Physics of Gamma-Ray Bursts: Turbulence, Energy Transfer and Reconnection

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1 Introduction

Understanding of the nature of gamma-ray bursts (GRBs) is one of the challenging problems facing the astrophysics community. The field of gamma-ray bursts is a rapidly developing one and we expect that new missions, like SWIFT and GLAST will bring the field to a new quantitative level.

As a result of the intensive observational and theoretical efforts a basic picture of the gamma-ray bursts has emerged. The long standing question about the distances of these sources has been resolved and their cosmological origin is by now well established. As has been known for some times now, this combined with their short time scales, point to the relativistic fireball (or jet) as the most likely model (see e.g. [1, 2]). Such fireballs could arise from rapid and episodic accretion onto a stellar size black hole produced by either a hypernova-collapsar ([3, 4]) or mergers of compact objects. The investigations of the spectra of both the prompt gamma-rays and the afterglows seem to point to the synchrotron process as the most likely source of the radiation ([5, 6, 7]) but other models involving Compton scattering have also been proposed. These models require presence of relativistic electrons and either strong magnetic fields or a source of soft photons. The particle acceleration takes place either in internal shocks arising from the episodic nature of the accretion, for the gamma-ray emission, or in an external shock arising from interaction of the fireball with the surrounding medium, for the afterglows ([7, 8]). Some of these aspects of GRBs have received some scrutiny but most of them are still at the stage of a back-of-an-envelop calculations. The detail of the explosion, the formation of the fireball, its propagation, the generation of the shocks, the source of the magnetic fields and soft photons, the particle acceleration process, and the details of the radiation process are all still outstanding questions.

The goal of this review is to attract the attention of the community to a number of physical processes that seem to be very relevant for these sources. The next step toward the building of a credible model is the determination of the mechanism for the acceleration of nonthermal relativistic particles. Most current
studies of GRBs do not deal with the mechanism of the acceleration; the common practice is to assume presence of an isotropic distribution of electrons with a either a simple power law spectrum, \( f(\gamma) \propto \gamma^{-\delta} \), with the Lorentz factors in the range \( \gamma_{\text{min}} < \gamma < \gamma_{\text{max}} \), a broken power law, or one with an exponential cutoff. More importantly, the crucial question of how the energy of the fireball, which is carried mainly by protons, is transferred to the radiating electrons has not received adequate attention. In this review we shall discuss the possibility that \textit{stochastic acceleration by plasma turbulence} is the agent for this energy transfer and for the acceleration of the electrons. In the next section (\$2) we shall present a simple model of the gamma ray burst and in \$3 we discuss the stochastic mechanism for particle acceleration. Recent \textit{advances in understanding of the nature of turbulence} provide solid foundations for addressing those questions. These results are discussed in \$4 and \$5. Finally, in view of the emerging interest in the role \textit{magnetic reconnection}in GRBs, we shall briefly discuss how turbulence may make reconnection both fast and bursty (\$6). The mechanism of stochastic reconnection that we discuss releases most of its energy in the form of turbulent motions. Therefore our scenario of stochastic acceleration of electrons should be relevant to both shock-induced fireball and to reconnection mechanism. The summary is provided in \$7.

2 A Possible Scenario

The large distances and rapid variability of GRBs require release of a large amount of energy \( \mathcal{E} > 10^{53}(\Omega_b/4\pi) \) ergs, where \((\Omega_b/4\pi)\) is the beaming factor, in a small region and within a short time. This in turn indicates presence of a high energy density and a relativistic outflow with a bulk Lorentz factor \( \Gamma \sim 10^3 \) involving a baryon mass of \( M = \mathcal{E}/c^2\). The light curves of GRBs show a varied structure sometimes involving many pulses, indicating the episodic nature of the process. After the initial fireballs associated with these episodes cool and become transparent, the result is a series of relatively cold “shells” of masses \( M_i \) moving with Lorentz factors \( \Gamma_i \). It is believed that the interaction of these shells with each other (faster ones running into slower ones) and with the surrounding medium (interstellar or preburst stellar wind) converts a large fraction of the kinetic energy \((\Gamma - 1)Mc^2\) into prompt gamma-rays and afterglow photons. These interactions will clearly produce shocks. The shocks produced by shell-shell interactions are referred to as internal and those produced by shell-medium interactions are referred to as external shocks. It is generally believed that the burst proper is produced by the internal shocks and that the afterglow is produced by an external shock ([9]), although a model of gamma-ray production via external shocks also exists ([10]).

