Charged pion pair production by two photons from a chiral sum rule

N. F. Nasrallah

Faculty of Science, Lebanese University, Tripoli, Lebanon

and K. Schilcher

Institut für Physik, Johannes-Gutenberg-Universität, Staudinger Weg 7, D-55099 Mainz, Germany

Abstract: The cross-section for a charged pion pair production by two photons is evaluated by using the low energy expression previously obtained from current algebra and PCAC which involves an integral over vector and axial-vector spectral functions. Data on the latter obtained from \( \tau \)-decay as measured by the ALEPH collaboration is then inserted in the integral which is appropriately modified in order to eliminate contributions near the cut in the duality contour integral. The experimental behavior of the cross-section for \( \gamma \gamma \to \pi^+ \pi^- \) is well reproduced at low energies.
Two photon processes for charges pion pair production have recently become a subject of experimental investigation by the use of $e^+e^-$ colliding beams. Measurements of the cross-section for the two-photon production of charged pion and kaon pairs have been performed by the Mark II[1] and TPC/Two-Gamma[2] collaborations and, more recently, by the CLEO[3] collaboration.

On the theoretical side pion pair production by photons at low energies may is best described by of chiral perturbation theory (CHPT)[4]. The Born term dominates the behavior of the cross-section at low energies. Next-to-leading corrections (one loop graphs) change the result very little[5]. Two-loop contributions to the amplitude have recently been presented by Burgi[6]. As is well known, this elaborate calculation involves new unknown coupling constants (3 in this case), which were estimated in[6] by use of resonance saturation.

An alternative approach based on dispersion theory has also reached a considerable degree of sophistication [7]. In this note we want to present a simple analysis of the process $\gamma\gamma \rightarrow \pi^+\pi^-$ based on a chiral sum rule proposed by Terazawa[8]. This sum rule was an early application of current-algebra and the partially conserved axial-vector current (PCAC) hypothesis. The method of ref.[8] gives an elegant expression of the cross-section for $\gamma\gamma \rightarrow \pi^+\pi^-$ in terms of vector and axial-vector spectral function integrals. An update of the calculation presented in ref.[8] appeared recently[9] in which $\rho$ and $A_1$ meson pole dominance of the spectral function integrals was used and the result was expressed in terms of an effective vector meson mass $m_v$ which was determined by a best fit to be $m_v \simeq 1.4 GeV$, a value somewhat larger than the $\rho$-mass of 0.77 GeV. This result is just a manifestation of the fact that the assumption of vector meson dominance is notoriously unreliable if, as is the case in Terazawa’s sum rule, the small difference of two large spectral functions enters.

In the meantime the ALEPH[10] collaboration has obtained detailed and precise experimental information about the vector and axial-vector spectral function from $\tau$ decay, which can be used in Terazawa’s sum rule. We shall argue that the sum rule has to be appropriately modified in order to minimize the contribution of the contour near the cut in the duality integral.

The PCAC hypothesis and current algebra yield the following expression for the differential cross-section of the reaction $\gamma\gamma \rightarrow \pi^+\pi^-$ in terms of the scattering angle $\theta$

$$\frac{d\sigma}{d(cos\theta)} = \frac{\pi\alpha^2}{s} \left(1 - \frac{4m_n^2}{s}\right)^{1/2} F^2(s/2)$$

(1)

where $s$ is the invariant mass-squared of the photoproduced pion pair and $F(s)$ is the pion structure function

$$F(s) = \frac{1}{4\pi f_\pi^2} \int_0^\infty dt \frac{t}{(s+t)} (\rho_V(t) - \rho_A(t))$$

(2)

where $f_\pi = 92.4$ MeV is the pion decay constant and where $\rho_V$ and $\rho_A$ are the spectral functions of the vector and axial-vector currents respectively. We subdivide the interval into two regions, one from threshold to $R = 3 GeV^2$, where the ALEPH data are measured with good precision, and a tail from $R$ to $\infty$. As there is no singularity for $|s| > R$, eq.(2) can be written by Cauchy’s theorem as

$$F(s) = \frac{1}{4\pi f_\pi^2} \int_0^R dt \frac{t}{(s+t)} (\rho_V(t) - \rho_A(t)) - \frac{1}{2i} \frac{1}{4\pi f_\pi^2} \oint dt \frac{t}{(s+t)} (\Pi_V(t) - \Pi_A(t))$$

(3)

where $\rho(t) = \text{Im}\Pi(t)$ and the second integral is extended over a circle of radius $R$. For the first integral the ALEPH data can be used, and in the second integral one is tempted to use the QCD expressions for $\Pi_{V,A}$. From a study of the Weinberg sum rules [13] it is known that the QCD expressions cannot be trusted in the vicinity of the positive real axis for $t \simeq R$.  

2
In order to minimize the uncertainty introduced by the inadequate QCD expression near the positive real axis we make use of the second Weinberg sum rule[11] which holds in QCD

\[
\frac{1}{4\pi f_\pi^2} \int_0^\infty dt \quad t(\rho_V(t) - \rho_A(t)) = 0 .
\] (4)

This allows us to write

\[
F(s) = \frac{1}{4\pi^2 f_\pi^2} \int_0^R dt \quad t(\frac{1}{s+t} - \frac{1}{s+R})[\rho_V(t) - \rho_A(t)]
\] (5)

\[
+ \frac{1}{4\pi^2 f_\pi^2} \int_R^\infty dt \quad t(\frac{1}{s+t} - \frac{1}{s+R})[\rho_V^{QCD}(t) - \rho_A^{QCD}(t)]
\] (6)

The second integral above extends to infinity but the integrand vanishes rapidly with increasing \(t\). The contribution of the tail can be estimated by using the QCD result [12]

\[
\rho_V^{QCD} - \rho_A^{QCD} = \frac{32\pi}{9} \alpha_s(t) \langle \alpha_s(\bar{q}q)^2 \rangle \frac{1}{t^3}
\] (7)

Using the standard value \(\langle \alpha_s(\bar{q}q)^2 \rangle \sim 1.0 \times 10^{-4} GeV^6\) shows that this contribution is negligible.

In order to compare with the experiment we have to make use of the fact that the measured values of the scattering angle are limited to \(|\cos \theta| \leq 0.6\). The total cross-section for the process \(\gamma\gamma \to \pi^+\pi^-\) is thus

\[
\sigma(|\cos \theta| \leq 0.6) = \frac{\pi\alpha^2}{s} (1 - \frac{4m_\pi^2}{s})^{1/2} |F(s/2)|^2 \times 1.2
\] (8)

This expression for \(\gamma\gamma \to \pi^+\pi^-\) cross section the is plotted in Fig. (1) where the data are also represented. From the figure we see that a simple analysis based on the chiral sum rule of Terazawa combined with the ALEPH data on vector and axial-vector spectral functions yields a consistent description of the low energy behavior of the photo production of charged pions.
References