Quasi-2D sediment transport model combined with Bagnold-type bed load transport

Sabaruddin Rahman†, Akira Mano‡ and Keiko Udo‡

†Department of Ocean Engineering, Hasanuddin University, Makassar, 90245, Indonesia
sabaruman5@gmail.com
‡International Research Institute of Disaster Science, Tohoku University, Sendai, 980-8579, Japan
mano@civil.tohoku.ac.jp
udo@irides.tohoku.ac.jp

ABSTRACT


Suspended load and bed load sediment transport are important component for the sediment transport in surf zone. The purpose of this study is to obtain a good model for sediment transport in surf zone by combining a quasi-2D sediment transport model and the Bagnold-type sediment transport model. The quasi-2D sediment transport model is used to simulate the suspended load transport, while the Bagnold-type for the bedload transport. A quasi-2D numerical wave model called Funwave was expanded to accommodate the sediment transport model. The model is validated by the published data for sediment transport in a wave flume. Two mode of morphological change is compared to evaluate the influence of wave-current (mode A) and instantaneous bottom velocity (mode B) in the third and forth velocity moment of Bagnold-type sediment transport model. Four sets of bed load transport parameters are evaluated to calculate bed level change. The evaluation shows that although $e_0$ exceeds one, it can produce bed level change similar to that by using the parameters proposed by Bailard. Parameters by van der Molen calculated very high bed level change, while Gallagher’s parameter produced relatively small bed level change. The performance of two modes of morphological change shows that mode A produce much better morphological change than mode B in surf zone for the bedload transport component. While for the suspended load component, mode B produces very high erosion in surf zone. Coupling of mode B and a wave motion-induced suspended load transport gives comparable morphological change to the experimental data.

ADDITIONAL INDEX WORDS: Quasi-2D, suspended load, bed load.

INTRODUCTION

In recent years, prediction of morphological change in the coastal region with acceptable accuracy has become increasingly important to engineers. Due to present computational constraints, time-domain modeling of large-scale seabed evolution in the nearshore zone requires approximate equations. Quasi-2D model is becoming more popular for this task. In this model, computation is discretised in horizontal space, while an approximation is made in the vertical direction. Boussinesq-type equations (BTE) are increasingly used by researchers in wave field. These equations have been used to accurately predict wave evolution across large basins, wave breaking over irregular topography, wave-structure interaction, and wave-induced current patterns. To simulate nearshore morphodynamics, some method must be employed to approximate breaking, wave-induced current and suspended sediment distribution.

Several artificial viscosity models to determine dissipation term for breaking wave based on BTE have been proposed by researchers. Zelt (1991) proposed an artificial viscosity to produce the dissipation term due to turbulence generated by wave breaking and bore propagation. It was treated by a diffusion term in the momentum conservation equation. Kennedy et. al (2000) used a momentum conserving eddy viscosity technique to model breaking. This is somewhat like the artificial viscosity formulated by Zelt (1991), but with extensions to provide a more realistic description of the initiation and cessation of wave breaking. Since this artificial viscosity is only useful for the horizontal diffusion, relationship of that to the vertical diffusion has been proposed by Rahman et al. (2010). However, further development to the evaluation of morphological change is required.

Hence, a quasi-2D sediment transport model using Boussinesq-type equations of wave model is proposed that allows the assessment of morphological change by incorporating of both suspended load and bed load sediment transport.

BOUSSINESQ-TYPE WAVE MODEL

Numerical model for wave propagation based on Boussinesq-type equations have become an important tool in coastal engineering, especially in applications where reflection and diffraction as well as nonlinear wave-wave interactions are important. In order to obtain an efficient formulation, Boussinesq-type wave models assume a vertical flow structure, i.e. the velocity profile across the depth is modeled analytically instead of calculated numerically (Wenneker, et.al. 2011).