We could envision the following model for production of the gamma-rays and

\[ \text{\footnotesize 1This surge of interest was quite clear during the conference.} \]
afterglows. The interaction of two shells with Lorentz factors $\Gamma_1$ and $\Gamma_2$ (or a shell and the external medium; $\Gamma_2 = 1$) gives rise to a relativistic shock front. (For a description of structure of relativistic shocks see [13]). In the frame of the shock the particles from the slow shell (or the external medium) will enter the shocked region, essentially as a monoenergetic beam, with a relative Lorentz factor $\Gamma_{rel} \simeq (\Gamma_1/\Gamma_2 + \Gamma_2/\Gamma_1)/2$. This is subject to the well known two stream instability and can give rise to plasma turbulence which dissipates the bulk of the kinetic energy of the system. Most of the energy is transferred to MHD turbulent motions (Alfvén waves or fast modes) by the instability induced by the proton beam ([12]). Fast modes and Alfvén waves in magnetically dominated plasma form distinct weakly coupled cascades which transfer energy to small scales and possibly to whistler waves (see [14, 16]). Transit time damping and direct scattering by the whistler waves can accelerate the electrons. In this scenario the electrons behind the shock are continually being accelerated as they also radiate, or lose energy by other ways, until the shells are completely merged (or stopped by the external medium) and the shocks are dissipated.

3 Acceleration Mechanism

The three acceleration processes most usually proposed in astrophysical sources are acceleration by static electric fields parallel to the magnetic field, first order Fermi acceleration by shocks, and stochastic or second order Fermi acceleration by turbulence (see reviews [17, 18, 19]). Large scale Electric Fields needed to accelerate charged particles to significant energies can be maintained only if the electrical resistivity is anomalously large ([20]) or if the plasma density $n$ is high and the Dreicer field is large ([21]). This does not seem to be a natural mechanism for GRBs.

**Shocks** are the most commonly considered mechanism of acceleration because they can quickly accelerate particles to very high energies. In a nonrelativistic shock, acceleration to high energies takes place by repeated passages of the particles across the shock. This requires the existence of some scattering agents. The most likely agent for scattering charged particles is plasma turbulence. Presence of turbulence in the downstream region is natural but in the upstream region is problematic; self generation of turbulence by the accelerated particles is often assumed. In any case, the rate of energy gain is governed by this scattering rate which (for charged particles tied to magnetic fields) is proportional to the pitch angle, diffusion coefficient $D_{\mu\mu}$; $\mu$ is the cosine of the pitch angle. For a relativistic shock of Lorentz factor $\Gamma \gg 1$ most of the energy gain (equal to $\Gamma mc^2$) occurs at the first passage. Subsequent crossings, if any, increase this energy by a factor of at most a few. It has been shown (e.g. [22]) that this results in a power law spectrum with a well defined index, $\delta = -2.23$, which would give a synchrotron photon index of -1.62 (or -2.12 for cooling spectra). Early observations of afterglows seem to suggest such values of the index. However, this is
not the case for all afterglows and it certainly is not true for the prompt gamma-rays, where the spectra involve at least two indexes both of which show a wide dispersion extending from \( > 0 \) to \( < -3 \) (see [23]). In addition, shock acceleration cannot be the sole mechanism for the GRBs for the following reason.

The turbulence needed for the scattering can also accelerate particles **stochastically** at a rate \( D_{pp}/p^2 \), where \( D_{pp} \) is the momentum diffusion coefficient. In most studies of astrophysical sources, and at high energies, the pitch angle scattering rate is higher than the momentum diffusion rate \( (D_{\mu\mu} \gg D_{pp}/p^2) \) so that shock acceleration is more efficient than direct acceleration by turbulence. However, **stochastic acceleration** becomes more efficient when the above inequality is reversed, \( D_{pp}/p^2 \gg D_{\mu\mu} \) ([24]), which occurs at low energies, but more importantly for our purpose here, is true for high \( B \) and low \( n \) plasmas, where the formally defined Alfvén velocity (in units of \( c \) ) \( \beta_A = B/[(4\pi nm\rho c^2)^{1/2}] \) exceeds unity. In this case the electric field fluctuations \( \delta E \sim \beta_A \delta B \) are larger than the magnetic field fluctuations \( \delta B \), which means a faster acceleration \( (\propto \delta E^2) \) than scattering rate \( (\propto \delta B^2) \). This trend can be seen in results published in [25, 27]. Thus, under these conditions, once the particle crosses the shock front into the turbulent region behind the shock, it will undergo stochastic acceleration much faster than it can be turned around to cross the shock again.

These conditions are what is present in GRBs. The observed values of photons energies require magnetic fields \( B \geq 10^4 \)G. Various arguments (see review [28]) suggest that \( n \leq 10^8 \) cm\(^{-3}\) and therefore \( \beta_A \gg 1 \).

### 4 Magnetic Turbulence and Particle Acceleration

The rate of stochastic acceleration of electrons depends on the properties of magnetic turbulence. Almost everyone would agree that incorporation of this aspect is necessary for a realistic model, but the complexity of the problem has made many researchers wary of MHD turbulence and has resulted in a tendency to avoid dealing with the phenomenon. However, recently a substantial progress has been achieved in the field enabling us to deal with this problem adequately. First of all, simple scaling model of incompressible MHD turbulence developed by Goldreich & Shridhar ([14]) has been successfully tested and extended recently (see review [29]). Moreover, important advances in understanding of MHD turbulence in compressible media ([30, 31, 32]) make an adequate quantitative investigation of turbulence possible for the first time.

