation.

HYDRODYNAMICS OF THE INTERFACE OF SEMARANG AQUIFER

Hydrogeological System of Semarang Aquifers

The Semarang area is a flat terrain with average surface elevation of about 5.0 m above mean sea level (MSL) and becoming gently undulating in the southern part. The groundwater system is characterized with interaction between coastal area and hilly area.

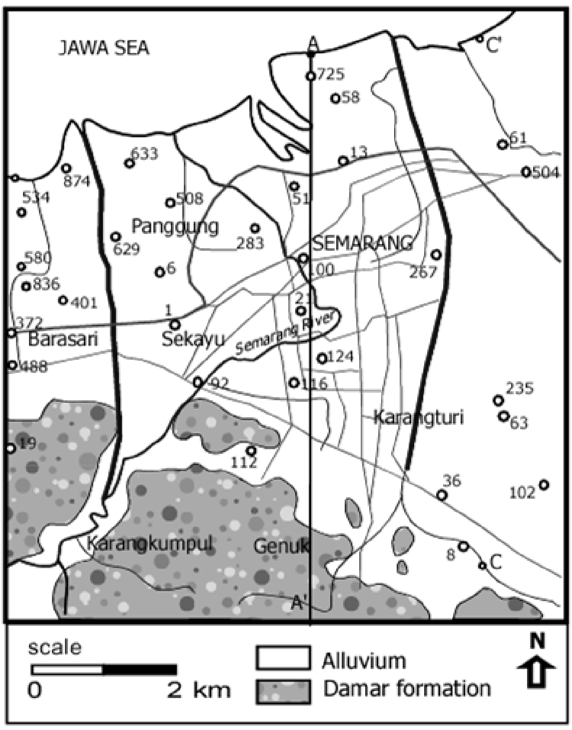

Fig. 3. Geological map of Semarang – Indonesia (GRDC in Arifin, et al. 2000)

Geological condition consists of two types of geological formation (fig. 3); Alluvium (Qa), and Damar Formation (Qtd). The Alluvium is composed coastal plain, river, and lake deposits. The materials arrange alternating layers of medium grained sand with clayey materials, with thickness of 50 m or more. The Damar formation is composed tuffaceous sandstone, conglomerate, and volcanic breccia. The groundwater extraction from unconfined aquifer was recorded about 6.20 x 10^4 m^3/year in 1990. In order to support modeling effort, trend of the exploitation by dug wells was be interpolated with a linear increment of 1.17 % based on population growth.

Two-Dimensional Profile Modeling

A two-dimensional profile model was aimed to achieve two objectives; to help facilitate development of 3D model, and to simulate the groundwater flow and solute transport patterns in detail with a fine level of spatial resolution. Despite of the profile model simulates only two dimensions problem, the model is better suited for calibrating certain aquifer parameters. This simulation was designed to obtain average piezometer heads and salinity concentrations from 1984 to 1998 in which was selected according to data availability.

A two-dimensional profile model was constructed along groundwater flow lines toward central of the Semarang city (fig. 3) at line A-A'. The cross section extends from inland boundary at 7,000 meter of coastline to sea boundary at 500 m of coastline. This physical domain was discretized 1 and 5 meter vertically, and 20 and 25 meter laterally. The discretization produces 9,410 nodes, and 9,024 elements.

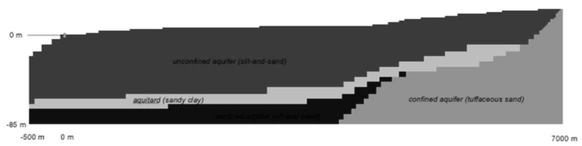

Fig. 4. The 2D-Cartesian mesh of Semarang Aquifer

Geological materials can be categorized into three aquifers; an unconfined aquifer and two confined aquifers conceptualized at figure (4). The boundary conditions consists of; Neumann type of no-flux boundary condition was assigned to the bottom of the aquifer, and above groundwater table; Neumann influx boundary condition was assigned at the land surface with net recharge of 3.5 x 10^{-4} m/day and salt concentration is 9 mg/liter; and Dirichlet type to the below MSL at seaward, and below 5 m at inland boundary.

Several internal sinks were assigned (fig. 5) to simulate the effect of pumping stress on the aquifer. Each internal sink does not act for actual position of production well in the field, but to represent a group of wells with relative pumping to distance from coastline.