The Boussinesq-type equations of Gobbi, et al. (2000) for conservation of mass may be written as:

$$ \eta + \mathbf{v} \cdot \mathbf{M} = 0 $$

where $\eta$ is the free surface elevation, the subscript $t$ denotes partial derivative with respect to time, and:

$\mathbf{M}$

$\mathbf{v}$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$

$\Theta$
In the present paper, the model is expanded to the suspended load transport model to evaluate suspended sediment transport rate at a point by the following expression:

\[ q_s = \int u(z)C(z)dz \]  

(7)
The transport rate is separated into two terms: mean and oscillatory term, respectively by the following formula:

\[ q = \overline{uC} + u'C' \]  

(8)
where the first term on the right-hand side of the equation is suspended sediment flux due to mean currents, while the second term is the flux coupling between oscillatory wave motions and sediment concentrations.

**Bagnold-type Bedload Transport Model**

Bagnold (1966) proposed a theory for bed-load transport based on the work done by the fluid to transport the sediment. He considered the stress equilibrium in steady flow, introducing the concept of a dispersive grain pressure on the bed surface, and assumed that at low bed-load concentrations the fluid component of the turbulent bed-shear stress equals the critical bed-shear stress at the threshold of sediment motion, while at high bed-load concentrations the fluid component of the turbulent bed-shear stress may be neglected. Luque (1974) found experimentally that at low bed-load concentrations the fluid component of the turbulent bed-shear stress is practically equal to the total bed-shear stress, and concluded on a theoretical basis that at high bed-load concentrations, during erosion with or without simultaneous deposition, the fluid component of the turbulent bed-shear stress must be practically equal to the critical bed-shear stress at the initiation of non-ceasing scour in the absence of a bed load.

Bagnold (1966) derived a stream-based sediment transport model. In that model, Bagnold assumed the sediment is transport in two models, i.e, the bedload transport and suspended transport. The bedload sediment is transported by the flow via grain to grain interactions; the suspended sediment transport is supported by fluid flow through turbulent diffusion. The expression to calculate sediment transport in term of the immersed weight vertically averaged total sediment transport rate becomes (Bagnold, 1966):

\[ i = \frac{\varepsilon_s}{\tan \phi - \tan \beta} + \frac{\varepsilon_c(1 - \varepsilon_s)}{w_s/u - \tan \beta} \omega \]  

(9)
where \( \omega \) is the available fluid power, \( w_s \) is the fall velocity of sediment, \( \varepsilon_s \) and \( \varepsilon_c \) are the bedload and suspended load efficiencies, respectively. They both are smaller than one. \( \tan \beta \) is the slope bottom, and \( \phi \) is the particle internal friction angle. A decrease of \( \phi \) means that the particles are eroded more easily (larger bed load) at a downward sloping bottom and less easily (smaller bed load) at an upward sloping bottom. The available fluid power is the work done by the bottom stress \( \tau_b \) 

\[ \omega = \tau_b \cdot u_b \]  

(9)

**SEDIMENT TRANSPORT MODEL**

In this section, developed morphology modules are described. Sediment transport consists of two components: suspended load transport and bed-load transport. To evaluate the suspended sediment transport, an advection diffusion equation for the suspended sediment transport is incorporated in the model.

**Quasi-2D Sediment Transport Model**

Rahman et al. (2011) proposed suspended sediment concentration model derived from classical convection-diffusion equation to compute the equilibrium concentration profile in steady flow. The model can be written as follow:

\[ C(z) = C_0 \exp \left( \frac{w_s(z - a)}{e_c} \right) \]  

(6)
where \( C_0 \) is reference concentration, \( w_s \) is settling velocity, \( z \) is arbitrary elevation, \( a \) is reference elevation and \( e_c \) is sediment diffusivity. Briefly discussion of those parameters can be found in Rahman et al. (2011).
Table 1. Important parameter of bed load transport

