Truly Minimal Left-Right Model of Quark and Lepton Masses

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Abstract

We propose a left-right model of quarks and leptons based on the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, where the scalar sector consists of only two doublets: $(1,2,1,1)$ and $(1,1,2,1)$. As a result, any fermion mass, whether it be Majorana or Dirac, must come from dimension-five operators. This allows us to have a common view of quark and lepton masses, including the smallness of Majorana neutrino masses as the consequence of a double seesaw mechanism.
In the standard model of electroweak interactions, neutrinos are massless. On the other hand, recent experimental data on atmospheric [1] and solar [2] neutrinos indicate strongly that they are massive and mix with one another [3]. To allow neutrinos to be massive theoretically, the starting point is the observation of Weinberg [4] over 20 years ago that a unique dimension-five operator exists in the standard model, i.e.

\[ \mathcal{L}_\Lambda = \frac{f_{ij}}{2\Lambda} (\bar{\nu}_i \phi^0 - e_i \phi^+) (\bar{\nu}_j \phi^0 - e_j \phi^+) + H.c. \] (1)

which generates a Majorana neutrino mass matrix given by

\[ (\mathcal{M}_\nu)_{ij} = \frac{f_{ij} v^2}{\Lambda}, \] (2)

where \( v \) is the vacuum expectation value of \( \phi^0 \). This also shows that whatever the underlying mechanism for the Majorana neutrino mass, it has to be “seesaw” in character, i.e. \( v^2 \) divided by a large mass [5].

If the particle content of the standard model is extended to include left-right symmetry [6], then the gauge group becomes \( G_{LR} \equiv SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \), whose diagonal generators satisfy the charge relationship

\[ Q = T_{3L} + T_{3R} + \frac{(B - L)}{2} = T_{3L} + \frac{Y}{2}. \] (3)

Quarks and leptons transform as

\[ q_L = (u, d)_L \sim (3, 2, 1, 1/3), \] (4)
\[ q_R = (u, d)_R \sim (3, 1, 2, 1/3), \] (5)
\[ l_L = (\nu, e)_L \sim (1, 2, 1, -1), \] (6)
\[ l_R = (N, e)_R \sim (1, 1, 2, -1), \] (7)

where a new fermion, i.e. \( N_R \), has been added in order that the left-right symmetry be maintained.
In all previous left-right models, a scalar bidoublet transforming as \((1, 2, 2, 0)\) is then included for the obvious reason that we want masses for the quarks and leptons. Suppose however that we are only interested in the spontaneous breaking of \(SU(2)_L \times SU(2)_R \times U(1)_{B-L}\) to \(U(1)_{em}\) with \(v_R \gg v_L\), then the simplest way is to introduce two Higgs doublets transforming as

\[
\Phi_L = (\phi^+_L, \phi^0_L) \sim (1, 2, 1, 1),
\]

\[
\Phi_R = (\phi^+_R, \phi^0_R) \sim (1, 1, 2, 1).
\]

Suppose we now do not admit any other scalar multiplet. This is analogous to the situation in the standard model, where \(SU(2)_L \times U(1)_Y\) is spontaneously broken down to \(U(1)_{em}\) by a Higgs doublet and we do not admit any other scalar multiplet. In that case, we find that quark and charged-lepton masses are automatically generated by the existing Higgs doublet, but neutrinos obtain Majorana masses only through the dimension-five operator of Eq. (1). In our case, in the absence of the bidoublet, all fermion masses, be they Majorana or Dirac, must now have their origin in dimension-five operators, as shown below.

Using Eqs. (4) to (9), it is clear that

\[
(l_L \Phi_L) = \nu_L \phi^0_L - e_L \phi^+_L
\]

and

\[
(l_R \Phi_R) = N_R \phi^0_R - e_R \phi^+_R
\]

are invariants under \(G_{LR}\). Hence we have the dimension-five operators given by

\[
\mathcal{L}_M = \frac{f_{ij}^L}{2 \Lambda_M^2} (l_{iL} \Phi_L)(l_{jL} \Phi_L) + \frac{f_{ij}^R}{2 \Lambda_M^2} (l_{iR} \Phi_R)(l_{jR} \Phi_R) + \text{H.c.},
\]

which will generate Majorana neutrino masses proportional to \(v^2_L/\Lambda_M\) for \(\nu_L\) and \(v^2_R/\Lambda_M\) for \(N_R\). In addition, we have

\[
\mathcal{L}_D = \frac{f_{ij}^D}{\Lambda_D} (l_{iL} \Phi_L)(l_{jR} \Phi_R) + \text{H.c.}
\]
and the corresponding dimension-five operators which will generate Dirac masses for all the quarks and charged leptons.

