Rapidity and Transverse Momentum Distributions of DIS 1-jet Inclusive Cross Section — A Next-to-Leading Order Monte-Carlo Prediction

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ABSTRACT: We have implemented in the RAPGAP program a previously derived subtraction method for next-to-leading order (NLO) corrections in Monte-Carlo (MC) event generators, and we show results for jet production in deep inelastic scattering. At small $x_{bj}$, NLO corrections are comparable to the LO cross section, because of the large gluon density. We devise a jet observable that is particularly sensitive to the treatment of parton kinematics, which our method is intended to treat correctly. We compare the results of the calculation with LO calculations and with the previously existing treatment of NLO processes in RAPGAP. Substantial corrections to the event distribution are found.

KEYWORDS: Subtraction Method, QCD, NLO Computations, RAPGAP.
1. Introduction

In contrast to “fixed order” calculations, Monte-Carlo (MC) event generators attempt to predict the bulk of the exclusive components of perturbative QCD subprocesses and to give the details of the final states. While this is at the expense of model dependence in the non-perturbative hadronization, it is also the case that MC event generators incorporate a useful approximation to the resummation of large logarithms in their parton showering. This provides a successful phenomenology of infrared sensitive cross sections.

Now, it is quite hard to treat non-leading corrections in MC event generators. However, this is also quite essential. For example, in deep inelastic scattering (DIS) at small $x_{bj}$ or in diffractive DIS, the gluon density is substantially larger than the quark densities, and the next-to-leading order (NLO) gluon-induced process is comparable to the leading order (LO) quark-induced process. Therefore, to get a sensible phenomenology, the gluon-induced NLO corrections must be included.

Precisely because MC event generators treat exclusive components of the cross section, standard “fixed order” calculations of hard scattering coefficients are not well adapted for use in event generators. Some symptoms of this are that in LO, the coefficients contain $\delta$-functions of parton kinematics, and that in NLO the coefficients are singular distributions (generalized functions). They only give results that can be compared with hadronic level cross sections after an integration over a wide range of parton kinematics, which provides a kind of infrared safety. Standard fixed-order calculations approximate the partons as on-shell with zero transverse momentum. For cross sections that do not include a wide integration over parton kinematics, only a correct treatment of parton kinematics allows a correct conversion of the generalized functions to ordinary functions of kinematic variables. In fact, the standard approximations used in hard-scattering coefficients cause complications and difficulties in the derivation of the algorithms used in event generators.

In this paper, we will examine the rapidity and transverse momentum distribution of the 1-jet inclusive cross section in DIS. Even though this is a jet cross section, it is not infra-red safe because its LO hard scattering coefficient is a $\delta$-function, and in the approximation that parton virtuality and transverse momentum are ignored, the jet momentum can be calculated from the measured 4-momentum of the scattered electron.
Thus the $\delta$-function appears in the hadron-level jet cross section, in this simple parton-model approximation. Similarly, the conventional NLO corrections are singular at the parton-model values. Because we only integrate over a narrow range of parton kinematics around these singularities, it is not clear how to apply these conventional NLO hard scattering coefficients to this cross section. However, it is crucial for a good NLO MC method to consistently treat this kind of cross section with NLO accuracy.

As we argued in [1, 2, 3], a proper treatment of higher order (NLO and beyond) corrections in event generators requires (a) that the parton kinematics should be treated more exactly and (b) that the hard scattering coefficients should be ordinary functions, not generalized functions. This is in contrast to other proposals [4] for matrix element corrections to event generators. In [1] our approach is proposed in general and is designed to be generalizable to all orders in perturbation theory.

More concretely, in [2] this method was applied to DIS in a form specifically designed for use in event generators using the Bengtsson–Sjöstrand [5] algorithm for showering. Such event generators include RAPGAP [6] and PYTHIA [7]. We have now implemented this method in RAPGAP, a widely used event generator for both inclusive and diffractive DIS. The implementation is based on earlier work by Schilling [8].

