On supersymmetric contributions to the CP asymmetry of the $B \rightarrow \phi K_S$

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Abstract

We analyse the CP asymmetry of the $B \rightarrow \phi K_S$ process in general supersymmetric models. In the framework of the mass insertion approximation, we derive model independent limits for the mixing CP asymmetry. We show that chromomagnetic type of operator may play an important role in accounting for the deviation of the mixing CP asymmetry between $B \rightarrow \phi K_S$ and $B \rightarrow J/\psi K_S$ processes observed by Belle and BaBar experiments. A possible correlation between the direct and mixing CP asymmetry is also discussed. Finally, we apply our result in minimal supergravity model and supersymmetric models with non-universal soft terms.

1 Introduction

With the advent of experimental data from the $B$ factories, the Standard Model (SM) will be subject to a very stringent test, with the potential for probing virtual effects from new physics. In particular, various CP asymmetries in $B$ decays will be measured, and in the SM all of them have to be accommodated with a single parameter, namely the phase in the Cabbibo–Kobayashi–Maskawa mixing matrix $\delta_{KM}$ [1].

The BaBar [2] and Belle [3] measurements of time dependent asymmetry in $B \rightarrow J/\psi K_S$ have provided the first evidence for the CP violation in the $B$ system. The world average of these results, $S_{J/\psi K_S} = \sin 2\beta/(2\phi_1) = 0.734 \pm 0.054$, is in a good agreement with the SM prediction. Therefore, it was concluded that CP is significantly violated in nature and the KM mechanism is the dominant source of CP violation. However, for the process $B \rightarrow J/\psi K_S$ the SM contribution is at tree level and any new physics contributions are at one loop level, hence they are expected to be naturally suppressed. As shown in Ref.[4], in order to have a significant supersymmetric contribution to $S_{J/\psi K_S}$, a
large flavour structure and/or large SUSY CP violating phases are required. For instance, in order to account for the above experimental results by supersymmetric contributions, the imaginary parts of the relevant mass insertions \((\delta_{LL}^d)_{31}\) and \((\delta_{LR}^d)_{31}\) should be of order \(10^{-1}\) and \(10^{-2}\), respectively for average scalar mass and gluino mass are of order 400 GeV. Such large mixings, usually, do not exist in most of the supersymmetric models, especially in SUSY models with minimal flavour violation or in SUSY models with non-minimal flavour and hierarchical Yukawa couplings [4]. In this class of models, the typical SUSY contributions to the \(B^0 - \overline{B}^0\) mixing and the CP asymmetry \(S_{J/\psi K_S}\) are founded to be small and the SM contribution gives the dominant effect.

Unlike the \(B \rightarrow J/\psi K_S\) decay, the process \(B \rightarrow \phi K_S\) has no tree level decay amplitude. In the SM and at the quark level, the decay \(b \rightarrow s \bar{s}s\), which contribute to \(B \rightarrow \phi K_S\), is induced only at the one loop level. Thus, it is tempting to expect that the SUSY contributions to this decay are more significant [5–8]. Based on the KM mechanism of CP violation, both CP asymmetries of \(B \rightarrow \phi K_S\) and \(B \rightarrow J/\psi K_S\) processes should measure \(\sin 2\beta\) with negligible hadronic uncertainties (up to \(\mathcal{O}(\lambda^2)\) effects, with \(\lambda\) being the Cabibbo mixing). However, the recent measurement by BaBar and Belle collaborations show a 2.7\(\sigma\) deviation from the observed value of \(S_{J/\psi K_S}\) [3, 9]. The average of these two measurements implies

\[
S_{\phi K_S} = -0.39 \pm 0.41. \tag{1}
\]

This difference between \(S_{J/\psi K_S}\) and \(S_{\phi K_S}\) is considered as a hint for new physics, in particular for supersymmetry. Several works in this respect are in the literature with detail discussion on the possible implications of this result [10–20].

As known, in supersymmetric models there are additional sources of flavour structures and CP violation with a strong correlation between them. Therefore, SUSY emerges as the natural candidate to solve the problem of the discrepancy between the CP asymmetries \(S_{J/\psi K_S}\) and \(S_{\phi K_S}\). However, the unsuccessful searches of the electric dipole moment (EDM) of electron, neutron, and mercury atom impose a stringent constraint on SUSY CP violating phases [21]. It was shown that the EDM can be naturally suppressed in SUSY models with small CP phases [21] or in SUSY models with flavour off–diagonal CP violation [21, 22]. It is worth mentioning that the scenario of small CP phases (\(< 10^{-2}\) in supersymmetric models is still allowed by the present experimental results [23]. In this case of models, the large flavour mixing is crucial to compensate for the smallness of the CP phases.

The aim of this paper is to investigate, in a model independent way, the question of whether supersymmetry can significantly modify the CP asymmetry in the \(B \rightarrow \phi K_S\) process. We focus on the gluino contributions to the CP asymmetry \(S_{\phi K_S}\) for the following two reasons. First, it is less constrained by the experimental results on the branching ratio of the inclusive transitions \(B \rightarrow X_s \gamma\) and \(B \rightarrow X_s l^+ l^-\) than the chargino contributions [24]. Second, it includes the effect of the chromomagnetic operator which, as we will show, has a huge enhancement in SUSY models. We perform this analysis at the NLO accuracy
in QCD by using the results of Ali and Greub [25]. We also apply our result in minimal supergravity model where the soft SUSY breaking terms are universal and general SUSY models with non–universal soft terms and Yukawa couplings with large mixing. We show that in the case of non–universal $A$–terms, the gluino contributions can account for the experimental results of $S_{\phi K_S}$.