<table>
<thead>
<tr>
<th>Author</th>
<th>$\epsilon_f$</th>
<th>$\tan \phi$</th>
<th>$\epsilon_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bailard</td>
<td>0.12-0.017</td>
<td>0.63</td>
<td>0.21</td>
</tr>
<tr>
<td>van der Molen</td>
<td>-</td>
<td>0.63</td>
<td>0.1</td>
</tr>
<tr>
<td>Gallagher</td>
<td>0.003</td>
<td>0.63</td>
<td>0.135</td>
</tr>
<tr>
<td>Drake</td>
<td>0.003</td>
<td>0.47</td>
<td>1.03</td>
</tr>
</tbody>
</table>

Morphological Change

The schematic idea to model morphological change is shown in Figure 1. Based on the initial topography, wave model generates the wave-current information, the second is to use diffusion module with the latest wave-current information to calculate the concentration distribution and further evaluate the bed level evolution by means of the bed morphology equation and the seabed profile is updated with the new bed level at the end of one time step marching.

Morphological changes are governed by the equation for conservation of sediment mass, which can be written as (Long, 2008):

$$\frac{\partial}{\partial t}(1 - n_b) \frac{\partial z}{\partial t} + \nabla \cdot \mathbf{q} = 0$$

(14)

where, $\mathbf{q}$ is the volumetric sediment transport rate. Here, we consider both suspended load transport rate ($\mathbf{q}_s$) and bed load transport rate ($\mathbf{q}_b$) as:

$$\mathbf{q} = \mathbf{q}_s + \mathbf{q}_b$$

(15)

Morphological changes can also be calculated from the continuity equation for the total sediment transport by considering erosion and deposition flux of suspended load sediment as follow (Wu, 2000):

$$\frac{\partial}{\partial t} \left( 1 - n_b \right) \frac{\partial z}{\partial t} + D_b - E_b + \nabla \cdot \mathbf{q}_b = 0$$

(16)

where $n_b$ is the bed porosity, $D_b - E_b$ is the sediment flux at the lower boundary of suspended-load layer which can be calculated as:

$$D_b - E_b = w_b (C_b - C_s)$$

(17)

where $C_s$ is sediment concentration at the bottom. Since the vertical distribution of sediment concentration in the previous section is considered from reference level to wave surface, we derived equation (6) calculate $C_b$ and found a formula as follow:

$$C_b = C_s \frac{w_b}{g} \left( \frac{z_s - z_b}{1 - \exp \left( - \frac{C_s w_b}{g} (z_s - z_b) \right)} \right)$$

(18)

From this equation, we found that $C_b$ is strongly depends on depth.

Figure 1. Sediment transport mechanism (after Wu, et al. 2000).
averaged sediment concentration \( (\bar{C}) \). Depth averaged sediment concentration can be evaluated by solving sediment continuity equation as follow:

\[
\frac{\partial \bar{C}}{\partial t} + \frac{1}{h} \frac{\partial q_s}{\partial x} = \frac{\partial}{\partial x} \left( \varepsilon_b \frac{\partial \bar{C}}{\partial x} \right) + \frac{w_e}{h} (C_s - C_a)
\]

where \( q_s \) and \( q_a \) are the sediment and the water flux, respectively.

**VALIDATION**

We shall now test our developed model derived in the previous section. Laboratory experiment data collected by Ikeno and Shimizu (1997) is used. The experiment was conducted in a wave tank that was 205 m long, 3.4 m wide and 6.0 m deep. The initial beach slope was approximately 1/20. The measured initial profile is used in the following computation. The median sand diameter was 1.0 mm. The specific gravity of the sand was \( \rho_s = 2.65 \text{ ton/m}^3 \). The porosity of the sand bed differed slightly along the beach profile, but \( n_p = 0.4 \) is assumed in the following comparison. The wave period, wave height and water depth at the constant depth were 5.0 s, 1.0 m, and 4.0 m, respectively.