Turbulence is ubiquitous in astrophysics. All turbulent systems have one thing in common. They have a large “Reynolds number”, which is the ratio of the viscous drag time on the largest scales \( L^2/\nu \) to the eddy turnover time of a parcel of gas \( L/V \); \( Re \equiv LV/\nu \), where \( L \) is the characteristic scale or driving scale of the system, \( V \) is the velocity difference over this scale, and \( \nu \) = viscosity). A similar parameter,
the “magnetic Reynolds number”, $R_m \equiv LV/\eta$ (with $\eta$ as the magnetic diffusion) is the ratio of the magnetic field decay time $L^2/\eta$ to the eddy turnover time $L/V$. The properties of the flows on all scales depend on $Re$ and $R_m$. Flows with $Re < 100$ are laminar; chaotic structures develop gradually as $Re$ increases. For the gamma-ray bursts both Reynolds numbers are $>>> 1$ and fluid motions are expected to be extremely chaotic.

*Hydrodynamic turbulence* of an incompressible fluid (or Kolmogorov turbulence) is the simplest example of turbulence. For instance, an obstacle of size $L$ in a flow excites motions on scales of the order $L$. The turbulent energy injected at this scale cascades to progressively smaller and smaller scales at the eddy turnover rate, with negligible energy losses along the cascade. Ultimately, the energy reaches the molecular dissipation scale $l_d$, i.e. the scale, where the local $Re \sim 1$, and is dissipated there. The scales between $L$ and $l_d$ are called the *inertial range* which typically covers many decades. The motions over the inertial range are self similar and this provides tremendous simplification for theoretical description.

*MHD turbulence* is more complex because the frozen-in magnetic fields alter substantially the dynamics of fluid. GRBs happen in magnetically dominated plasma with $\beta_p = 4\pi P_{\text{gas}}/B^2 \ll 10^{-4}$, where $P_{\text{gas}}$ is the gas pressure. Turbulence for a low beta regime has been studied in [31] (see Fig. 1).

The large scale Alfvénic turbulence can interact with protons and only with high energy electrons. For a more efficient acceleration or heating of the electrons one must rely on the cascade of the generated turbulence down to small scales. The details of this cascade for low beta plasma ($\beta_p \ll 1$) are discussed in [31], where it is demonstrated that the incompressible Alfvén waves and compressible (slow and fast) waves have distinct scaling relations. In [14] it was stated that incompressible Alfvénic modes should transfer energy to small scales over a hydrodynamic eddy turnover time $l_\perp/v_l$, where $l_\perp$ eddy size measured perpendicular to magnetic field direction. It was proved in [29] that the motions perpendicular to magnetic field are identical to hydrodynamic motions. In other words, while magnetic fields resist bending, they allow mixing hydrodynamic-type motions. Therefore, the power spectrum of Alfvénic motions perpendicular to magnetic field lines is Kolmogorov-type (i.e. $v_l \sim l_\perp^{1/3}$). Those mixing motions induce waves propagating along magnetic field lines. The corresponding motions involve bending of magnetic field over scales $l_\parallel$. The relation between the bending and mixing motions for an eddy is given by the condition that the period of the wave propagating along magnetic field lines is equal to the period of the mixing motions on a scale that excites this wave $l_\parallel/V_A \sim l_\perp/v_l$ ([14]). As the result of this coupling the eddies get more anisotropic (i.e. $l_\perp \sim l_\parallel^{2/3}$) at small scales, i.e. $l_\perp \ll l_\parallel$.