The paper is organised as follows. In section 2, we present the CP violation master formulae in $B$–system including the SUSY contribution. In section 3, we discuss the effective Hamiltonian for $\Delta B = 1$ transition. Section 4 is devoted to the study of the supersymmetric contributions to the mixing and direct CP asymmetry $S_{\phi K_S}$ and $C_{\phi K_S}$. We show that the chromomagnetic operator plays a crucial role in explaining the observed discrepancy between $S_{\phi K_S}$ and $S_{\psi K_S}$. In section 5, we analyse the SUSY contributions to $S_{\phi K_S}$ in explicit models. We show that only in SUSY models with non–universal soft breaking terms and large Yukawa mixing, one can get significant SUSY contributions to $S_{\phi K_S}$. Our conclusions are presented in section 6.

2 The CP Violation in $B \to \phi K_S$ Process

We start the sections by summarising our convention for the CP asymmetry in B system. The time dependent CP asymmetry for $B \to \phi K_S$ can be described by [26]:

$$a_{\phi K_S}(t) = \frac{\Gamma(\bar{B}^0(t) \to \phi K_S) - \Gamma(B^0(t) \to \phi K_S)}{\Gamma(\bar{B}^0(t) \to \phi K_S) + \Gamma(B^0(t) \to \phi K_S)}$$  \hfill (2)$$

$$= C_{\phi K_S} \cos \Delta M_{B_d} t + S_{\phi K_S} \sin \Delta M_{B_d} t$$  \hfill (3)$$

where $C_{\phi K_S}$ and $S_{\phi K_S}$ represent the direct and the mixing CP asymmetry, respectively and they are given by

$$C_{\phi K_S} = \frac{|\overline{\rho}(\phi K_S)|^2 - 1}{|\overline{\rho}(\phi K_S)|^2 + 1}, \quad S_{\phi K_S} = \frac{2 \text{Im}\left[\frac{q}{p} \overline{\rho}(\phi K_S)\right]}{|\overline{\rho}(\phi K_S)|^2 + 1}. \hfill (4)$$

The parameter $\overline{\rho}(\phi K_S)$ is defined by

$$\overline{\rho}(\phi K_S) = \frac{\overline{A}(\phi K_S)}{A(\phi K_S)}. \hfill (5)$$

where $\overline{A}(\phi K_S)$ and $A(\phi K_S)$ are decay amplitudes of $\bar{B}^0$ and $B^0$ meson which can be written in terms of the matrix element of the $\Delta B = 1$ transition as

$$\overline{A}(\phi K_S) = \langle \phi K_S | \mathcal{H}_{\Delta B=1}^{\text{eff}} | \bar{B}^0 \rangle, \quad A(\phi K_S) = \langle \phi K_S | \mathcal{H}_{\Delta B=1}^{\text{eff}}^\dagger | B^0 \rangle. \hfill (6)$$

The mixing parameters $p$ and $q$ are defined by $|B_1\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$, $|B_2\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$ where $|B_{1(2)}\rangle$ are mass eigenstates of $B$ meson. The ratio $q/p$ can be written by using the
off-diagonal element of the mass matrix and its non-identity \( q/p \neq 1 \) is the signature of
the CP violation through mixing:

\[
\frac{q}{p} = \sqrt{\frac{M_{12}^2 - \frac{i}{2} \Gamma_{12}}{M_{12}^2 + \frac{i}{2} \Gamma_{12}}}.
\] (7)

The off-diagonal element of the mass matrix is given by the matrix element of the \( \Delta B = 2 \) transition as

\[
\langle B^0|H_{\Delta B=2}^{\text{eff}}|B^0\rangle \equiv M_{12} - \frac{i}{2} \Gamma_{12}.
\] (8)

In SM, the major contribution to this matrix element is obtained from the box diagram
with \( W \)-gauge boson and top quark in the loop. As a result, we obtain:

\[
\frac{q}{p} = V_{tb}^* V_{td} V_{tb} V_{td}^*.
\] (9)

where we ignored terms \( O(\Gamma_{12}/M_{12}) \). Since \( p(\phi K_S) = \frac{A^{SM}(\phi K_S)}{A^{SM}(\phi K_S)} = \frac{V_{tb} V_{td}^*}{V_{tb} V_{td}} = 1 \), the mixing
CP asymmetry in \( B \to \phi K_S \) process is found to be

\[
S_{\phi K_S} = \sin 2\beta.
\] (10)

Therefore, the mixing CP asymmetry in \( B \to \phi K_S \) is same as the one in \( B \to J/\psi K_S \)
process in SM.

In supersymmetric theories, there are new contributions to the mixing parameters
through other box diagrams with gluinos and charginos exchanges. These contributions
to the \( \Delta B = 2 \) transition are often parametrised by [27, 28]

\[
\sqrt{M_{12}/M_{12}^{SM}} \equiv r_d e^{i\theta_d},
\] (11)

where \( M_{12} = M_{12}^{SM} + M_{12}^{SUSY} \). In this case, the ratio of the mixing parameter \( q/p \) can be
written as

\[
\frac{q}{p} = e^{-2i\theta_d} \frac{V_{tb}^* V_{td}^*}{V_{tb} V_{td}}.
\] (12)

Thus, in the framework of SUSY, the mixing CP asymmetry in \( B \to J/\psi K_S \) is modified
as

\[
S_{J/\psi K_S} = \sin(2\beta + 2\theta_d).
\] (13)

