Abstract. We review how RHIC is expected to deepen our understanding of the spin structure of longitudinally polarized nucleons. After briefly outlining the current status of spin-dependent parton densities and pointing out open questions, we focus on theoretical calculations and predictions relevant for the RHIC spin program. Estimates of the expected statistical accuracy for such measurements are presented, taking into account the acceptance of the RHIC detectors.

Exploring Nucleon Spin Structure in Longitudinally Polarized Collisions

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I LESSONS FROM POLARIZED DEEP INELASTIC SCATTERING

Before reviewing the prospects for spin physics at the BNL-RHIC we briefly turn to longitudinally polarized deep-inelastic scattering (DIS) and what we have learned from more than twenty years of beautiful data [1]. To next-to-leading order (NLO) in the strong coupling $\alpha_s$, the DIS structure function $g_1$, which parametrizes our ignorance about the nucleon spin structure, can be expressed as

$$g_1(x, Q^2) = \frac{1}{2} \sum_{q=u,d,s} e_q^2 \left[ (\Delta q + \Delta \bar{q}) \otimes \left( 1 + \frac{\alpha_s}{2\pi} \Delta C_q \right) + \frac{\alpha_s}{2\pi} \Delta g \otimes \Delta C_g \right] (x, Q^2) .$$

(1)

The symbol $\otimes$ denotes the usual convolution, and $\Delta C_{q,g}$ are the perturbatively calculable coefficient functions, which are known even up to next-to-next-to-leading order [2]. The $\Delta f$, $f = (q, \bar{q}, g)$, are the spin-dependent parton distributions, defined as

$$\Delta f(x, \mu) \equiv f^+(x, \mu) - f^-(x, \mu) .$$

(2)
FIGURE 1. Available information on $g_1(x, Q^2)$ as collected by fixed-target experiments [1] compared to results of a typical NLO QCD fit (solid lines). The indicated rectangular and triangular regions contain data which would not pass kinematical cuts of $Q^2 > 4 \text{ GeV}^2$ and $W^2 > 10 \text{ GeV}^2$, respectively, usually imposed in all fits to unpolarized DIS data.

where $f^+ (f^-)$ is the number density of a parton type $f$ with helicity “+” (“−”) in a proton with positive helicity, carrying a fraction $x$ of the proton’s momentum. Once they are known at some initial scale $\mu_0$, their scale $\mu$ dependence is governed by a set of evolution equations

$$
\mu \frac{d}{d\mu} \left( \frac{\Delta q(x, \mu)}{\Delta g(x, \mu)} \right) = \left( \frac{\Delta P_{qq}}{\Delta P_{qg}} \Delta P_{gq} \right) \otimes \left( \frac{\Delta q}{\Delta g} \right)(x, \mu) . \tag{3}
$$

So far, the spin-dependent $j \rightarrow i$ splitting functions entering these evolution equations have been calculated up to NLO accuracy [3].

Figure 1 compares the available information on $g_1(x, Q^2)$ for DIS off a proton target to results of a typical NLO QCD fit. From such types of analyses a pretty good knowledge of certain combinations of different quark flavors has emerged, and it became clear that quarks contribute only a small fraction to the proton’s spin. However, there is still considerable lack of knowledge regarding the polarized gluon density $\Delta g$, which is basically unconstrained by present data, the separation of quark and antiquark densities and of different flavors, and the orbital angular momentum of quarks and gluons inside a
nucleon. With the exception of orbital angular momentum RHIC can address all of these questions as will be demonstrated in the following [4].

There is also an important difficulty when analyzing polarized DIS data in terms of spin-dependent parton densities: compared to the unpolarized case the presently available kinematical coverage in \( x \) and \( Q^2 \) and the statistical precision of polarized DIS data are much more limited [1]. As a consequence, one is forced to include data into the fits from \((x, Q^2)\)-regions where corresponding fits of unpolarized leading-twist parton densities start to break down, see Fig. 1. Data from RHIC, taken at “resolution” scales \( \mu \) where perturbative QCD and the leading-twist approximation are supposed to work, can shed light on the possible size of unwanted higher-twist contributions in presently available sets of polarized parton distributions.