Rapid transfer of energy within the Alfvénic cascade makes the much slower non-linear interaction of Alfvén and fast modes unimportant. Therefore the Alfvén modes follow the Goldreich-Shridhar scalings even in compressible media. Slow waves in
Figure 1: Statistics of MHD turbulence in low beta plasma (mostly from [31, 33]). Results of driven compressible turbulence with Mach number \( V/V_s \), where \( V_s \) is the sound velocity, equal to 2.2 and the Alfvénic Mach number \( V/V_A \) equal to 0.7. The procedure of separation of various modes is explained in [31]. (a) The Alfvén modes show Kolmogorov-type scaling; (c) Kolmogorov-type scaling is also true for slow modes; (e) Scaling of fast modes is argued in [31] to follow the spectrum of acoustic turbulence, i.e. \( E(k) \sim k^{-3/2} \). While the differences in the slope of the fast, slow and Alfvén modes are not so different, they exhibit very different anisotropies. The isocontours of equal correlation (measured with structure functions \( \langle (v(x_1) - v(x_2))^2 \rangle \)) are stretched along the direction of magnetic field for slow and Alfvén modes (see (b) and (d)), but isotropic for fast modes (see (f)). By now similar results have been confirmed for plasmas with different Mach numbers [32] and ratios of gas to magnetic pressure. (g) Decay of Alfvén turbulence. The decay rate of Alfvén turbulence is not strongly affected by the presence of slow and fast modes. In the solid line, slow and fast modes are not present at the beginning of the simulation. In the dotted line, we include slow and fast modes at the beginning. \( \beta < 1 \). From Cho & Lazarian ([32]). (h) The ratio of \( (\delta V)^2 \) to \( (\delta V_A)^2 \). Initially, only Alfvén modes are present. The ratio is measured at \( t \approx 3 \) for all simulations. Generation of fast modes (or \( \sim \)radial modes) is not very efficient. Pluses are for low-\( \beta \) cases and diamonds are for high-\( \beta \) cases. From Cho & Lazarian ([32]). (i) Whistler mode turbulence. Initially Fourier modes in the dotted box are excited. The contours show energy distribution after the turbulence evolves for about one eddy turnover time. From an upcoming paper.
magnetically dominated plasma move with sound velocity \( v_s \ll V_A \) and are passively mixed up by much faster Alfvén waves. Therefore, similar to the passive scalar, slow modes exhibit the Goldreich-Shridhar\[14\] scaling/anisotropies. Fast modes in magnetically dominated plasmas move with the velocity (equal to \( V_A \)) that does not depend on the local direction of the magnetic field. Therefore shearing motions that arise from Alfvénic modes modify the fast modes only marginally. As the result, the fast modes form a distinct cascade of their own \[31\]. Motions associated with this cascade are isotropic.

The transfer of energy to electrons happens through collisionless damping for compressible mode and through the transformation of the Alfvén modes into the whistler modes. The later process requires more investigation. The unclear issues are related to (a) the efficiency of the energy transfer from Alfvén modes to whistlers, (b) possible modification of the Alfvén cascade if such a transfer presents a bottleneck effect, (c) the tranfer of energy from whistlers to protons. While the issues (a) and (b) are still at the early stages of numerical investigation, preliminary results have been obtained for (c). It is clear that how efficiently whistler turbulence heats protons depends on the degree of the anisotropy associated with whistler modes. The original whistlers are likely to have the anisotropy similar to that of the Alfvén modes that give rise to them. Results of our calculations shown in Fig. 1(i) indicate that the whistler modes injected with a high degree of anisotropy preserve their anisotropy for sufficiently long time. This may mean that the energy transfer from whistlers to protons may not be efficient. Longer integration will provide a more definitive answer.

5 Interactions of Turbulence and Particles

The resonant interaction of energetic particles with MHD turbulence has been suggested as a mechanism for scattering of cosmic rays and for acceleration of the radiating electrons and protons in many astrophysical plasmas (e.g. in solar flares; see \[26\]). Specifically, the resonance condition is \( \omega - k_{\parallel}v_{\mu} = n\Omega \), \( (n = 0, \pm1, 2...) \), where \( \omega \) is the wave frequency, \( k_{\parallel} \) is the parallel component of wave vector along the magnetic field, \( v \) is the particle velocity, \( \mu \) is the pitch angle cosine to the magnetic field, \( \Omega \) is the Larmor frequency of the particle. Basically there are two main types of resonant interaction: transit and gyroresonance.

For the transit acceleration, the energy exchange corresponds to resonance at \( n = 0 \). It is the resonant interaction with parallel magnetic field perturbations, and therefore only concerning compressible waves. In the wave frame, the perturbations are stationary, particles will be affected by the magnetic mirror force \(- (mv_{\perp}^2/2B) \nabla_{\parallel} B \). For small amplitude waves, particles must be in phase with the wave in order to be reflected by the compression. This requires the particles to have component of velocity \( v_{\perp} \mu \) equal to the Alfvén velocity \( V_A \). Particles gain energy in
head-on collisions and lose energy in on-tail collisions. Since the frequency of head-on collisions is greater than that of trailing collisions, particles will gain a net amount of energy. ([35]).

*Gyroresonance* is a resonant interaction between a particle and the transverse electric field of a wave. It occurs when the Doppler shifted frequency of the wave in the particle’s guiding center rest frame \( \omega_{gc} = \omega - k_{\parallel}v_{\mu} \) is a multiple of the particle gyrofrequency, and the rotating direction of wave electric vector is the same with the direction for Larmor gyration of the particle. Thus from this resonance condition, we know that the most important interaction occurs at \( k_{\parallel} = k_{res} \sim \Omega / v_{\parallel} \sim r_L \), the Larmor radius of the high-energy particles.