In this paper we show the results of the 1-jet inclusive cross section calculated by our subtraction method. We also compare our method with a method previously implemented in RAPGAP, which uses a $k_t$-cut applied to the simple unsubtracted NLO cross section. The $k_t$-cut method is intended mainly for the di-jet rate calculation of high $p_T$ jet events.

We also compare our results to those of a LO calculation from RAPGAP, where the $\delta$-function in the 1-jet inclusive cross section is smeared out only by the parton showering and the hadronization. At small $x_{bj}$ the gluon-induced NLO contributions are enhanced, and we expect the cross section with NLO accuracy to be a very broad distribution, and the three methods should give very different results. At large $x_{bj}$ the NLO corrections are suppressed by $O(\alpha_s)$, we expect to see similar results by these methods.

The organization of this paper is the following. In Sec. 2 we briefly describe the subtraction method and the $k_t$-cut method in RAPGAP. In Sec. 3 we show and discuss the results of the 1-jet inclusive cross sections for DIS at small $x_{bj}$ and large $x_{bj}$ predicted by the three different methods. In Sec. 4, we discuss the possible applications and future work of the NLO RAPGAP with subtraction method.

1Only gluon-induced NLO corrections are implemented, as in [2]. The quark-induced NLO terms have the complication of a soft divergence, which we plan to deal with in a later paper. Gluon-induced NLO corrections are important in DIS, because they are not suppressed compared to the LO terms whenever the gluon distribution is large.
2. NLO hard scattering

In [2], we devised a subtraction method for NLO corrections, and obtained the coefficient function for the theoretically simple yet phenomenologically significant gluon-induced correction to DIS. It was assumed that the Bengtsson-Sjöstrand (BS) [5] algorithm is used for the initial state parton shower, as in RAPGAP. A point-by-point subtraction in the phase space ensures that the hard-scattering coefficient is non-singular [2]. The pdfs needed are specific to the showering algorithm [9], and those needed to match the BS algorithm were calculated in [3]. The quark-induced NLO corrections have the complication of a soft divergence, and we plan to treat this case, matched to the BS algorithm, in a later paper; the quark-induced term is a genuine correction, order $\alpha_s$ compared with the LO process, whereas the gluon-induced term is not suppressed at all when the gluon density is large.

In RAPGAP, the previously implemented method for NLO was the $k_t$-cut method, designed mainly to give the correct di-jet rate at large $Q^2$. Here the unsubtracted parton-level matrix elements of the NLO processes are used with a cut on the transverse momentum $k_t$ of the outgoing partons in the center of mass frame of $\gamma^* +$ parton scattering. $k_t$ cut is required to be large enough so that the NLO contributions are smaller than the total cross section. Usually $k_t^2_{\text{cut}} = 4 \text{GeV}^2$. This gives us a large logarithm $\log(k_t^2_{\text{cut}}/Q^2)$ in the integrated NLO cross section when $Q^2$ is large.

3. Rapidity and Transverse Momentum Distribution of DIS 1-jet Inclusive Cross Section

3.1. Theoretical and experimental considerations

To probe the differences between different treatments of NLO corrections in an event generator, we need an infrared sensitive cross section that potentially gives us the maximum difference between the different methods. We choose the rapidity and transverse momentum distribution of the 1-jet inclusive cross section, $d^2\sigma/(d\hat{\eta}dx_t)$ in the laboratory frame, with the rapidity and transverse momentum measured relative to the simplest parton-model values. At the LO parton level, the scattered quark has a transverse momentum of $E_t^{\text{PM}}$ and pseudorapidity $\eta^{\text{PM}}$, both of which can be calculated from the momentum of the scattered electron, and so we define

$$x_t \equiv \frac{E_t^{\text{jet}}}{E_t^{\text{PM}}}, \quad \text{(3.1)}$$

$$\hat{\eta} \equiv \eta^{\text{jet}} - \eta^{\text{PM}}. \quad \text{(3.2)}$$

If we ignore parton virtuality and transverse momentum, the LO cross section is a simple
\[
\frac{d^2\sigma}{d\eta dx_t} \propto \delta(x_t - 1)\delta(\hat{\eta}).
\] (3.3)

In real QCD, with effects from higher order QCD subprocesses and the hadronization, the \(\delta\)-function will be smeared out. The shape of the smeared \(\delta\)-function predicted by a MC event generator will provide us information about its treatment of the NLO corrections.