II SPIN PHYSICS AT RHIC

A Prerequisites

The QCD-improved parton model has been successfully applied to many high energy scattering processes. The predictive power of perturbative QCD follows from the universality of the parton distribution and fragmentation functions which is based on the factorization theorem. To be specific, let us consider the inclusive production of a hadron \( H \), e.g., a pion, in longitudinally polarized pp collisions at a c.m.s. energy \( \sqrt{S} \). The cross section can be written in a factorized form as a convolution of perturbatively calculable partonic cross sections \( d\hat{\sigma}_{ab} \) describing the hard scattering \( ab \rightarrow cX \) and appropriate combinations of parton densities \( \Delta f_{a,b} \) and fragmentation functions \( D^H_c \) embodying the non-perturbative physics:

\[
\frac{d\Delta \sigma^H}{d\Gamma} = \sum_{abc} \int dx_a dx_b dz \Delta f_a(x_a, \mu) \Delta f_b(x_b, \mu) \frac{d\hat{\sigma}_{ab}^c(x_a, x_b, z, S, \Gamma, \mu)}{d\Gamma} D^H_c(z, \mu).
\]

(4)

Here, \( \Gamma \) stands for any appropriate set of kinematical variables like the transverse momentum \( p_T \) and/or rapidity \( y \) of the observed hadron. The \( D^H_c \) are the parton-to-hadron fragmentation functions. Their scale \( \mu \)-dependence is governed by a set of equations very similar to (3). The factorization scale \( \mu \), introduced on the r.h.s. of (4), separates long- and short-distance phenomena. \( \mu \) is completely arbitrary but usually chosen to be of the order of the scale characterizing the hard interaction, for instance \( p_T \). Since the l.h.s. of (4) has to be independent of \( \mu \) (and other theoretical conventions), any residual dependence of the r.h.s. on the actual choice of \( \mu \) gives an indication of how well the theoretical calculation is under control and can be trusted. In particular, leading order (LO) estimates suffer from a strong, uncontrollable
scale dependence and hence are not sufficient for comparing theory with data. Figure 2 shows a typical factorization scale dependence for various “high-\(p_T\)” processes and experiments as a function of \(p_T\). Clearly, the situation is only acceptable at collider experiments where one can easily reach \(p_T\) values in excess of 5 GeV. \(p_T\) values of the order of 1-2 GeV, accessible at fixed-target experiments, are not sufficient to provide a large enough scale \(\mu\) for performing perturbative calculations reliably. For simplicity we have not distinguished between renormalization and initial/final-state factorization scales in (4) which can be chosen differently.

FIGURE 2. Typical factorization scale dependence for various “high-\(p_T\)” processes and experiments as a function of \(p_T\). Shown is the cross section ratio for two choices of scale, \(p_T\) and \(p_T/2\).

In practice, spin experiments do not measure the polarized cross section \(d\Delta\sigma/d\Gamma\) itself, but the longitudinal spin asymmetry, which is given by the ratio of the polarized and unpolarized cross sections, i.e., for our example above, Eq. (4), it reads

\[ A_{LL}^H \equiv \frac{d\Delta\sigma^H/d\Gamma}{d\sigma^H/d\Gamma} . \]  

The unpolarized cross section \(d\sigma^H/d\Gamma\) is given by Eq. (4) with all polarized quantities replaced by their unpolarized counterparts. At RHIC one can also study doubly transverse spin asymmetries [4] but here we will focus on longitudinal polarization only.

B Accessing \(\Delta g\)

The main thrust of the RHIC spin program [4] is to pin down the so far elusive gluon helicity distributions \(\Delta g(x, \mu)\). The strength of RHIC is the
possibility to probe $\Delta g(x, \mu)$ in a variety of hard processes [4], in each case at sufficiently large $p_T$ where perturbative QCD is expected to work. This not only allows to determine the $x$-shape of $\Delta g(x, \mu)$ for $x \gtrsim 0.01$ but also verifies the universality property of polarized parton densities for the first time. In the following we review the status of theoretical calculations for processes sensitive to $\Delta g$, experimental aspects can be found, e.g., in [5].