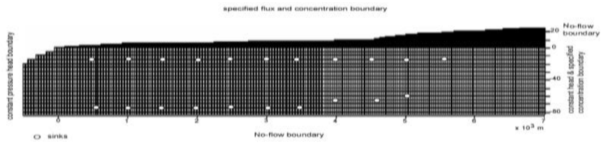

Fig.5. Boundary conditions are applied in the modeling

Initial condition was obtained by using two extra simulations. The first run with initial condition is arbitrary freshwater pressure head and 0.9 kg/m^3 of groundwater salinity. This run set boundaries without groundwater pumping and rainfall. Simulation was run for time step of one month, and numerical convergence of pressure head is 1.0 x 10^2 kg/m^2, and salinity concentration is 1.0 x 10^{-4} kg/m^3. The second running by using the pressure head and the salinity obtained on the first run, and setting the sinks with approximated pumping discharge on 1982 to achieve steady state.

This profile model was calibrated by adjusting the boundary stress and aquifer parameters, within a range of reasonable values, until simulated conditions generally matched with the conditions observed in the field. The calibration was based on data of monitoring wells located on or near the model transect. The calibrated parameters show in table (1).

Table 1. Calibrated hydrogeologic parameter for Semarang aquifer

Parameter	Value	Unit
Intrinsic permeability (silty sand)	1.02 x 10^{-11}	m^2
Intrinsic permeability(sandy clay)	1.02 x 10^{-17}	m^2
Intrinsic permeability (silty sand)	3.57 x 10^{-11}	m^2
Intrinsic perm.(tuffaceous sand)	4.08 x 10^{-12}	m^2
Porosity of aquifer	0.30	-
Porosity of aquitard	0.40	-
Viscosity	0.001	m s^{-2}
Gravity	9.81	kg m^{-2}
Longitudinal dispersivity	50	m
Transversal dispersivity	10	m
Molecular diffusion	3.565 x 10^{-3}	m^2 s^{-1}
Tortuosity	1	-
Density of freshwater	1000	kg m^{-3}
Density of seawater	1025	kg m^{-3}
Compressibility of water	4.47 x 10^{-10}	m s^2 kg^{-1}
Compressibility of soil matrix	1.00 x 10^{-8}	m s^2 kg^{-1}

Three-Dimensional Modeling

Spatial and Temporal Discretization

To simulate groundwater flow of Semarang area, a regularly spaced, Cartesian model grid was constructed. The x-axis would roughly parallel the coast. Each cell is 500 m by 100 m in the horizontal plane. The grid consists of 31 columns and 51 rows. In the vertical direction (z-axis) is discretized 2 m from land surface to MSL, and 5 m from the MSL to bottom boundary of the modeling area. As result, after eliminating non-active background mesh, the computational domain consists of 61,421 nodes and 55,358 elements. Input data into the 3D-model used calibrated parameter of the 2D-model with time discretization of one month.

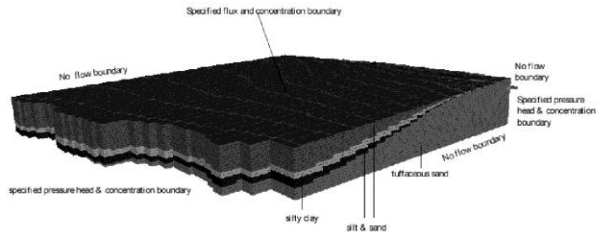

Fig. 6. The 3D Cartesian mesh of Semarang aquifer

Boundaries and Initial Condition

For most simulations of groundwater flow, boundaries of the model are extended to locations in the aquifer where hydrogeologic boundaries reside. Ideally, these hydrogeological boundaries are persistent flow linier, impermeable barriers, or areas that can be represented do not exist for the inland portion of the model domain. The boundaries conditions of modeling area can be described as;

a. *Seaward boundary;* was specified with a constant pressure head (0.0 kg.m^{-2}.s^{-1}), and salinity concentration boundary (0.0357 kg.m^{-3})
b. *Inland boundary;* was assumed not to be affected by withdrawal due to exploitation, so that specified pressure head and concentration boundary (0.0009 kg/m^3) was assigned to each node in the southern boundary.
c. *Lower model boundary*; represents the base of the Semarang aquifer as an impermeable layer.
d. *Upper model boundary*; represents water recharge due to rainfall evidence according to calibrated recharge in the 2D-model.