The quasi-linear theory (QLT) can be used to describe the interaction between particles and MHD waves ([35]). As we know, GRBs require very strong background magnetic field \( B_0 \). Therefore it’s very likely that the perturbation on the resonant scale \( \delta B \ll B_0 \). The diffusion coefficients of the Fokker-Planck equation that describe the evolution of the distribution function of the particles are determined by the statistical properties of the MHD turbulence in the medium. Frequently, for practical calculations the MHD turbulence spectrum is assumed to be isotropic and Kolmogorov. However, our considerations above testify that this is an erroneous assumption. The correlation tensors of the perturbations for Alfvén modes and compressible modes that account for the actual properties of MHD turbulence have been obtained in [29, 31, 36].

The calculations in [36] that made use of the tensors provided the scattering efficiency of anisotropic Alfvénic turbulence (see Fig.1a). We see from Figure 2a that the scattering is substantially suppressed, compared to the Kolmogorov turbulence that is usually used for scattering calculations (see also [38]). This happens, first of all, because most turbulent energy in GS95 turbulence goes to \( k_{\perp} \) so that there is much less energy left in the resonance point \( k_{\parallel} = (\mu r_L)^{-1} \). Furthermore, \( k_{\perp} \gg k_{\parallel} \) means \( k_{\perp} \gg r_L^{-1} \) so that energetic particles sample many eddies during one gyration ([38]). This random walk decreases the scattering efficiency by a factor of \( (\Omega / k_{\perp}v_{\perp})^{1/2} = (r_L/l_{\perp})^{1/2} \), where \( l_{\perp} \) is the turbulence scale perpendicular to magnetic field.

Thus the gyroresonance with Alfvénic turbulence is not an effective scattering mechanism if turbulence is injected on the large scales, since the degree of anisotropy increases on smaller scales. However, if energy is injected isotropically at small scales, the resulting turbulence would be more isotropic and scattering will be more efficient. Scattering by fast modes can be more efficient since they are isotropic. Yan & Lazarian [36] (see also [37] for a review) performed calculations taking into account the collisionless damping of fast modes and showed that the scattering by fast modes is the dominant scattering mechanism (see Fig.1b).

In general, the Fokker-Planck equation is complicated. However, we can get an approximate solution for acceleration in certain conditions. For instance, if the particle distribution is isotropic, then the acceleration process can be described by the
Figure 2: Momentum diffusion $D_{pp}$ from scattering (a) by Alfvénic turbulence, (b) by fast modes. In (a), the dash-dot line refers to the energy diffusion in isotropic turbulence. The cross(x) line represents our result for anisotropic turbulence. In (b), the dashed line represents the gyroresonance with fast modes, the solid line are the results for transit time acceleration.

so-called diffusion-convection equation ([22]). This is applicable at high energies and specifically to cosmic rays. As pointed out above, for GRBs the acceleration is faster than scattering as we pointed out earlier, in which case the Fokker-Planck equation can be simplified as ([27])

$$\frac{\partial f^\mu}{\partial t} + v^\mu \frac{\partial f^\mu}{\partial z} = \frac{1}{p^2} \frac{\partial}{\partial p} p^2 D^\mu_{pp} \frac{\partial f^\mu}{\partial p} + S^\mu, \quad (1)$$

where $S^\mu$ is the source term. This expression is similar to the isotropic case except that now everything is a function of $\mu$.

Nevertheless, there are still several problems left if we want to apply this result to GRBs. The important difference comes from the unusually strong magnetic field for GRBs. As mentioned above, the Alfvénic velocity approaches the velocity of light. With such a high phase speed, the dynamics of the MHD turbulence should be taken into account. Instead of the sharp $\delta$ function, we should use the Breit-Wigner-type function ([35], [36]) to describe resonance interactions. Weeding out the spurious contributions that arise from particle interaction with large scale magnetic fluctuations is a necessary requirement for accounting for MHD turbulence dynamics within the QLT [36]. Additional work is required for describing damping in relativistic plasmas.

### 5.1 Towards Self-consistent Model

A usual assumption for the studies of propagation of energetic particles is that the turbulence properties are given by external sources. However, it is clear that if a substantial part of the plasma particles is going to be accelerated, this assumption
cannot be true. Instead, the back-reaction of the accelerated particles on the turbulence is essential. In practical terms that means that a system of two Fokker-Planck equations, one describing the evolution of the distribution function of particles and another describing the evolution of the turbulence energy should be solved. The coupling of the equations is provided by the fact that the coefficients of the Fokker-Planck equation describing particle acceleration depend on the turbulence energy at a particular wavenumber, while the rates of damping in the equation describing turbulence do depend on the particle distribution (see [41]).

