Citations:

N. Bohr:
Notwithstanding the difficulties which are involved in the formulation of quantum theory it seems that its essence may be expressed in the so called quantum postulate, which attributes to any atomic process an essential discontinuity, or rather individuality, completely foreign to classical physics and symbolized by the Planck’s quantum action.

P. Dirac:
An act of observation is thus necessarily accompanied by some disturbance of the object observed... If a system is small, we cannot observe it without producing a serious disturbance and, hence we cannot expect to find any causal connection between the results of our observation.

R. Feynman:
But far more fundamental was the discovery that in nature the laws of combining probabilities were not those of the classical probability of Laplace.

A. Peres:
What we call quantum paradoxes are quantum phenomena that lead to paradoxes if we try to interpret them in a classical way.

S. Gudder:
Where does the Hilbert space \( H \) come from? Why does the probability have the postulated form? Why does a physical theory which must give real-valued results involve a complex amplitude state? Why must a quantum particle exhibit wave behaviour (wave-particle dualism)? Must quantum mechanics be nonrealistic (a quantum system only has properties when they are observed)? Is there a realistic solution of the EPR problem?

A. Plotnitsky:
Bohr’s interpretation of quantum mechanics as complementarity may be seen, or interpreted, in informational terms. The main reason for this view is that this interpretation is grounded in the epistemologically radical assumption that no physical properties of quantum objects or of processes involving them are describable by means of quantum theory or, it appears, any theory, and still more radically that no such properties are ascribable to them. The formalism of quantum theory and its physical interpretation are seen as referring to certain, statistically predictable, effects of the (quantum) interaction between quantum objects and the
measuring instruments (partially described in terms of classical physics) upon the latter.
1 Preface

This volume\(^1\) constitutes the proceedings of the Conference "Quantum Theory: Reconsideration of Foundations" held in Växjö (Smaland, Sweden), 17-21 June, 2001.

The organizing committee of the conference: C. Fuchs (Bell's Laboratory, USA), A. Khrennikov (Växjö University, Sweden), P. Lahti (Turku University, Finland).

The purpose of the conference (the fourth in the series of Växjö conferences) was to bring together scientists (physicists, mathematicians and philosophers) who are interested in foundations of quantum physics. An emphasis was made on both theory and experiment, the underlying objective being to offer to the physical, mathematical and philosophic communities a truly interdisciplinary conference as a privileged place for a scientific interaction. Due to the actual increased role of foundations in the development of quantum information theory as well as the necessity to reconsider foundations at the beginning of the new millennium, the organizers of the conference decided that it was just the right time for taking the scientific risk of trying this.

Quantum theory was created at the beginning of XXth century. For many years it is considered as the a well established scientific discipline with its own philosophy, logic, methodology, probability, geometry and having numerous experimental confirmations. Moreover, quantum theory has a lot of applications: physics of elementary particles, nuclear weapon and energy and the last years also - quantum information, computing, cryptography and teleportation. It must be underlined that the latter applications of quantum theory (quantum information and so on) are essentially stronger related to foundations than, for example, nuclear physics. My contacts with Soviet nuclear physicists demonstrated that they were merely interested in computational apparatus of quantum theory. An interpretation of these computations was not yet a subject of deep investigations. But quantum information, computing, cryptography are deeply connected with many fundamental problems in foundations of the quantum theory. It seems that it is no longer the case that we have restrict ourself just to the use of computational methods of quantum mechanics.

\(^1\)To order this volume please contact Kerstin.Broden@adm.vxu.se
Some fundamental problems are still unsolved (despite enormous efforts from physicists, mathematicians, philosophers). I think that there are following main reasons for such a situation in foundations of quantum theory:

(A) Great complexity of problems under considerations.
(B) Some wrong pathways that were chosen in the 1920s and ‘cemented’ by the authority of the fathers - creators of the quantum theory.
(C) Insufficient knowledge in mathematics by physicists and insufficient knowledge in physics by mathematicians working in quantum physics.

One of the main problems with wrong pathways is the huge overestimation of the authority of the fathers - creators of the quantum theory. At the moment it is really impossible (or at least very hard) to discuss consciously fundamental problems such as e. g. presented in Gudder’s list of questions, see citations. The standard arguments are citations of Bohr, Heisenberg and Pauli or their adherents as well as pointing out that there are great experimental confirmations of quantum theory and, finally, (especially last years) recalling Bell’s story.

First we pay attention to the experimental confirmation argument. Yes, quantum computations give right predictions that are confirmed by experiments. But we have to split quantum computation machinery and interpretations of results of such computations as well as functioning of quantum computation apparatus.

One of the main purposes of the present conference was to discuss the views of the fathers - creators to foundations of quantum mechanics. We were not oriented to criticize ideas of the fathers - creators. The main attitude was to understand better their ideas via deep studying of their works. In fact, some wrong pathways as well as prejudices were not consequences of e.g. Bohr’s original views, but a rather vague understanding of these views. I think that philosophers H. Folse and A. Plotnitsky in their talks made great contributions to the correct understanding of Bohr’s views. Many things that are rigidly associated with Bohr’s name were, in fact, never directly presented by Bohr. Even so called orthodox Copenhagen interpretation of quantum mechanics was not originally formulated by N. Bohr: ‘The wave function provides the complete description of an individual quantum system’.