The “classical” tool for determining the gluon density is high-$p_T$ prompt photon production due to the dominance of the LO Compton process, $qg \rightarrow \gamma q$. Exploiting this feature, both RHIC experiments, PHENIX and STAR, intend to use this process for a measurement of $\Delta g$. Apart from “direct” mechanisms like $qg \rightarrow \gamma q$, the photon can also be produced by a parton, scattered or created in a hard QCD reaction, which fragments into the photon. Such a contribution naturally arises in a QCD calculation from the necessity of factorizing final-state collinear singularities into a photon fragmentation function $D_\gamma$. However, since photons produced through fragmentation are always accompanied by hadronic debris, an “isolation cut” imposed on the photon signal in experiment, e.g., a “cone”, strongly reduces such contributions to the cross section.

![Graph](image)

**FIGURE 3.** $A_{LL}$ for prompt photon production in NLO QCD as a function of $p_T$ for different sets of parton densities. The “error bars” indicate the expected statistical accuracy $\delta A_{LL}$, Eq. (6), for the PHENIX experiment. Figure taken from [8].

The NLO QCD corrections to the direct (non-fragmentation) processes have been calculated in [6] and lead to a much reduced factorization scale dependence as compared to LO estimates. In addition, Monte Carlo codes have been developed [7,8], which allow to include various isolation criteria and to study also photon-plus-jet observables. The latter are relevant for $\Delta g$ measurements planned at STAR [4,5]. Since present comparisons between experiment and theory are not fully satisfactory in the unpolarized case, in particular in the fixed-target regime, considerable efforts have been made to push calculations beyond the NLO of QCD by including resummations of large logarithms [9].
It is hence not unlikely that a better understanding of prompt photon production can be achieved soon. Figure 3 shows $A_{\gamma LL}^t$ as predicted by a NLO QCD calculation [8] as a function of the photon’s transverse momentum $p_T$. The applied rapidity cut $|\eta| \leq 0.35$ matches the acceptance of the PHENIX detector. The important result is that the expected statistical errors $\delta A_{\gamma LL}$ are considerably smaller than the changes in $A_{\gamma LL}^t$ due to different spin-dependent gluon densities over a wide range of $p_T$. Hence RHIC should be able to probe $\Delta g$ in prompt photon production. $\delta A_{\gamma LL}$ may be estimated by the formula

$$\delta A_{\gamma LL} = \frac{1}{P^2 \sqrt{L \sigma_{\text{bin}}}},$$

where $P$ is the polarization of one beam, $L$ the integrated luminosity of the $pp$ collisions, and $\sigma_{\text{bin}}$ the unpolarized cross section integrated over the $p_T$-bin for which the error is to be determined. Unless stated otherwise, $P = 0.7$ and $L = 320 (800) \text{ pb}^{-1}$ is used in Eq. (6) for $pp$ collisions at $\sqrt{S} = 200 (500)$ GeV [4].

Jets are another key-process to pin down $\Delta g$ at RHIC: they are copiously produced at $\sqrt{S} = 500$ GeV, even at high $p_T$, $15 \lesssim p_T \lesssim 50$ GeV, and gluon-induced $gg$ and $qg$ processes are expected to dominate in accessible kinematical regimes. Due to limitations in the angular coverage, jet studies will be performed by STAR only. As jet surrogates, PHENIX can look for high-$p_T$ leading hadrons, such as pions, whose production proceeds through the same partonic subprocesses as jet production. Hadrons have the advantage that they can be studied also at $\sqrt{S} = 200$ GeV and down to lower values in $p_T$ than jets as they do not require the observation of clearly structured “clusters” of particles (jets). Contrary to jet production, a fragmentation function has to be introduced into the theoretical framework, cf. Eq. (4), to take care of final-state collinear singularities. In case of pion production, the $D_{\pi}^{m}$ are, however, fairly well constrained by $e^+e^-$ data. It should be also emphasized that

![Figure 4](image-url)
in the unpolarized case, the comparison between NLO theory predictions with jet production data from the Tevatron is extremely successful. The same is true for first preliminary data on the $p_T$-spectrum for pions at $\sqrt{S} = 200$ GeV from PHENIX [10].