In \( B \to \phi K_S \) process, we have to additionally consider the SUSY contributions to the
\( \Delta B = 1 \) transition. The supersymmetric contributions to the \( \Delta B = 1 \) transition comes
from the penguin diagrams with gluinos and charginos in the loop (see Fig.1). We can
parametrise this effect in the same manner [5, 28]:

\[
\frac{A(\phi K_S)}{A^{SM}(\phi K_S)} = S_A e^{i\theta_A},
\] (14)
The Effective Hamiltonian for the $\Delta B = 1$ transitions through penguin process in general can be expressed as

\[
\mathcal{H}_{\text{eff}}^{\Delta B=1} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ \sum_{i=3}^{6} C_i \mathcal{O}_i + C_g O_g \sum_{i=3}^{6} \tilde{C}_i \tilde{\mathcal{O}}_i + \tilde{C}_g \tilde{O}_g \right],
\]

where $A(\phi K_S) = A^{SM}(\phi K_S) + A^{SUSY}(\phi K_S)$. Therefore, we obtain $\mathcal{P}(\phi K_S) = e^{-2i\theta_A}$, hence Eq. (4) leads to

\[
C_{\phi K_S} = 0, \quad S_{\phi K_S} = \sin(2\beta + 2\theta_d + 2\theta_A). \quad (15)
\]

However, this parametrisation is true only when we ignore the so-called strong phase. Since the Belle collaboration observed nonzero value for $C_{\phi K_S}$ [3] we should consider the strong phase in the analysis. In this respect, we reparametrise the SM and SUSY amplitudes as

\[
A^{SM}(\phi K_S) = |A^{SM}| e^{i\delta_{SM}}, \quad A^{SUSY}(\phi K_S) = |A^{SUSY}| e^{i\theta_{SUSY}} e^{i\delta_{SUSY}}, \quad (16)
\]

\[
\overline{A}^{SM}(\phi K_S) = |\overline{A}^{SM}| e^{i\delta_{SM}}, \quad \overline{A}^{SUSY}(\phi K_S) = |\overline{A}^{SUSY}| e^{-i\theta_{SUSY}} e^{i\delta_{SUSY}}, \quad (17)
\]

where $\delta_{SM(SUSY)}$ is the strong phase (CP conserving) and $\theta_{SUSY}$ is the CP violating phase. By using this parametrisation, Eq. (4) leads to

\[
S_{\phi K_s} = \frac{\sin 2\beta + 2 \left( \frac{|A^{SUSY}|}{|A^{SM}|} \right) \cos \delta_{12} \sin(\theta_{SUSY} + 2\beta) + \left( \frac{|A^{SUSY}|}{|A^{SM}|} \right)^2 \sin(2\theta_{SUSY} + 2\beta)}{1 + 2 \left( \frac{|A^{SUSY}|}{|A^{SM}|} \right) \cos \delta_{12} \cos \theta_{SUSY} + \left( \frac{|A^{SUSY}|}{|A^{SM}|} \right)^2}, \quad (18)
\]

\[
C_{\phi K_s} = -\frac{2 \left( \frac{|A^{SUSY}|}{|A^{SM}|} \right) \sin \delta_{12} \sin \theta_{SUSY}}{1 + 2 \left( \frac{|A^{SUSY}|}{|A^{SM}|} \right) \cos \delta_{12} \cos \theta_{SUSY} + \left( \frac{|A^{SUSY}|}{|A^{SM}|} \right)^2}, \quad (19)
\]

where $\delta_{12} \equiv \delta_{SM} - \delta_{SUSY}$. Assuming that the SUSY contribution to the amplitude is smaller than the SM one, we can simplify this formula by expanding it in terms of $|A^{SUSY}|/|A^{SM}|$:

\[
S_{\phi K_s} = \sin 2\beta + 2 \cos 2\beta \sin \theta_{SUSY} \cos \delta_{12} \left| \frac{A^{SUSY}}{A^{SM}} \right|, \quad (20)
\]

\[
C_{\phi K_s} = -2 \sin \theta_{SUSY} \sin \delta_{12} \left| \frac{A^{SUSY}}{A^{SM}} \right|, \quad (21)
\]

where $\mathcal{O}(\left| \frac{A^{SUSY}}{A^{SM}} \right|^2)$ is ignored. However, as can be seen from these formulae that the Belle measurements

\[
S_{\phi K_s} = -0.73 \pm 0.66, \quad (22)
\]

\[
C_{\phi K_s} = -0.56 \pm 0.43 \quad (23)
\]

require large value of $|A^{SUSY}|/|A^{SM}|$. In our analysis, we consider the complete expressions for $S_{\phi K_s}$ and $C_{\phi K_s}$ as given in Eqs. (18) and (19), respectively.

### 3 Effective Hamiltonian for $\Delta B = 1$ transitions

The Effective Hamiltonian for the $\Delta B = 1$ transitions through penguin process in general can be expressed as

\[
\mathcal{H}_{\text{eff}}^{\Delta B=1} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ \sum_{i=3}^{6} C_i \mathcal{O}_i + C_g O_g \sum_{i=3}^{6} \tilde{C}_i \tilde{\mathcal{O}}_i + \tilde{C}_g \tilde{O}_g \right], \quad (24)
\]
\begin{align*}
O_3 &= \bar{s}_\alpha \gamma^\mu L b_\alpha \bar{s}_\beta \gamma^\mu L s_\beta, \\
O_4 &= \bar{s}_\alpha \gamma^\mu L b_\beta \bar{s}_\beta \gamma^\mu L s_\alpha, \\
O_5 &= \bar{s}_\alpha \gamma^\mu L b_\alpha \bar{s}_\beta \gamma^\mu L s_\beta, \\
O_6 &= \bar{s}_\alpha \gamma^\mu L b_\beta \bar{s}_\beta \gamma^\mu R s_\alpha, \\
O_g &= \frac{g_s}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} R^A b_\beta G^A_{\mu\nu}.
\end{align*}