We now pay attention to the (C)-source of the unsatisfactory situation in foundations, namely the insufficient level of exchange of ideas between physicists, mathematicians and philosophers. It is in our power to improve (C) - to
organize a series of meetings oriented to the collaboration of physicists (theoreticians as well as experimenters), mathematicians and philosophers. The present conference, "Quantum Theory: Reconsideration of Foundations", was the fourth meeting with such an aim that has been taken place in Växjö the last three years: "Quantum days in Växjö", November 1999: p-adic numbers and space-time, Bell’s inequality and foundations of probability theory, quantum information; "Bohmian Mechanics - 2000", May 2000: computer simulation of Bohmian trajectories, noncommutative geometry and Bohmian mechanics, typically and probability, Bohmian model for mental processes; "Foundations of Probability and Physics", November-2000: the role of probability in EPR-Bell considerations, probability and information, p-adic probability and p-adic reality, Kolmogorov complexity, von Mises’ probability theory and quantum mechanics.

The role of probability in foundations of quantum mechanics was one of the most important problems discussed during the last Växjö conference. I would like to pay attention to an extended discussion on the possibility to use Bayesian (subjective) probability theory in quantum information induced by talks of C. Fuchs and R. Schack, see the fundamental paper of C. Fuchs in this volume.

This discussion played an important role in the clarifying of the probabilistic structure of quantum information theory. It also attracted the attention to the role of a mathematical model of probability theory in quantum formalism. In particular, I presented some reasons in the favour of the frequency (von Mises) probabilistic model. The ‘Orthodox Bayesian approach’ of Fuchs–Schack was criticized from the frequency point of view. There are no doubts that we can use Bayesian method of statistical hypotheses and, moreover, it is very convenient in quantum information theory. However, it is very doubtful that quantum probabilities can be introduced as a measure of our personal belief. Well, it may be belief, but belief based on frequency information. On the other hand, C. Fuchs and I. Pitowsky presented strong critical arguments against von Mises’ frequency probability theory (including the impossibility to verify statements depending on \( N \to \infty \) number of trials).

In many talks and during the round table it was discussed the role of probability in the EPR-Bell framework: L. Accardi - theory of chameleons and analogy between Bell’s inequality and inequalities for the sum of angles in a triangle for various geometric models, W. De Baere - fluctuations of hid-
den variables, A. Khrennikov - non-Kolmogorov models of probability theory, e.g. frequency (von Mises) or \( p \)-adic and Bell’s inequality, I. Pitowsky - Bell’s inequality as an example of an inequality for random variables derived by J. Boole in XXth century, generalizations of Bell’s inequality. Investigations on the probabilistic structure of Bell’s assumptions performed e.g. by L. Accardi, W. De Baere, S. Gudder, A. Khrennikov, W. De Muynck, I Pitowsky are the good illustrations to the (C)-problem in the study of foundations of quantum mechanics. In fact, by taking into account very important mathematical assumption - the use of Kolmogorov’s probability model by J. Bell, we can easily see that Bell’s inequality does not contradict to local realism. The above authors presented various models in that Bell’s inequality is violated. However, all these investigations are considered by the majority of the quantum community as pure mathematical, non-relevant to real physics.

The crucial role of a mathematical probabilistic model in the foundations of quantum mechanics, namely quantum interference, was discussed in the talk of A. Khrennikov. It was demonstrated that ‘wavelike’ interference of probabilities of alternatives can be derived without using wave arguments. We must only leave the domain of applications of the conventional probabilistic model, Kolmogorov - 1933, and use the frequency or contextual models of probability theory, see the paper in this volume.

One of the most important problems, intensively discussed during the conference, was the problem of an interpretation of a wave function. It was quite surprising that the orthodox Copenhagen interpretation was denied by the majority of participants. Personally I (as well as W. De Baere) supported a contextual statistical realist interpretation, ‘Växjö interpretation’, by that quantum randomness is context (=complex of experimental conditions) depending randomness. Such an interpretation (in the opposite to Bohr’s informational/contextualism, see citation of A. Plotnitsky and further considerations) does not imply the impossibility to create finer description of physical reality (if you like hidden variables) than given by the quantum theory, see my text in this volume. The individual realistic interpretation used in operational quantum physics was strongly supported by P. Lahti during the round table of the Conference.

During the round table, the large group of participants, J. Bub, C. Fuchs, D. Mermin, A. Plotnitsky,…, supported various forms of information-oriented interpretation of quantum mechanics. To some degree, such interpretations may be seen as following Bohr’s interpretation (or a certain type of interpre-
tation of Bohr interpretation). According to this type of view, in a rough outline, quantum mechanics does not refer to objective properties of physical objects themselves under investigation, but deals with the relationships between and predictions concerning informational quantities.