We calculate the cross section from the event generator RAPGAP by three methods:

1. LO hard scattering, where jet structure is generated exclusively by parton showering and hadronization.
2. With NLO corrections implemented by the \(k_t\) cut method.
3. With NLO corrections implemented by our subtraction method.

Due to the enhanced gluon density at small \(x_{bj}\), the cross section in the small \(x_{bj}\) region is likely to give us the maximum difference between the different methods. At large \(x_{bj}\) the NLO corrections are suppressed by \(\alpha_s\), and therefore in this region the cross section is not sensitive to the difference between the NLO methods. So in this paper we calculate this cross section in two different regions, \(10^{-4} < x_{bj} < 10^{-2}\), where we expect to see very different results from the three calculations, and \(0.02 < x_{bj} < 1\), where similar results are expected.

Our calculations are performed under the following conditions, with hadron kinematics appropriate to the collider experiments at HERA:

- The positron is moving in the \(-z\) direction with \(P^e_z = -27.5\) GeV.
- The proton is moving in the \(+z\) direction with \(P^p_z = 820\) GeV.
- \(20 < Q^2 < 50\) GeV\(^2\).
- In LO and \(k_t\)-cut calculations, the LO pdfs Cteq4L are used.
- In RAPGAP with our subtraction method, we use the BS-algorithm specific pdf calculated by a scheme change \([3]\) from the \(\overline{\text{MS}}\) pdf, Cteq4M.
- The jet-finder is PXCON, with \(E_t\)-mode and cone radius= 1.
- The running scale of the pdfs and \(\alpha_s\) is \(Q\).
- Initial state showering is turned on, with \(Q^2\) ordering.
- Final state showering is on.
- Hadronization and proton remnant are required.
• The LO subprocess and (for the NLO calculations) the gluon-induced NLO subprocesses are chosen.

• Only light quarks are considered.

Figs. 1, 2 and 3 give the results for \(10^{-4} < x_{\text{bj}} < 10^{-2}\), while Figs. 4 and 5 give the results for \(0.02 < x_{\text{bj}} < 1\).

3.2. Cross Section at small \(x_{\text{bj}}\)

![Figure 1: Transverse momentum and rapidity distribution of 1-jet inclusive cross section for \(10^{-4} < x_{\text{bj}} < 10^{-2}\). (a) RAPGAP with the subtraction method, (b) RAPGAP with the \(k_t\)-cut method and (c) LO RAPGAP. Note that the vertical scales for these three graphs are different.](image)

In Fig. 1 we plot \(d^2\sigma/(d\hat{\eta}dx_t)\) as a function of \(x_t\) and \(\hat{\eta}\) for the small \(x_{\text{bj}}\) region, \(10^{-4} < x_{\text{bj}} < 10^{-2}\). It clearly shows that the LO (graph c) prediction is strongly peaked at the PM value \(x_t = 1\) and \(\hat{\eta} = 0\), as expected. The cross section predicted by the \(k_t\)-cut method (graph b) is somewhat broader but still peaked at the PM values. In contrast, the prediction from our subtraction method (graph a) is broadly distributed; although it has a maximum close to the PM position, the peak is not at all \(\delta\)-function like. This indicates that when the gluonic NLO term is comparable to the LO term, a correct treatment of off-shell parton kinematics has a substantial effect on the shape of the cross section.