where \( L = 1 - \gamma_5 \) and \( R = 1 + \gamma_5 \). The terms with tilde are obtained from \( C_{i,g} \) and \( O_{i,g} \) by exchanging \( L \leftrightarrow R \). The Wilson coefficient \( C_{i(g)} \) includes both SM and SUSY contributions. In our analysis, we neglect the effect of the operator \( O_\gamma = \frac{\epsilon}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} R b_\alpha F_{\mu\nu} \) and the electroweak penguin operators which give very small contributions. In this paper, we follow the work by Ali and Greub [25] and compute the \( B \to \phi K \) process by including the NLL order precision for the Wilson coefficients \( C_{3-6} \) and at the LL order precision for \( C_g \). The Wilson coefficient at a lower scale \( \mu \approx O(m_b) \) can be extrapolated by

\[ C_i(\mu) = \hat{U}(\mu, \mu_W) C_i(\mu_W) \quad i = 1 \sim 6 \]

where the evolution matrix at NLO order is given by

\[ \hat{U}(\mu, \mu_W) = \hat{U}^{(0)}(\mu, \mu_W) + \frac{\alpha_s}{4\pi} \left( \hat{J} \hat{U}^{(0)}(\mu, \mu_W) - \hat{U}^{(0)}(\mu, \mu_W) \hat{J} \right) \]

where \( \hat{U}^{(0)} \) is obtained by the \( 6 \times 6 \) LO anomalous dimension matrix and \( \hat{J} \) is obtained by the NLO anomalous dimension matrix. The explicite forms of these matrices can be found for example, in [30]. Since the \( O_g \) contribution to \( B \to \phi K_S \) is order \( \alpha_s \) suppressed in the matrix element the Wilson coefficient \( C_g(\mu) \) should include, for consistency, only LO corrections:

\[ C_g(\mu) = \hat{U}^0(\mu, \mu_W) C_g(\mu_W) \]

where \( \hat{U}^0(\mu, \mu_W) \) is obtained by the \( 8 \times 8 \) anomalous dimension matrix of LO.

The anomalous dimension matrix at NLO does depend on regularisation scheme. To avoid this problem, QCD corrections are carefully included in the literature [25]. As a result, the matrix element of \( B^0 \to \phi K^0 \) process is given by the effective Wilson coefficient,

\[ \langle \phi K^0 | \mathcal{H}_{\Delta B=1}^{\text{eff}} | B^0 \rangle \]

where

\[ \mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ \sum_{i=3}^6 C_i^{\text{eff}} O_i + \sum_{i=3}^6 \tilde{C}_i^{\text{eff}} \tilde{O}_i \right] \]

The detailed expression of the effective Wilson coefficient can be found in [25]. The effective Wilson coefficient \( C_i^{\text{eff}}(\tilde{C}_i^{\text{eff}}) \) includes all the QCD corrections mentioned above. We
must emphasizes that these corrections include the contribution from the chromomagnetic type operator given in Eq. (29). Note that the LLO Wilson coefficient $C^{(SM)}_g$ itself is an order of magnitude larger than the others, however it enters as a QCD corrections so that $\alpha_s/(4\pi) \approx 1/50$ suppressed. As a result, the effect of $O_g$ in SM is less than 10% level in the each effective Wilson coefficient $C^{(SM)}_{\text{eff}}$. However, we will show that in supersymmetric theories, the Wilson coefficient for the operator $O_g(\tilde{O}_g)$ is very large and its influence to the effective Wilson coefficients $C^{(SUSY)}_{\text{eff}}$ and $\tilde{C}^{(SUSY)}_{\text{eff}}$ are quite significant.

Employing the naive factorisation approximation [31], where all the colour factor $N$ is assumed to be 3, the amplitude can be expressed as:

$$\mathcal{A}(\phi K) = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ \sum_{i=3}^{6} C^{(\text{eff})}_i + \sum_{i=3}^{6} \tilde{C}^{(\text{eff})}_i \right] \langle \phi K^0 | O_i | B^0 \rangle.$$

The matrix element is given by:

$$\langle \phi K^0 | O_3 | B^0 \rangle = \frac{4}{3} X,$$

$$\langle \phi K^0 | O_4 | B^0 \rangle = \frac{4}{3} X,$$

$$\langle \phi K^0 | O_5 | B^0 \rangle = X,$$

$$\langle \phi K^0 | O_6 | B^0 \rangle = \frac{1}{3} X$$

with

$$X = 2 F_1^{B\rightarrow K} (m_\phi^2) f_\phi m_\phi (p_K \cdot \epsilon_\phi).$$

where $F_1^{B\rightarrow K}$ is the $B - K$ transition form factor and $f_\phi$ is the decay constant of $\phi$ meson. Note that the matrix elements for $O_{i(g)}$ and $\tilde{O}_{i(g)}$ are same for $B \rightarrow \phi K$ process. We use the following values for the parameters appearing in the above equation, $m_\phi = 1.02$ GeV, $f_\phi = 0.233$ GeV, $(p_K \cdot \epsilon_\phi) = \frac{m_B}{m_\phi} \sqrt{\left[ \frac{1}{2m_B} \left( m_B^2 - m_K^2 + m_\phi^2 \right) \right]^2 - m_\phi^2} \approx 13$ GeV, and $F_1^{B\rightarrow K} = 0.35$ GeV [32]. Finally, we discuss on the matrix elements of the chromomagnetic operator $O_g$ which is given by:

$$\langle \phi K^0 | O_g | B^0 \rangle = -\frac{\alpha_s m_b}{\pi q^2} \left( \bar{\sigma} \gamma_\mu g(1 + \gamma_5) \frac{\lambda^{A \beta}}{2} b_\beta \right) \left( \bar{\sigma} \gamma_\mu \frac{\lambda^{A \beta}}{2} \right).$$

where $q^\mu$ is the momentum carried by the gluon in the penguin diagram. As discussed above, this contribution is already included in $C^{(eff)}_{3-6}$. In fact, this is possible only when the matrix element of $O_g$ is written in terms of the matrix element of $O_{3-6}$. It was achieved by using an assumption [25]:

$$q^\mu = \sqrt{\langle q^2 \rangle} \frac{P_0^\mu}{m_b}$$

where $\langle q^2 \rangle$ is an averaged value of $q^2$. We treat $\langle q^2 \rangle$ as an input parameter in a range of $m_b^2/4 < \langle q^2 \rangle < m_b^2/2$. As we will see in the next section, the SUSY contribution to the $S_{\phi K_S}$ is quite sensitive to the value of $\langle q^2 \rangle$. 