In the computation domain, the origin is set on the slope where the still water depth over toe of bottom slopes as shown in Figure 2. The internal wave-maker is located two wavelength in front of the bottom slope. The left boundary is made to be a radiation boundary that is behind an artificial sponge layer with a length of \( x_s = 1.5L \). Grid size (\( \Delta x = 0.2 \text{ m} \)) and time step (\( \Delta t = 0.1 \text{ s} \)) are set to meet stability computation condition with the Courant number not exceed 0.4. The simulations are set to one hour.

Several evaluation of bed level change will be discussed in this section. Four sets of bed load transport parameters shown in Table 1 have been evaluated to calculate bed level change applying eq. (12) (bed load transport of mode B). Figure 3 shows bed level change after 1 hour simulation. Since bed level change measured by Ikeno and Shimizu (1997) was available for 14 hours wave simulation, we obtained one hour measured data by adopting Kobayahshi and Johnson (2001) method by dividing the available data by 14. This method neglects bed evolution effect on wave motion and sediment transport. Figure 3 shows that although \( \varepsilon_{\phi} \) exceeds one (Drake, 2001), it can produce bed level change similar to that by using the parameters proposed by Bailard (1981). Parameters proposed by van der Molen (2003) calculated very high bed level change, while Gallagher’s parameter produced relatively small bed level change.

Comparison of bed level change due to mode A and mode B is shown in Figure 4. It can be concluded that mode B can produce more erosion in surf zone area than mode A for bedload transport. Mode A was produce unrealistic bed level change due to suspended load as shown in Figure 4.b. This fact concludes that mode B is better than mode A simulating sediment transport.

We compare two methods of bed level change, equation (14) and equation (16). Bed level change due to suspended load after 1 hour simulation is presented in Figure 5. This figure shows that both methods produce similar bed level change. However the last
method gives more disturbance in bed level change. Even though first method has to solve continuity of sediment, time consuming is not significant during the simulation.

Figure 6 shows the proportion of 14 hours bed level change due to suspended load transport rate, bed load transport rate using parameters proposed by Bailard (1981) and total sediment transport rate. Suspended load component is the dominant factor around the breaking point, erodes an accumulation of sand bed to the shoreward direction. While in surf zone to swash zone, bedload transport component erodes sand material seaward direction. In surf zone, bedload transport becomes the dominant transport. However, it is not adequate to transport sediment landward in swash zone. The discrepancy in this area because of the infiltration-exfiltration during run up and rundown processes haven’t been considered in this model. The infiltration can make the settling velocity higher producing accretion in swash zone as observed in laboratory. Turner and Masselink (1998) found that swash infiltration-exfiltration across a beach face enhances the net upslope transport of sediment.

CONCLUDING REMARKS

Coupling between quasi-2D suspended load transport and Bagnold-type bed load transport has been discussed in this paper. Results indicated that both suspended load and bed load transport has its own dominancy in coastal area. Bedload transport is dominant in surf zone, erodes sediment to seaward direction. Return flow play an important role for this mechanism. Turbulence due to wave breaking makes suspended load transport dominant near the breaking point, erodes sediment to onshore direction. Applicability of friction factor, bedload and suspended load efficiency has been shown. The selected bed load efficiency coefficient produces significant erosion before the crest of sand bar. However, unexpected erosion in swash zone is occurred. The discrepancy is occurred because of the infiltration-exfiltration in laboratory during the run up process has not been considered in the present model.

ACKNOWLEDGEMENT

This study is financially supported by the Grant-in-aid for scientific research (B) (22360193) and by the GRANDE Project in the framework of JST/JICA, SATPERS.

LITERATURE CITED


Figure 5. Bed level change due to suspended load after 1 hour. Blue line: pick up rate; red line: suspended load rate only.

Figure 6. Bed level change after 14 hours. Bed load (red line), suspended load (green line), total sediment transport (blue line) and measured (circle) after 14.0 hours.