For a more quantitative comparison between these methods, we now concentrate on the one parameter cross sections \(d\sigma/d\hat{\eta}\) and \(d\sigma/dx_t\), shown in Figs. 2 and 3. In each figure, we show in graphs (a), (b) and (c) the components of the 1-jet inclusive cross section separated according to the number of jets in the final state: 1 jet, 2 jets and \(\geq 3\) jets. Then in graph (d), we give the total 1-jet inclusive cross section. The solid curves, dashed curves and the dotted curves are the predictions of our subtraction method, the \(k_t\)-cut method, and the LO calculation, respectively.
Figure 2: Rapidity distribution of 1-jet inclusive cross section for $10^{-4} < x_{bj} < 10^{-2}$. The four graphs show the following contributions: (a) 1-jet events, (b) 2-jet events, (c) $\geq 3$-jet events, (d) total cross section. The solid curve, dashed curve and dotted curve are predictions from the subtraction method, the $k_t$-cut method and the LO RAPGAP, respectively.

The differences between the predictions are evident. The subtraction method gives the broadest distribution, while the $k_t$-cut method and the LO results show progressively more strongly peaked distributions. A comparison with data should fairly easily test our claim that the subtraction method is more correct. We also see that the number of 1-jet events and the number of 2-jets events predicted by our subtraction method are of the same order, while the $k_t$-cut method and LO predict more dominance of 1-jet events.$^2$ Also note that, in Fig. 2, the peaks of the curves are not at the PM position $\hat{\eta} = 0$. Even though the peak corresponds to a smeared out $\delta(\hat{\eta})$, the peak has shifted to $\hat{\eta}$ around 1/2. The reason is that showering and hadronization of the jet changes a massless on-shell quark into a massive quark, and the kinematics of 2-body collisions shows that this moves the rapidity of the quark towards the proton’s rapidity. Since the proton is chosen

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$^2$The number of events is roughly proportional to the area under the curve divided by the number of jets in an event.
to move in the +z direction, this means that the rapidity of the jet is biased to a positive value relative to the simple parton model value.

3.3. Cross Section at large $x_{bj}$

In Figs. 4 and 5 we show the results for large $x_{bj}$. The parameters are the same as before, except that we choose to analyze events with $0.02 < x_{bj} < 1$. In contrast to the small $x_{bj}$ results, the three different methods give quite similar results, all peaked near $\hat{\eta} = 0$ and $x_t = 1$. In addition, most of the events are 1-jet events. This is because gluon distribution is now much smaller, and the LO process dominates.

4. Applications and Future Work

We have implemented our subtraction method [2] in RAPGAP for DIS processes. The subtraction method is designed to give perturbative QCD predictions for the hard scattering with a correct treatment of parton kinematics. We chose to calculate an infra-red
Figure 4: Rapidity distribution of 1-jet inclusive cross section for $0.02 < x_{bj} < 1$. The coding of the curves is the same as in Fig. 2.

sensitive observable $d^2\sigma/dx_t d\hat{\eta}$ using three different methods, and to perform the calculation in a region where the NLO correction is large because of a large gluon density. We see large differences between the subtraction method and the $k_T$-cut method, thereby demonstrating the importance of the treatment of parton kinematics.

There is no experimental analysis on $d^2\sigma/dx_t d\hat{\eta}$ available yet, current analyses \cite{10,11} being in the Breit frame. Since our subtraction method, combined with the parton shower, is intended to give more accurate perturbative QCD predictions for infrared sensitive cross sections, it would be interesting to do the experimental analysis of the jet cross section in the laboratory frame relative to the position of parton-model jet, and to compare it with the MC calculations using our subtraction method.

In the future we would like to work on two directions. One is to implement the subtraction method for diffractive DIS where there is also a large gluon distribution. Another is to work on the quark-induced NLO subprocess in order to get a complete NLO MC event generator.
Figure 5: Transverse momentum distribution of 1-jet inclusive cross section for \( 0.02 < x_{bj} < 1 \). The coding of the curves is the same as in Fig. 2.

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References