The NLO QCD corrections to polarized jet production are available as a Monte Carlo code [11]. Apart from a significant reduction of the scale dependence, they are also mandatory for realistically matching the procedures used in experiment in order to group final-state particles into jets. For single-inclusive high-$p_T$ hadron production the task of computing the NLO corrections has been completed only very recently [12,13]. Figure 4 shows $A_{LL}$ for single-inclusive jet production at the NLO level as a function of the jet $p_T$. A cut in rapidity, $|\eta| \leq 1$, has been applied in order to match the acceptance of STAR. The asymmetries turn out to be smaller than for prompt photon production, but thanks to the much higher statistics one can again easily distinguish between different spin-dependent gluon densities. Results for single-inclusive $\pi^0$ production are presented in Fig. 5. Note that here the expected statistical accuracy refers to only a very moderate integrated luminosity and beam polarization as targeted for the upcoming run of RHIC. Even under these assumptions a first determination of $\Delta g$ can be achieved.

![Figure 5](image)

**FIGURE 5.** As in Fig. 3 but now for high-$p_T$ $\pi^0$-production. Note that here the statistical “error bars” $\delta A_{LL}$ have been estimated by assuming only $P = 0.4$ and $L = 7$ pb$^{-1}$ in Eq. (6) which is a realistic target for the next RHIC run. Figure taken from [13].

The last process which exhibits a strong sensitivity to $\Delta g$ is heavy flavor production. Here, the LO gluon-gluon fusion mechanism, $gg \rightarrow QQ$, dominates unless $p_T$ becomes rather large. Unpolarized calculations have revealed that NLO QCD corrections are mandatory for a meaningful quantitative analysis. In the polarized case they have been computed recently in case of single-inclusive heavy quark production [14]. Again, one observes a strongly reduced scale dependence for charm and bottom production at RHIC energies. It turns out that the major theoretical uncertainty stems from the unknown precise...
values for the heavy quark masses [14]. Since the heavy quark mass already sets a large scale, one can perform calculations for small transverse momenta or even for total cross sections which, in principle, give access to the gluon density at smaller $x$-values than relevant for high-$p_T$ jet, hadron or prompt photon production.

Heavy flavors are not observed directly at RHIC but only through their decay products. Possible signatures for charm/bottom quarks at PHENIX are inclusive-muon or electron tags or $\mu e$-coincidences. The latter provide a much better $c/b$-separation which is an experimental problem. In addition, lepton detection at PHENIX is limited to $|y| \leq 0.35$ and $1.2 \leq |y| \leq 2.4$ for electrons and muons, respectively. Since heavy quark decays to leptons proceed through different channels and have multi-body kinematics, it is a non-trivial task to relate, e.g., experimentally observed $p_T$-distributions of decay muons to the calculated $p_T$-spectrum of the produced heavy quark. One possibility is to model the decay with the help of standard event generators like PYTHIA [15] by computing probabilities that a heavy quark with a certain $(p_T, y)$ is actually seen within the PHENIX acceptance for a given decay mode. Figure 6 shows a prediction for the charm production asymmetry $A_{LL}$ at PHENIX in NLO QCD for the inclusive-electron tag. The sensitivity to $\Delta g$ is less pronounced than for the processes discussed above. It remains to be checked if heavy flavor production at RHIC can be used to extend the measurement of $\Delta g$ towards smaller $x$-values.

![Figure 6](image_url)

FIGURE 6. NLO single-inclusive charm production asymmetry (rescaled by $1/x_T^{\text{min}}$) as a function of $x_T^{\text{min}} = p_T^{\text{min}}/p_T^{\text{max}}$ for different sets of parton densities. The “error bars”, Eq. (6), are for the PHENIX experiment and include a detection efficiency for the channel $c \rightarrow eX$. Figure taken from [14].

C Further Information on $\Delta q$ and $\Delta \bar{q}$

Inclusive DIS data only provide information on the sum of quarks and antiquarks for each flavor, i.e., $\Delta q + \Delta \bar{q}$. At RHIC a separation of $\Delta u$, $\Delta \bar{u}$, $\Delta d$, and $\Delta \bar{d}$ can be achieved by studying $W^\pm$-boson production. Exploiting the
parity-violating properties of $W^\pm$-bosons, it is sufficient to measure a single spin asymmetry, $A_L^W$, with only one of the colliding protons being longitudinally polarized. The idea is to study $A_L^W$ as a function of the rapidity of the $W$, $y_W$, relative to the polarized proton [16]. In LO it is then easy to show [16,4] that for $W^+$-production, $u\bar{d} \rightarrow W^+$, and large and positive (negative) $y_W$, $A_L^W$ is sensitive to $\Delta u/u$ ($\Delta \bar{d}/\bar{d}$). Similarly, $W^-$-production probes $\Delta d/d$ and $\Delta \bar{u}/\bar{u}$. The NLO QCD corrections for $A_L^W$ as well as the factorization scale dependence are small [17]. Experimental complications [4] arise, however, from the fact that neither PHENIX nor STAR are hermetic, which considerably complicates the reconstruction of $y_W$. Therefore it is important to understand $A_L^W$ on the decay-lepton level. Here, fully differential NLO cross sections are available as a MC code [18]. The anticipated sensitivity of PHENIX on the flavor decomposed quark and antiquark densities is illustrated in Fig. 7.