7
4 Supersymmetric contributions to \( B \to \phi K_S \) decay

As advocated above, the general amplitude \( \overline{A}(\phi K) \) can be written as

\[
\overline{A}(\phi K) = \overline{A}^{\text{SM}}(\phi K) + \overline{A}^{\tilde{g}}(\phi K) + \overline{A}^{\tilde{\chi}^\pm}(\phi K),
\]

where \( \overline{A}^{\text{SM}}, \overline{A}^{\tilde{g}}, \) and \( \overline{A}^{\tilde{\chi}^\pm} \) refer to the SM, gluino, and chargino contributions, respectively. In our analysis we consider only the gluino exchanges through \( \Delta B = 1 \) penguin diagrams which gives the dominant contribution to the amplitude \( \overline{A}^{\text{SUSY}}(\phi K) \). In Fig. 1 we exhibit the leading diagrams for \( B \to \phi K_S \) decay. At the first order in the mass insertion approximation, the gluino contributions to the Wilson coefficients \( C_{i,g} \) at SUSY scale \( M_S \) are given by

\[
C_3(M_S) = \frac{\sqrt{2} \alpha_S^2}{G_F V_{tb} V_{ts}^* m_q^2} (\delta^{dLL}_{23}) \left[ -\frac{1}{9} B_1(x) - \frac{5}{9} B_2(x) - \frac{1}{18} P_1(x) - \frac{1}{2} P_2(x) \right],
\]

\[
C_4(M_S) = \frac{\sqrt{2} \alpha_S^2}{G_F V_{tb} V_{ts}^* m_q^2} (\delta^{dLL}_{23}) \left[ -\frac{7}{3} B_1(x) + \frac{1}{3} B_2(x) + \frac{1}{6} P_1(x) + \frac{3}{2} P_2(x) \right],
\]

\[
C_5(M_S) = \frac{\sqrt{2} \alpha_S^2}{G_F V_{tb} V_{ts}^* m_q^2} (\delta^{dLL}_{23}) \left[ -\frac{10}{9} B_1(x) + \frac{1}{18} B_2(x) - \frac{1}{18} P_1(x) - \frac{1}{2} P_2(x) \right],
\]

\[
C_6(M_S) = \frac{\sqrt{2} \alpha_S^2}{G_F V_{tb} V_{ts}^* m_q^2} (\delta^{dLL}_{23}) \left[ -\frac{2}{3} B_1(x) + \frac{7}{6} B_2(x) + \frac{1}{6} P_1(x) + \frac{3}{2} P_2(x) \right],
\]
$$C_g(M_S) = -\frac{\sqrt{2}\alpha_S\pi}{G_F V_{tb} V_{ts}^* m_t^2} \left[ (\delta^d_{LL})_{23} \left( \frac{1}{3} M_3(x) + 3M_4(x) \right) + (\delta^d_{LR})_{23} \frac{m_\tilde{g}}{m_b} \left( \frac{1}{3} M_3(x) + 3M_2(x) \right) \right],$$

and the coefficients $\tilde{C}_{i,g}$ are obtained from $C_{i,g}$ by exchanging $L \leftrightarrow R$. The functions appear in these expressions can be found in Ref.[33] and $x = m_b^2/m_t^2$. As in the case of the SM, the Wilson coefficients at low energy $C_{i,g}(\mu)$, $\mu \simeq O(m_b)$, are obtained from $C_{i,g}(M_S)$ by using the evolution matrix at NLO given in eq.(31).

From the above expressions for the gluino contributions to the Wilson coefficients, it is remarkable to notice that the LR and RL effects in $C_g$ and $\tilde{C}_g$ are enhanced due to the factor $m_\tilde{g}/m_b$. Therefore, in the case of $\Delta S = 1$ transition (and in particular, the direct CP violation parameter in the kaon system $\varepsilon'/\varepsilon$ [34]), this enhancement makes the LR and RL mass insertions natural candidates to saturate the experimental results of $S_{\phi K_S}$. However, we should note that the experimental results for the branching ratio of the decay $B \to X_S \gamma$ impose constraints on the absolute value of the mass insertions $(\delta^d_{AB})_{23}$, with $A, B = (L, R)$ [33, 35]. These constraints are very week on the $LL$ and $RR$ mass insertions and the only limits we have on these mass insertions, $|\langle \delta^d_{LL,RR} \rangle_{23}| < 1$, which arise from their definition. The $LR$ and $RL$ mass insertions are more constrained by the decay $B \to X_S \gamma$, for instance with $m_\tilde{q} \simeq m_\tilde{q} \simeq 500$ GeV $|\langle \delta^d_{LR,RL} \rangle_{23}| \simeq 1.6 \times 10^{-2}$. Nevertheless, as we will show below, in order to have significant SUSY contributions to $S_{\phi K_S}$, one just needs $|\langle \delta^d_{LR,RL} \rangle_{23}|$ to be of that order.