The complication arises from the fact that the actual MHD turbulence is anisotropic and this calls for revision of the earlier studies making use of the recent theoretical and numerical insights. While the generalization of the particle diffusion equation is straightforward, the equation that describes the turbulent cascade requires more care. Generalizing expression in [42] we can describe the three-dimensional spectral density cascade process as \( \frac{\partial W(k)}{\partial t} = -\nabla_k \cdot F(k) \), where the flux \( F_i(k) = -D_{ij}\nabla_j W(k) \) and \( D_{ij} \) is the diffusion tensor. Here, \( \nabla_i \equiv \frac{\partial}{\partial k_i} \) and summation over repeated indices is assumed. With addition of sources and losses the wave transport equation becomes

\[
\frac{\partial W(k)}{\partial t} = \dot{Q}_p(k) - [\gamma_e(k) + \gamma_p(k)] W(k) + \nabla_i [D_{ij}\nabla_j W(k)] - \frac{W(k)}{T_{\text{esc}}}.
\]

Here \( \dot{Q}_p(k) \) is the rate of wave generation, \( \gamma_e \) and \( \gamma_p \) are the rates of wave damping by the electrons and protons, and as for particle transport \( T_{\text{esc}} \) describes wave leakage from the source region, if any. The damping coefficients depend not only on the background plasma but also on the distribution of the accelerated particles. This is how this equation is coupled to the particle transport equation (e.g. eq.[1]), where the diffusion coefficients depend on \( W(k) \).

As shown in [31], the fast mode cascade is isotropic (i.e. \( D_{ij} \) is a scalar function). Therefore, we have a one-dimensional spectral energy density distribution; \( \tilde{W}(k) = 4\pi k^2 W(|k|) \) and \( D/k^2 \sim c\beta_A k (8\pi \tilde{W}(k)/B^2) \) gives the rate of the cascade. The resultant isotropic case of equation (2) describes an acoustic type cascade of fast waves (see [41]).

For Alfvénic cascade the losses are negligible till the scale of the motions \( \sim k^{-1} \) reaches much lower values than \( L \), possibly till it approaches the proton Larmor radius. In this case the wave equation reduces to \( \nabla_i [D\nabla_i W(k)] = \dot{Q}_p(k) \), which is independent of the electron distribution. In other words, one can assume that almost all of the injected energy \( \dot{Q}_p(k) \) reaches the Larmor radius of a proton. However, this cascade will yield a non isotropic wave spectrum, favoring perpendicular propagation, even if it is isotropic at injection. This anisotropy increases with increasing \( k \) (or decreasing scale) approximately as \( (kl_0)^{1/3} \), where \( l_0 \) is the energy injection scale (see [34]). At scales near the proton Larmor radius, whistler waves will be generated. Quataert & Gruzinov ([43]) speculated that due to the anisotropy of the Alfvénic turbulence cascade they expect the whistler turbulence to be highly anisotropic. Our pre-
liminary analysis indicates that whistler turbulence will eventually become isotropic. (see also [15]). Therefore, it can be described by an equation similar to (2), where according to [42] the cascade rate is given by $D/k^2 \sim c\beta_A k(8\pi W(k)/B^2)^{1/2}$. This means that most of the energy from the Alfvénic cascade goes into the whistler mode, which can accelerate electrons of essentially all energies and pitch angles very efficiently ([25, 27]).

6 Gamma Ray Bursts and Magnetic Reconnection

An alternative model for energizing GRBs, that we heard about at the conference, is reconnection, i.e. the process of magnetic field annihilation. This will require a rapid rate of energy release. Without elaborating on the details, we briefly discuss the conditions under which magnetic reconnection can provide rapid bursts of energy release.

In the existing models of Gamma Ray bursts magnetic field play important role. If magnetic field is sufficiently high its energy may be sufficient to feed the Gamma Ray burst itself (see other papers in the volume that explore this possibility). For this purpose a mechanism of a fast release of energy stored in magnetic field is necessary. The problem that one encounters here is similar to the one in solar flare research where a relatively slow accumulation of the oppositely directed flux is required to be followed by a catastrophic release of energy on a short time scale. Therefore to understand the reconnection one should understand why the reconnection can be both fast and slow.

It is trivial to understand why reconnection can be slow in most astrophysical environments. Indeed, the ratio of the advection term to the diffusion term in the induction equation is given by the so-called Lundquist number $R_L = (V_A L/\eta)$, where $V_A$ is the Alfvén velocity, $L (> 10^{13}$ cm) is the scale of the system and $\eta = c^2/(4\pi \sigma)$ is magnetic diffusivity. For plasma of temperature $T$, $\eta = (10^9/T)^{-3/2}$ cm$^2$ s$^{-1}$ and so typically $R_L \gg 1$ and therefore the magnetic field diffusion term is negligible compared to advection.