Semi-inclusive DIS measurements, $ep \rightarrow HX$, are another probe to separate quark and antiquark densities. HERMES has recently published first preliminary results [19]. The accessible $x$-range for the $\Delta q$ and $\Delta \bar{q}$ densities is comparable to that of RHIC, see Fig. 7, but at scales $Q \simeq 1 − 2$ GeV rather than $M_W$. The combination of both measurements can provide an important test of the QCD scale evolution for polarized parton densities and of the possible relevance of higher twist contributions at low scales.

D Towards a Global Analysis of Upcoming Data

Having available at some point in the near future data on various different reactions, one needs to tackle the question of how to set up a “global QCD analysis” for spin-dependent parton densities. The strategy is in principle clear from the unpolarized case: an ansatz for the densities, Eq. (2), at some initial scale $\mu_0$, given in terms of some functional form with a set of free parameters, is evolved, Eq. (3), to a scale $\mu$ relevant for a certain data point. A $\chi^2$-value is assigned that represents the quality of the comparison of the theoretical calculation to the experimental point. The parameters are varied until eventually a global minimum in $\chi^2$ is reached mutually for all data points. In practice, this approach is not fully viable since the numerical evaluations of the cross sections in NLO QCD are usually time-consuming as they require several tedious integrations. Hence the computing time for a QCD fit easily becomes excessive.

In the unpolarized case, the wealth of DIS data already provides a pretty good knowledge of the parton densities, and reasonable approximations can be made for the most time-consuming processes. For instance, one can absorb all NLO corrections into some pre-calculated “correction factors” $K$, and simply multiply them in each step of the fit to the LO approximation for the cross sections which can be evaluated much faster. In the polarized case, it is in general not at all clear whether such a strategy will work. Here, par-
FIGURE 7. Expected statistical accuracy for $\Delta q/q$ from $A_L$ overlayed on two sets of parton densities. The full [open] circles refer to $A_L(W^+)$ [$A_L(W^-)$]. Figure taken from [4].

Spin observables are also an interesting tool to uncover important new physics. One idea is to study single spin asymmetries $A_L$ for large-$p_T$ jets. In the standard model $A_L$ can be only non-zero for parity-violating interactions, i.e., QCD-electroweak interference contributions, which are fairly small. The existence of new parity-violating interactions could lead to sizable modifications [22] of $A_L$. Possible candidates are new quark-quark contact interactions, characterized by a compositeness scale $\Lambda$. RHIC is surprisingly sensitive to quark substructure at the 2 TeV scale, and is competitive with the Tevatron despite the much lower c.m.s. energy [22]. Other candidates for new physics are possible new gauge bosons, e.g., a leptophobic $Z'$. Of course, high luminosity and precision as well as a good knowledge of polarized and unpolarized

III  EXPLORING PHYSICS BEYOND THE STANDARD MODEL

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parton densities and of the standard model “background” are mandatory. For
details, see [22,4].

IV SUMMARY AND OUTLOOK

With first data from RHIC hopefully starting to roll in soon, we can address
many open, long-standing questions in spin physics like the longitudinally
polarized gluon density. With data from many different processes taken at high
energies, where perturbative QCD should be at work, a first global analysis
of spin-dependent parton densities will be possible. For a long time to come
RHIC will provide the best source of information on polarized parton densities,
certainly much improving our knowledge of the spin structure of the nucleon,
and, perhaps, the next “spin surprise” is just round the corner. Future projects
like the EIC [23], which is currently under scrutiny, would help to further
depen our understanding by probing aspects of spin physics not accessible
in hadron-hadron collisions. The structure function $g_1$ at small $x$ or the spin
content of circularly polarized photons are just two examples.

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