From Eq. (13), it is clear that

$$ (m_D)_{ij} = \frac{f^D_{ij} v_L v_R}{\Lambda_D}, $$

hence $\nu_L$ gets a double seesaw [7] mass of order

$$ \frac{m^2_D}{m_N} \sim \frac{v_L^2 v_R^2 \Lambda_M}{\Lambda_D^2 v_R^2} = \frac{v_L^2 \Lambda_M}{\Lambda_D^2}, $$

which is much larger than $v_L^2/\Lambda_M$ if $\Lambda_D << \Lambda_M$. Take for example $\Lambda_M$ to be the Planck scale of $10^{19}$ GeV and $\Lambda_D$ to be the grand-unification scale of $10^{16}$ GeV, then the neutrino mass scale is 1 eV (for $v_L$ of order 100 GeV). The difference between $\Lambda_M$ and $\Lambda_D$ may be due to the fact that if we assign a global fermion number $F$ to $l_L$ and $l_R$, then $L_M$ has $F = \pm 2$ but $L_D$ has $F = 0$.

Since the Dirac masses of quarks and charged leptons are also given by Eq. (14), $v_R$ cannot be much below $\Lambda_D$. This means that $SU(2)_R \times U(1)_{B-L}$ is broken at a very high scale to $U(1)_Y$, and our model at low energy is just the standard model. We do however have the extra singlet neutrinos $N_R$ with masses of order $v_R^2/\Lambda_M$, i.e. below $10^{13}$ GeV, which are useful for leptogenesis, as is well-known [8].

For $m_t = 174.3 \pm 5.1$ GeV, we need $v_R/\Lambda_D$ to be of order unity in Eq. (14). One may wonder in that case whether we can still write Eq. (13) as an effective operator. The answer is yes, as can be seen with the following specific example. Consider the singlets

$$ U_L, U_R \sim (3, 1, 1, 4/3), $$

with invariant mass $M_U$ of order $\Lambda_D$, then the $2 \times 2$ mass matrix linking $(\bar{t}_L, \bar{U}_L)$ to $(t_R, U_R)$ is given by

$$ M_{tU} = \begin{pmatrix} 0 & f^L_{tL} v_L \\ f^R_{tR} v_R & M_U \end{pmatrix}. $$

For $v_L << v_R, M_U$, we then have

$$m_t = \frac{f_t^L f_t^R v_L v_R}{M_U} \left[ 1 + \left( \frac{f_t^R v_R}{M_U^2} \right)^2 \right]^{-\frac{1}{2}},$$  \hspace{1cm} (18)

which is in the form of Eq. (14) even if $v_R/M_U \sim 1$.

A possible variant of our proposal is to add dimension-six operators, so that we can invoke the Bardeen-Hill-Lindner (BHL) dynamical mechanism [9] with a cutoff scale equal to $\Lambda_D$. We may assume that the effective dynamical BHL Higgs doublet [call it $\Phi_1 = (\phi_1^+, \phi_1^0)$] couples only to the top quark, whereas our fundamental $\Phi_L$ [call it $\Phi_2$] couples to all quarks and leptons. We thus have a specific two-Higgs-doublet model [10] with experimentally verifiable phenomenology, as described below.

Since the BHL model predicts $m_t = 226$ GeV for $\Lambda_D = 10^{16}$ GeV, the effective Yukawa coupling of $\tilde{t}_L t_R$ to $\phi_1^0$ is

$$f_t^{(1)} = (226 \text{ GeV})(2\sqrt{2}G_F)^{\frac{1}{2}} = 226/174 = 1.3,$$  \hspace{1cm} (19)

and for $\tan \beta = v_2/v_1$, we have

$$m_t = (1.3 \cos \beta + f_t^{(2)} \sin \beta)(174 \text{ GeV}).$$  \hspace{1cm} (20)