The orthodox Copenhagen interpretation was strongly supported by J. Summhammer. In some sense J. Summhammer presented the general point of view of experimenters working in neutron interferometry. Here neutron is imagined as a wavelike object that is split into two or more pathways with further interference of corresponding parts. Regarding to interpretations of a wave functions discussed during the conference, we should also notice the presentation of the Bohmian interpretation by B. Hiley. In Hiley’s presentation Bohmian mechanics was merely an attempt to testify how long we can proceed in quantum theory by using classical mechanical formalism. Finally, we remark that many worlds interpretation did not induce strong enthusiasm among the participants (despite very enthusiastic propaganda of this interpretation by L. Vaidman).

The round table discussion on interpretations was closely related to the discussion on foundations of quantum computing. Especially interesting problem was related to interpretations of quantum parallelism. It is really the hard problem. In majority of works on quantum computing there are really claimed that all values of a function under the computation are really calculated parallel. Regarding to quantum parallelism, it is important the remark of R. Jozsa. He pointed out that parallel computation of all values is merely a convenient mathematical picture for this quantum process. He also presented very important ideas on the role of entanglement of quantum computation and considered some possible interpretations.

Quantum information considerations demonstrated that there is a danger that manipulating with pure qubits we can forget real physics. In this way there might be produced results that are valid for pure qubits, but might not be applied to real quantum systems. I. Volovich underlined that such a problem we have already in Bohm-Bell framework for the EPR experiment. He presented strong arguments that by taking into account quantum mechanical processes in space-time we have to modify standard Bell’s inequality.

I would like to recall the words of Russian academician Krylov:

"Mathematics is a kind of mill. It mills all that we put in it."

It seems that quantum formalism is nothing than a quantum mathematical mill. The only difference from other mathematical mills that are used
in other domains of physics, e.g. Newtonian mill, is that we do not know well what we put into the quantum mill. We see the result of working of the quantum mill - very good mill that can be used for many purposes. But we cannot ‘see’ so called elementary particles, ‘quantum grain’, without to change their features (and, as a consequence, features of quantum mill at the output). Such a situation induces various prejudices. One of the strongest ones is the identification of some features of the quantum mathematical mill with physical features of quantum systems.

I have the feeling that, in fact, quantum theory was based on two distinct discoveries. One of them was in the domain of physics - discreteness of energy as well as some other observables. The second one was the purely mathematical discovery, namely discovery of the calculus of context depending probabilities (inducing interference rule for addition of probabilities) and the possibility to represent contextual probabilistic calculus as Hilbert space probabilistic calculus (see e.g. my paper in this volume).

In fact, contextual probabilistic calculus was only occasionally discovered in the connection with investigations of elementary particles. In principle, it might be discovered in the process of purely probabilistic investigations, e.g. in XVIII th or XIX th century. One of the problems was very preliminary stage of the development of the foundations of probability theory at the beginning of XXth century. In fact, the measure-theoretical approach to probability theory was developed at the same time when M. Planck and A. Einstein created foundations of quantum theory. If you read Einstein’s papers, you see that he should work with probabilities by using intuitive arguments - real mathematical theory of probability was created by Kolmogorov 25 years later! But even Kolmogorov’s theory of probability did not provide mathematical tools sufficient for describing of quantum statistical data. Kolmogorov’s probability model was the fixed context model. And in quantum physics we need to use a context-variable model for probability theory.

The absence of an adequate probabilistic theory induced the prejudice that quantum statistical data demonstrated extremely unusual (‘nonclassical’) features. Thus some features of the contextual probabilistic mill were interpreted as features of elementary particles. Moreover, quantum physicists, M. Born, W. Heisenberg, P. Dirac, did (rather unconsciously) the great mathematical discovery. They found that transformations of context depending
probabilities, interference of probabilities,

\[ P = P_1 + P_2 + 2\sqrt{P_1 P_2} \cos \theta \]

can be represented as linear transformations in a Hilbert space. So, instead of manipulating with nonlinear transformations of probabilities, we can work by using linear algebra. This Hilbert space probabilistic mill was not separated from, ‘quantum grain’, elementary particles. Some special features of the Hilbert space mill were considered as features of elementary particles.

In fact, an attempt to separate the quantum mill from quantum grain also was done by L. Hardy who derived quantum theory from five very reasonable axioms, see his paper in this volume. We underline that such a derivation might be in principle performed in XIXth century, far before quantum experimental discoveries.

The conference and the present volume give the good example of the fruitful collaboration between physicists, mathematicians and philosophers. We would like to thank the Swedish Science Foundation and Växjö University (through Rector’s "Strategic Investigations Foundation") for financial support. We would also like to thank Prof. Magnus Söderström, the Rector of Växjö University, for the support of the fundamental investigations, in particular, for his enormous efforts to create the "Mathematical Modeling in Physics, Engineering and Cognitive Sciences” specialization of Växjö University.

Andrei Khrennikov

Director of International Center for Mathematical Modeling in Physics, Engineering and Cognitive Sciences
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