As shown in Eq. (20), the deviation of $S_{\phi K_S}$ from $\sin 2\beta$ is governed by the size of $|A_{SUSY}^{SM}|$. Thus we start our analysis by discussing the gluino contribution to $|A_{SUSY}^{SM}|$. We choose the input parameters as

$$m_\tilde{q} = 500\text{GeV}, \ x = 1, \ q^2 = m_b^2/2, \ \mu = 2\text{GeV} \ (45)$$

then, we obtained

$$\frac{A_{SUSY}^{SM}}{A_{SM}} \simeq 0.25 \ (\delta^d_{LL})_{23} + 55.4 \ (\delta^d_{LR})_{23} + 55.4 \ (\delta^d_{RL})_{23} + 0.02 \ (\delta^d_{RR})_{23}. \ (46)$$

The largest theoretical uncertainty comes from the choice of $q^2$. We find that the smaller values of $q^2$ enhance the coefficients of each mass insertions and for the minimum value $q^2 = m_b^2/4$ gives

$$\frac{A_{SUSY}^{SM}}{A_{SM}} \simeq 0.43 \ (\delta^d_{LL})_{23} + 97.4 \ (\delta^d_{LR})_{23} + 97.4 \ (\delta^d_{RL})_{23} + 0.027 \ (\delta^d_{RR})_{23}. \ (47)$$

Using the constraints for each mass insertions described above, we obtain the maximum contribution from the individual mass insertions by setting the remaining three mass insertions to be zero:

$$\frac{A_{LL}^{SUSY}}{A_{SM}} < 0.43, \quad \frac{A_{LR(RL)}^{SUSY}}{A_{SM}} < 1.56, \quad \frac{A_{RR}^{SUSY}}{A_{SM}} < 0.027 \ (48)$$
It is worth mentioning that \((\delta_{LR}^d)_{23}\) and \((\delta_{RL}^d)_{23}\) contribute to \(S_{\phi K}\) with the same sign, unlike their contributions to \(\varepsilon'/\varepsilon\). Therefore, in SUSY model with \((\delta_{LR}^d)_{23} \simeq (\delta_{RL}^d)_{23}\), we will not have the usual problem of the severe cancellation between their contributions, but we will have a constructive interference which enhances the SUSY contribution to \(S_{\phi K}\).

Now using these maximum values, let us investigate whether any one of the mass insertions can accommodate the observed large deviation between \(S_{J/\psi K}\) and \(S_{\phi K}\). As can be seen from Eq. (20), a choice of the strong phase \(\cos \delta_{12} = \pm 1\) gives the largest deviation between \(S_{\phi K}\) and \(\sin 2\beta\). Inputting the measured central value of \(\beta\) and \(\cos \delta_{12} = \pm 1\), into the full formula in Eq. (18), we obtain the result for the \((\delta_{LL}^d)_{23}\) term:

\[
S_{\phi K} = 0.73 \pm 0.86 \sin(\arg(\delta_{LL}^d)_{23} + 0.818) + 0.185 \sin(2 \arg(\delta_{LL}^d)_{23} + 0.818) \\
1.185 \pm 0.86 \cos(\arg(\delta_{LL}^d)_{23})
\] (50)

Then the minimum value of \(S_{\phi K}\) is obtained by \(\sin(\arg(\delta_{LL}^d)_{23}) = \mp 0.90\) as

\[
S_{\phi K} = -0.071.
\] (50)

We find that if the experimental value for \(S_{\phi K}\) remains as small as the current central value, the SUSY models with \(LL\) mass insertion can not provide an explanation for this result.

The \((\delta_{RR}^d)_{23}\) contribution is much smaller due to small value of \(A_{SUSY}/A_{SM}\). The same condition as the case for \((\delta_{LL}^d)_{23}\) gives

\[
S_{\phi K} = 0.73 \pm 0.054 \sin(\arg(\delta_{RR}^d)_{23} + 0.818) + 0.00073 \sin(2 \arg(\delta_{RR}^d)_{23} + 0.818) \\
1.00 \pm 0.054 \cos(\arg(\delta_{RR}^d)_{23})
\] (51)

and the minimum value is obtained with \(\sin(\arg(\delta_{RR}^d)_{23}) = \mp 1.00\) as

\[
S_{\phi K} = 0.69.
\] (51)

Finally, we show that the \((\delta_{LR(RL)}^d)_{23}\) contribution can deviate significantly \(S_{\phi K}\) from \(\sin 2\beta\) as much as the experiments observed. The mixing CP asymmetry is expressed as

\[
S_{\phi K} = 0.73 \pm 1.95 \sin(\arg(\delta_{LR}^d)_{23} + 0.818) + 0.95 \sin(2 \arg(\delta_{LR}^d)_{23} + 0.818) \\
1.95 \pm 1.95 \cos(\arg(\delta_{LR}^d)_{23})
\] (53)

and the minimum value is obtained with \(\sin(\arg(\delta_{LR}^d)_{23}) = \mp 0.075\) as

\[
S_{\phi K} = -1.
\] (53)

In Fig.2, we present plots for the phase of \((\delta_{LL}^d)_{23}\), \((\delta_{RR}^d)_{23}\) and \((\delta_{LR}^d)_{23}\) versus the mixing CP asymmetry \(S_{\phi K}\) for \(\cos \delta_{12} = 1\). We choose the three values of the magnitude of these mass insertions within the bounds from the experimental limits in particular, from \(B \to X_s\gamma\). Each plot shows a contribution from an individual mass insertion by
setting the other three to be zero. As can be seen from these plots, the \( LR \) (same for \( RL \)) gives the largest contribution to \( S_{φK_S} \). The \( RR \) contribution is negligible even if the magnitude of the \( (δ^{d}_{RR})_{23} \) is of order one. In order to have a sizable effect from the \( LL \), the magnitude of \( (δ^{d}_{LL})_{23} \) has to be order one and furthermore, the imaginary part needs to be as large as the real part. In any case, it is very difficult to give negative value of \( S_{φK_S} \) from \( (δ^{d}_{LL})_{23} \) mass insertion. On the contrary, even if we reduce the magnitude of \( (δ^{d}_{LR})_{23} \) to the half of its maximum value, \( S_{φK_S} \) can still reach to a negative value. We also find that in the case of \( |(δ^{d}_{LR})_{23}| = 0.01 \), the minimum value of \( S_{φK_S} \) can be achieved without large imaginary part.