This shows that, with a second Higgs doublet, the correct value of $m_t$ may be obtained. Furthermore, $f_t^{(2)}$ may be assumed to be small, say of order $10^{-2}$. This allows $v_R/\Lambda_D \sim 10^{-2}$ in Eq. (14) and thus $v_R \sim 10^{14}$ GeV, so that $m_N \sim v_R^2/\Lambda_M$ is of order $10^9$ GeV, which may be more effective for leptogenesis, even with the reheating of the Universe after inflation. At the same time, using Eq. (20), this fixes

$$\tan \beta \simeq 0.83$$  \hspace{1cm} (21)

for the phenomenology of the two-doublet Higgs sector.
Since the $d, s, b$ quarks receive masses only from $v_2$, there is no tree-level flavor-changing neutral currents in this sector. This explains the suppression of $K_L - K_S$ mixing and $B - \bar{B}$ mixing. On the other hand, both $v_1$ and $v_2$ contribute to the $u, c, t$ quarks, so our model does predict tree-level flavor-changing neutral currents in this sector. Suppose the Yukawa interaction $f_1^{(1)} \phi_1^0 t_L t_R$ is replaced by $f_1^{(1)} (v_1/v_2) \phi_2^0 t_L t_R$, then the resulting mass matrix would be exactly proportional to the Yukawa matrix. This means that there would not be any flavor-nondiagonal interactions. Hence the term which contains all the flavor-changing interactions is given by [11]

$$f_1^{(1)} \left( \phi_1^0 - \frac{v_1}{v_2} \phi_2^0 \right) t_L t_R^* + H.c.,$$

(22)

where $t'_{L,R}$ are the original entries in the $u, c, t$ mass matrix before diagonalization to obtain the mass eigenstates. We thus expect contributions to, say $D - \bar{D}$ mixing, beyond that of the standard model. Let

$$t'_{L,R} \simeq t_{L,R} + \epsilon_{L,R}^L c_{L,R} + \epsilon_{L,R}^U u_{L,R},$$

(23)

where the $\epsilon$ parameters are at most of order $f_1^{(2)}/f_1^{(1)} \sim 10^{-2}$, then [11]

$$\frac{\Delta m_D^{D_0}}{m_D^{D_0}} \simeq \frac{B_D f_D^2 |f_1^{(1)}|^2}{3 m_{eff}^2 \sin^2 \beta} \epsilon_{L}^L \epsilon_{L}^{R} \epsilon_{U}^L \epsilon_{U}^{R},$$

(24)

where $m_{eff}$ is the effective normalized contribution from $\phi_1^0 - (v_1/v_2) \phi_2^0$. Using $f_D = 150$ MeV, $B_D = 0.8$, and the present experimental upper bound [12] of $2.5 \times 10^{-14}$ on this fraction, we then obtain

$$\frac{|\epsilon_{L}^L \epsilon_{L}^{R} \epsilon_{U}^L \epsilon_{U}^{R}|}{10^{-8}} \left( \frac{100 \text{ GeV}}{m_{eff}} \right)^2 < 1.$$

(25)

This shows that $D - \bar{D}$ mixing may be observable in this model, in contrast to the negligible expectation of the standard model.

Rare top decays such as $t \rightarrow c$ (or $u$) + neutral Higgs boson are now possible if kinematically allowed. Their branching fractions are of order $|\epsilon|^2 \sim 10^{-4}$. Once a neutral Higgs boson is produced at a future collider, its decay will also be a test of this model. Its dominant
decay is still $b\bar{b}$, but its subdominant decays will not just be $c\bar{c}$ and $\tau^-\tau^+$, but also $c\bar{u}$ and $u\bar{c}$. There should be observable $D^\pm\pi^\mp$ final states, for example.

In conclusion, we have proposed a truly minimal $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge model, with the simplest possible Higgs sector. All fermion masses, be they Majorana or Dirac, have a common origin, i.e. dimension-five operators. Whereas Dirac fermions have masses at the electroweak scale, the observed neutrinos have naturally small Majorana masses from a double seesaw mechanism. The existence of singlet right-handed neutrinos with masses in the range $10^9$ to $10^{13}$ GeV are required, and their decays establish a lepton asymmetry which is converted at the electroweak phase transition to the present observed baryon asymmetry of the Universe.

Since our proposed model is identical to the standard model below $10^{16}$ GeV (except for the $N_R$'s), the usual predictions of the latter also apply, including the expected occurrence of proton decay and neutron-antineutron oscillations from higher-dimensional operators due to new physics at or above $10^{16}$ GeV.

In the presence of dimension-six operators, we may invoke the Bardeen-Hill-Lindner mechanism to generate a dynamical Higgs doublet which renders the top quark massive. Since we also have a fundamental Higgs doublet, this allows us to have a realistic $m_t$ (which is not possible in the minimal BHL model) and an effective two-doublet Higgs sector at the electroweak scale with distinctive and experimentally verifiable flavor-changing phenomena.

The work of EM was supported in part by the U. S. Department of Energy under Grant No. DE-FG03-94ER40837. BB would like to thank the organisers of the flavour@cern series of talks, where some aspects of the double seesaw mechanism were discussed.
References