Initial condition was determined through a number of time steps of extra simulation with two running systems. The first running began to reach steady state condition through long-term transient simulation from arbitrary initial condition. The run was performed by applying above-mentioned boundary conditions except upper boundary in which was set to be zero recharge, and also there was no pumping discharge assigned. The second running was done with initial condition based on the obtained result from the first running. This simulation set pumping discharge equal to observation data at 1982 for 5 years of time periods. The final results of the second running system were assigned to be initial condition for simulation of the hydrodynamics at Semarang aquifer.

Simulation Results

Since the input parameter of this 3D-model used the calibrated parameter of 2D-model, no calibration effort had been performed. A different approach has been done in assignment pumping discharge to the model, in which in 2D-model converted the well discharge to a line by dividing with wide of aquifer interest, yet such kind assignment is not required in this 3D-model. To ensure that the variable density component of the model was working properly, special attention was focused on the salinity distribution that has simulated within the observation time. Within the observation periods, simulation was able to produce measured data to ensure the accuracy in representing the physical process within the hydrogeological system.

Numerical simulation results are visualized with maps (fig. 7) to represent iso-chlor 50 % of seawater salinity at depth of 35 below MSL. Obviously, the map shows that there is no seawater intrusion problem appears in 1985. In 1995 reflects a movement of seawater intrusion inland. The figure shows invasion of the seawater into aquifer at east side deeper than west side. Prediction of condition at 2010 by using assumption that the groundwater discharge increase according to trend 1982-1999, shows that the interface move far enough inland. The predicted interface has similar pattern that happened in 1995. These simulation results can be understood that a severe of the seawater intrusion problem will occur if the groundwater is continuously developed without mitigation efforts.

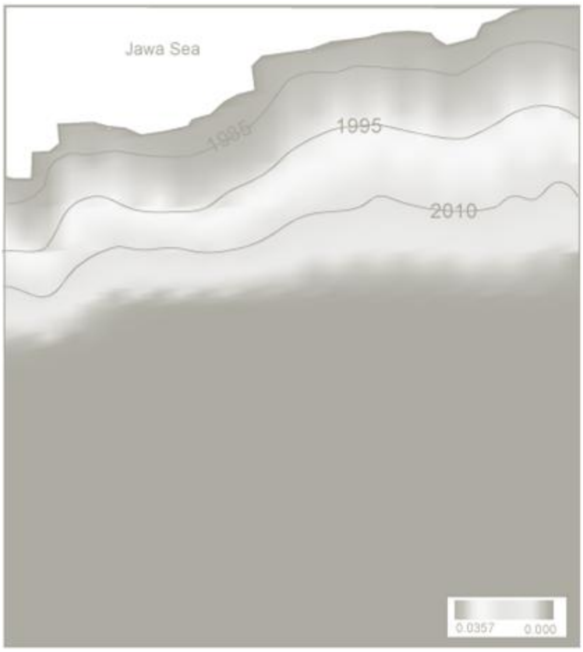

Fig. 7. The simulated interface form in 1985, 1995, and 2010 at depth of 35 m below MSL

CONCLUSIONS

A new numerical code of groundwater flow and solute transport employing the Finite Element Method with the Cartesian mesh has been developed. Verification by using benchmark problems of variable saturated-density flow of the Henry's seawater intrusion problem, and the circular island problem shows a good agreement with some others published numerical result.

Application of the code to simulate the seawater intrusion phenomena at Semarang aquifer could produce a reasonable interface form. The final calibration of the 2D-model shown an accurate the hydraulic head and the groundwater salinity compared with the observed data of 1982-1998. By using the calibrated parameter of the 2D-model, the 3D simulation produced reasonable form of the interface comparing with the observed data in 1982-1998. Extension the simulation periods to 2010 estimated that the interface invades too far inland. The thickness of aquifer has significant influence to seawater intrusion into the aquifer.

The Cartesian mesh system has contribution to discrepancy between the simulated with the observed of hydraulic head and groundwater salinity. These discrepancies are closely related to size of mesh, because an observation point will be defined into the nearest node.

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