At this stage, we should comment on the impact of the strong phase. So far, we have only considered the cases where the strong phase \( δ_{12} \) is given by \( \cos δ_{12} = ±1 \) so that the direct CP asymmetry \( C_{φK_S} \) was identically zero. By using the expanded formulae for \( S_{φK_S} \) and \( C_{φK_S} \) in Eqs. (20) and (21), we find that for any value of \( δ_{12} \), the plot of \( S_{φK_S} \)
$S_{\phi K_S}$ becomes an ellipse with its size proportional to $\sin \theta_{SUSY}$:

$$\frac{(S_{\phi K_S} - \sin 2\beta)^2}{\cos^2 2\beta \left(2 \sin \theta_{SUSY} \frac{|A_{SUSY}|}{|A_{SM}|}\right)^2} + \frac{C_{\phi K_S}^2}{\left(2 \sin \theta_{SUSY} \frac{|A_{SUSY}|}{|A_{SM}|}\right)^2} = 1 \quad (55)$$

In Fig. 3, we depict an example of the plot with $|A_{SUSY}|/|A_{SM}| \simeq 0.5$. Since we used the full formulae in Eqs. (18) and (19) to create this figure, different $\theta_{SUSY}$ does not give precisely rescaled ellipse. Nevertheless we can see the qualitative feature. The strong phase $\delta_{12} = 0$ corresponds to the point at the right most tip of the ellipse. As $\delta_{12}$ increases, it runs anti-clockwise and finishes a round when $\delta_{12} = 2\pi$.

![Figure 3: The mixing CP asymmetry $S_{\phi K_S}$ versus the direct CP symmetry $C_{\phi K_S}$ for strong phase $\delta_{12} \in [0, 2\pi]$ and five representative values of $\arg([\delta_{LR}^d]_{23})$.](image)

5 CP asymmetry $S_{\phi K_S}$ in explicit SUSY models

In this section we study the CP asymmetry of the $B \rightarrow \phi K_S$ in some specific SUSY models. We consider the minimal supersymmetric standard model (MSSM) (where minimal number of superfields is introduced and $R$ parity is conserved) with the following soft SUSY breaking terms

$$V_{SB} = m_{0a}^2 \phi_a^* \phi_a + \epsilon_{ab} (A_{ij}^{u} Y_{ij}^{u} H_2^{b \tilde{q}_{L_i}^{a} \tilde{u}_{R_j}^{*}} + A_{ij}^{d} Y_{ij}^{d} H_1^{a \tilde{q}_{L_i}^{a} \tilde{d}_{R_j}^{*}} + A_{ij}^{l} Y_{ij}^{l} H_1^{a \tilde{q}_{L_i}^{a} \tilde{e}_{R_j}^{*}} + B_{ij} H_1^{a} H_2^{b} + H.c.) - \frac{1}{2}(m_{3}^{-} g \tilde{g} + m_{2}^{-} W^{a} \tilde{W}^{a} + m_{1}^{-} \tilde{B} \tilde{B}), \quad (56)$$
where $i, j$ are family indices, $a, b$ are $SU(2)$ indices, $\epsilon_{ab}$ is the $2 \times 2$ fully antisymmetric tensor, with $\epsilon_{12} = 1$, and $\phi$ denotes all the scalar fields of the theory. We start with minimal supergravity model and then we consider general SUSY models with non-universal soft breaking terms. We also discuss the impact of the type of Yukawa couplings on the prediction of the later model.

### 5.1 minimal supergravity

In a minimal supergravity framework, the soft SUSY breaking parameters are universal at GUT scale, and we can write

\[
m^2_{0\alpha} = m_0^2, \quad m_i = m_{1/2}, \quad A_{ij}^a = A_0 e^{i\phi_A}.
\]  

In this model, there are only two physical phases: $\phi_A = \arg(A^* m_{1/2})$ and $\phi_\mu = \arg(\mu m_{1/2})$. In order to have EDM values below the experimental bounds, and without forcing the SUSY masses to be unnaturally heavy, the phases $\phi_A$ and $\phi_\mu$ must be at most of order $10^{-1}$ and $10^{-2}$ respectively [21].

It is clear that this class of models, where the SUSY phases are constrained to be very small and the Yukawa couplings are the main source of the flavour structure, can never generate a sizable contribution to the CP violating processes. As we will show, also our result for the CP asymmetry $S_{\phi K_S}$ confirms this conclusion and motivates the interest in supersymmetric models with non-universal soft breaking terms, if supersymmetry is meant to play any role in explaining the discrepancy between $S_{J/\psi K_S}$ and $S_{\phi K_S}$.

In fact, we find that even if we ignore the bounds from the EDM, and allow large values for SUSY phases, $\phi_A, \mu \simeq \pi/2$, still the SUSY contribution to $S_{\phi K_S}$ is negligible. This suppression is mainly due to the universality assumption of the soft breaking terms. For instance, with $m_{1/2} \simeq m_0 \simeq A_0 \simeq 200$ GeV we find the following values of the relevant mass insertions:

\[
(\delta^d_{LL})_{23} \simeq 0.009 + i 0.001,
\]

\[
(\delta^d_{RR})_{23} \simeq -2.1 \times 10^{-7} - i 2.5 \times 10^{-8},
\]

\[
(\delta^d_{LR})_{23} \simeq -2.5 \times 10^{-5} - i 1.9 \times 10^{-5}.
\]

Clearly these values are much smaller than the corresponding values mentioned in the previous section and the model give negligible contributions to the CP asymmetry $S_{\phi K_S}$. Indeed, we find that the total $S_{\phi K_S}$ in this example is given by $S_{\phi K_S} = 0.729$, which is essentially the value of $S_{J/\psi K_S}$.