The literature on magnetic reconnection is rich and vast (see, for example, [45] and references therein). We start by discussing a robust scheme proposed by Sweet and Parker ([46, 47]). In this scheme oppositely directed magnetic fields are brought into contact over a region of size $L_x$ (see Fig. 2). The diffusion of magnetic field takes place over the vertical scale $\Delta$ which is related to the Ohmic diffusivity by $\eta \approx V_r \Delta$, where $V_r$ is the velocity at which magnetic field lines can get into contact with each other and reconnect. Given some fixed $\eta$ one may naively hope to obtain fast reconnection by decreasing $\Delta$. However, this is not possible. An additional constraint posed by mass conservation must be satisfied. The plasma initially entrained on the magnetic field lines must be removed from the reconnection zone. In the Sweet-Parker scheme
Figure 3: Right Panel. Upper plot: Sweet-Parker scheme of reconnection. Middle plot: the new scheme of reconnection that accounts for field stochasticity, lower plot: a blow up of the contact region. Thick arrows depict outflows of plasma. Left Panel. The stochastic reconnection scheme requires 3D topology. In 2D the magnetic field lines cannot independently enter the reconnection region and this limits the reconnection speed. In the figure the magnetic fluxes intersect each other at 90 degrees.

this means a bulk outflow through a layer with a thickness of $\Delta$. In other words the entrained mass must be ejected, i.e. $\rho V_r L_x = \rho' V_A \Delta$, where it is assumed that the outflow occurs at the Alfvén velocity. If we ignore the effects of compressibility $\rho = \rho'$ and the resulting reconnection velocity allowed by Ohmic diffusivity and the mass constraint is $V_r \approx V_A R_L^{-1/2}$, i.e. very slow. Surely such reconnection cannot explain neither solar flares nor GRBs.

6.1 Fast Reconnection

So far, attempts to explain fast reconnection have not been supported by subsequent studies. The ‘X-point’ model of Petschek ([49]) collapses to a Sweet-Parker geometry after a short time ([48, 50]) and recent plasma reconnection experiments (see [51, 52]) show flat Sweet-Parker type current sheets. While collisionless effects can broaden the current sheet to roughly the ion skin depth ([53, 54, 55]), this, by itself, may not produce fast reconnection speeds in astrophysical contexts. For instance, such studies have not demonstrated the possibility of fast reconnection in the presence of the large scale forces acting to produce a large scale current sheet. Priest and Forbes ([56]) have stressed that even if fast Petschek reconnection is possible it will still be necessary to demonstrate “that it will apply to the turbulent MHD regime”. If magnetic fields are turbulent the boundary conditions for the current sheets are changing stochastically. On the other hand, boundary conditions need to be fine tuned for a Petschek reconnection scheme (see [56]).
A number of researchers have claimed that turbulence may accelerate reconnection. For instance, a turbulence-related concept of hyper-resistivity was put forward by Strauss ([57]). He correctly pointed out that the turbulence driven by current sheet instabilities may broaden the current sheets compared to the Sweet-Parker estimate. However, such instabilities, i.e. the tearing mode instability, do not allow us to evade the constraints on the global plasma flow that lead to slow reconnection speeds, a point which has been demonstrated numerically ([58]) and analytically (Lazarian & Vishniac [59] 1999, hereafter LV99).

Nevertheless, a further analysis of the stochastic reconnection resulted in a new model of fast reconnection that we shall briefly describe below. A detailed discussion is given in LV99.

MHD turbulence guarantees the presence of a stochastic field component properties of which depend on the admixture of compressible and incompressible modes ([31, 29]). We consider the case in which there exists a large scale, well-ordered magnetic field, of the kind that is normally used as a starting point for discussions of reconnection. This field may, or may not, be ordered on the largest conceivable scales. However, we will consider scales smaller than the typical radius of curvature of the magnetic field lines, or alternatively, scales below the peak in the power spectrum of the magnetic field, so that the direction of the unperturbed magnetic field is a reasonably well defined concept. In addition, we expect that the field has some small scale ‘wandering’ of the field lines. On any given scale the typical angle by which field lines differ from their neighbors is $\delta \phi \ll 1$, and this angle persists for a distance along the field lines $\lambda_\parallel$ with a correlation distance $\lambda_\perp$ across the field lines (see Fig. 2).

The modification of the mass conservation constraint in the presence of a stochastic magnetic field component is self-evident. Instead of being squeezed from a layer whose width is determined by Ohmic diffusion, the plasma may diffuse through a much broader layer, $L_y \sim \langle y^2 \rangle^{1/2}$ (see Fig. 2), determined by the diffusion of magnetic field lines. This suggests an upper limit on the reconnection speed of $\sim V_A \langle y^2 \rangle^{1/2}/L_x$. This will be the actual speed of reconnection the progress of reconnection in the current sheet itself does not impose a smaller limit. The value of $\langle y^2 \rangle^{1/2}$ can be determined once a particular model of turbulence is adopted, but it is obvious from the very beginning that this value is determined by field wandering rather than Ohmic diffusion as in the Sweet-Parker case.