### 5.2 SUSY models with non-universal soft terms

Now we consider SUSY models with non-universal soft terms. In particular, we focus on the models with non-universal $A$-terms in order to enhance the values of $(\delta^d_{LR})_{23}$ and
\( (\delta^d_{RL})_{23} \), which may give the dominant contributions to the CP asymmetry \( S_{\phi K_S} \). However, non-observation of EDMs leads to restrictive constraints on the non-degenerate \( A \)-terms and only certain patterns of flavour are allowed, such as the Yukawa and \( A \)-terms are Hermitian \cite{22}, or the \( A \)-terms are factorisable, \( i.e., \) \( (Y^A)^{ij} = A Y_i A \). \cite{34}. In the case of factorisation, the mass insertion \( (\delta^d_{LR})_{11} \) is suppressed by the ratio \( m_d/m_{\tilde{q}} \). Here we will consider this scenario with the following trilinear structure:

\[
A = m_0 \begin{pmatrix}
  a & a & a \\
  b & b & b \\
  c & c & c \\
\end{pmatrix}.
\]

(61)

As pointed out in Ref.\cite{23}, in the case of non-universal soft breaking terms, the type of the Yukawa couplings (hierarchical or nearly democratic) plays an important role and has significant impact on the predictions of these models. If we consider the standard example of hierarchical quark Yukawa matrices,

\[
Y_u = \frac{1}{v \sin \beta} \text{diag} (m_u, m_c, m_t),
\]

\[
Y_d = \frac{1}{v \cos \beta} K \text{diag} (m_u, m_c, m_t) K^+, \quad (62)
\]

where \( K \) is the CKM matrix, the relevant \( LR \) mass insertion is given by

\[
(\delta^d_{LR})_{23} \simeq \frac{v \cos \beta}{m_{\tilde{q}}^2} \left( K^+ Y^A_d K \right)_{ij} \quad (63)
\]

where \( (Y^A_d)^{ij} = Y^{d}_{ij} A^{d}_{ij} \). It is clear that the dominant contribution to this mass insertion is given by the term \( K_{22}(Y^A_d)_{23}K_{33}^+ \) which is still suppressed by the small entry \( Y^d_{23} \). The non-universality of the squarks can enhance the \( LL \) and \( RR \) mass insertions, however this non-universality is very constrained by the experimental measurements of \( \Delta M_K \) and \( \varepsilon_K \). Therefore, with the hierarchical Yukawa couplings we find that the typical values of the relevant mass insertions are at least two order of magnitude below the required values so that the splitting between the CP asymmetries \( S_{J/\psi K_S} \) and \( S_{\phi K_S} \) is again small.

Now we consider the same SUSY model but with converting the above hierarchical Yukawa matrices to democratic ones, which can be obtained by a unitary transformation. As emphasised in Ref.\cite{23} that these new Yukawa textures (and their diagonalising matrices \( S^{u,d}_{L,R} \)) have large mixing, which has important consequences in the SUSY results. Thus, the element \( (Y^A_d)^{ij} \) has no suppression factor as before and the magnitude of \( (\delta^d_{LR})_{23} \) can be of the desired order. As a numerical example, for \( m_0 = m_{1/2} = 200 \text{ GeV} \), \( i.e., m_{\tilde{q}} \simeq m_{\tilde{g}} \simeq 500 \text{ GeV} \) and assuming that \( |A_{ij}| \in [m_0, 4m_0] \) while the phases of the \( A \)-terms are chosen such that the bound of the EDMs are satisfied, we find that it is quite natural to obtain the following values of the mass insertion \( (\delta^d_{LR})_{23} \): \(|(\delta^d_{LR})_{23}| \simeq 0.005 \) and \( \text{Arg}[(\delta^d_{LR})_{23}] \simeq 1.2 \) which leads to \( S_{\phi K_S} \simeq -0.2 \).
6 Conclusions

In this paper, we have studied the supersymmetric contributions to the CP asymmetry of $B \to \phi K_S$ process. Using the mass insertion approximation method, we have derived model independent limits for the mixing CP asymmetry $S_{\phi K_S}$. We found that the $LR$ mass insertion gives the largest contribution to $S_{\phi K_S}$, while the $LL$ contributions are small and $RR$ contributions are negligible. Therefore, if the deviation between $S_{\phi K_S}$ and $S_{J/\psi K_S}$ observed by the B–factory experiments (Belle and BaBar) remains as large as its present central value, the SUSY models with large ($\sim 10^{-3}$) $LR$ mass insertions will be the only candidate, in this class of models, which can provide a consistent explanation for these measurements.

The Belle collaboration observed non–vanishing direct CP asymmetry $C_{\phi K_S}$ which can be obtained only by simultaneous non–vanishing of strong phase and SUSY CP violating phase. Thus, we studied the impact of the strong phase in our results for $S_{\phi K_S}$ and we have provided a correlation between $S_{\phi K_S}$ and $C_{\phi K_S}$.

We also applied our results to the minimal supergravity model and SUSY models with non–universal soft terms with two types of Yukawa couplings, namely hierarchal and nearly democratic Yukawa textures. We showed that only in SUSY models with large Yukawa mixing, the $LR$ mass insertions could be enhanced and reach the desired values to give significant contributions to the CP asymmetry $S_{\phi K_S}$. This result motivates the interest in SUSY models with non–universal soft terms and also shed the light on the type of the Yukawa flavour structure.

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References