What about limits on the speed of reconnection that arise from considering the structure of the current sheet? In the presence of a stochastic field component, magnetic reconnection dissipates field lines not over their entire length $\sim L_x$ but only over a scale $\lambda_\parallel \ll L_x$ (see Fig. 2), which is the scale over which magnetic field line deviates from its original direction by the thickness of the Ohmic diffusion layer $\lambda_\perp^{-1} \approx \eta/V_{\text{rec,local}}$. If the angle $\phi$ of field deviation does not depend on the scale, the local reconnection velocity would be $\sim V_A \phi$ and would not depend on resistivity. In LV99 we claimed that $\phi$ does depend on scale. Therefore the local reconnection rate
$V_{\text{rec,local}}$ is given by the usual Sweet-Parker formula but with $\lambda_\parallel$ instead of $L_x$, i.e. $V_{\text{rec,local}} \approx V_A(V_A \lambda_\parallel/\eta)^{-1/2}$. It is obvious from Fig. 2 that $\sim L_x/\lambda_\parallel$ magnetic field lines will undergo reconnection simultaneously (compared to a one by one reconnection process for the Sweet-Parker scheme). Therefore the overall reconnection rate may be as large as $V_{\text{rec,global}} \approx V_A(L_x/\lambda_\parallel)(V_A \lambda_\parallel/\eta)^{-1/2}$. Whether or not this limit is important depends on the value of $\lambda_\parallel$. The relevant values of $\lambda_\parallel$ and $(y^2)^{1/2}$ depend on the magnetic field statistics. This calculation was performed in LV99 using the Goldreich-Sridhar\[14\] model of MHD turbulence. The upper limit on $V_{\text{rec,global}}$ was greater than $V_A$, so that the diffusive wandering of field lines imposed the relevant limit on reconnection speeds. Thus

$$V_{r,up} = V_A \min \left[ \left( \frac{L_x}{l} \right)^{\frac{1}{2}} \left( \frac{l}{L_x} \right)^{\frac{1}{2}} \right] \left( \frac{v_l}{V_A} \right)^2,$$

(3)

where $l$ and $v_l$ are the energy injection scale and turbulent velocity at this scale respectively. In LV99 we also considered other processes that can impede reconnection and find that they are less restrictive. For instance, the tangle of reconnection field lines crossing the current sheet will need to reconnect repeatedly before individual flux elements can leave the current sheet behind. The rate at which this occurs can be estimated by assuming that it constitutes the real bottleneck in reconnection events, and then analyzing each flux element reconnection as part of a self-similar system of such events. This turns out to limit reconnection to speeds less than $V_A$, which is obviously true regardless. As the result we showed in LV99 that (3) is not only an upper limit, but is the best estimate of the speed of reconnection.

### 6.2 Flares of Reconnection

Evidently the reconnection rate given by eq. (3) is large, i.e. of the order of maximal possible velocity, which is $V_A$. Can the reconnection be slow at all? Naturally, when turbulence is negligible, i.e. $v_l \rightarrow 0$, the field line wandering is limited to the Sweet-Parker current sheet and the Sweet-Parker reconnection scheme takes over.

We also note that observations of solar flares seem to show that reconnection events start from some limited volume and spread as though a chain reaction from the initial reconnection region initiated a dramatic change in the magnetic field properties. Indeed, a solar flare happens as if the resistivity of plasma were increasing dramatically as plasma turbulence grows (see [60] and references therein). In the LV99 picture this is a consequence of the increased stochasticity of the field lines rather than any change in the local resistivity. The change in magnetic field topology that follows localized reconnection provides the energy necessary to feed a turbulent cascade in neighboring regions. This kind of nonlinear feedback is also seen in the geomagnetic tail, where it has prompted the suggestion that reconnection is mediated
by some kind of nonlinear instability built around modes with very small $k_\parallel$ (cf. [61] and references therein). The most detailed exploration of nonlinear feedback can be found in the work of Matthaeus and Lamkin ([62]), who showed that instabilities in narrow current sheets can sustain broadband turbulence in two dimensional simulations. Although the LV99 model is quite different, and relies on the three dimensional wandering of field lines to sustain fast reconnection, we note that the concept of a self-excited disturbance does carry over and may describe the evolution of reconnection between volumes with initially smooth magnetic fields.

To the best of our knowledge, there are no detailed calculations for GRBs using the stochastic reconnection scheme. Further research is necessary to show whether this is a viable alternative to the more traditional models of GRBs. Most of the energy released via stochastic reconnection goes into MHD turbulence. Therefore the particle acceleration will happen according to the model described in sections 4-5.

7 Summary

- Whether the GRB arises from the interaction of relativistic shocks or due to magnetic reconnection MHD turbulence is likely to play an essential role in transferring the energy to electrons.

- Recent understanding of fundamentals of MHD turbulence as well as particle acceleration by compressible and Alfénic modes of the turbulence allows quantitative description of GRBs.

- If magnetic reconnection plays a role in GRBs, the stochastic reconnection is the primary candidate to produce bursts.

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References


