On the stability of thick accretion disks around black holes

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ABSTRACT

Discerning the likelihood of the so-called runaway instability of thick accretion disks orbiting black holes is an important issue for most models of cosmic gamma-ray bursts. To this aim we investigate this phenomenon by means of time-dependent, hydrodynamical simulations of black hole plus torus systems in general relativity. The evolution of the central black hole is assumed to be that of a sequence of Kerr black holes of increasing mass and spin, whose growth rate is controlled by the transfer of mass and angular momentum from the material of the disk spiralling in through the event horizon of the black hole. The self-gravity of the disk is neglected. We find that when the black hole mass and spin are allowed to increase, constant angular momentum disks undergo a runaway instability on a dynamical timescale (a few orbital periods). However, our simulations show that a slight increase of the specific angular momentum of the disk outwards has a dramatic stabilizing effect. Our results, obtained in the framework of general relativity, are in broad agreement with earlier studies based both on stationary models and on time-dependent simulations with Newtonian and pseudo-Newtonian gravitational potentials.

Subject headings: Accretion — Accretion disks — Black hole physics — Gamma-rays: bursts — Hydrodynamics — Relativity
1. Introduction

The so-called runaway instability of thick accretion disks orbiting black holes was first noticed by Abramowicz et al. (1983). The origin of the instability is a dynamical process by which the cusp of a disk initially filling its Roche lobe moves outwards due to mass transfer from the disk to the accreting black hole. This process leads to the complete destruction of the disk on a dynamical timescale. Abramowicz et al. (1983) found that the runaway instability occurs for a large range of parameters such as the disk-to-hole mass ratio, $M_D/M_{BH}$, and the location of the disk inner radius. More detailed studies followed, most of which are based on stationary models in which a fraction of the mass and angular momentum of the initial disk is transferred to the black hole. The new gravitational field allows to compute the new position of the cusp, which controls the occurrence of the runaway instability.

The main conclusion of those studies is that the very existence of the instability remains uncertain. Taking into account the self-gravity of the disk seems to favor the instability, as shown from both, studies based on a pseudo-potential for the black hole (Khanna & Chakrabarti 1992; Masuda et al. 1998) and from fully relativistic calculations (Nishida et al. 1996). However, the rotation of the black hole has a stabilizing effect (Wilson 1984; Abramowicz et al. 1998), and the same happens when the disks are built with non-constant distributions of the specific angular momentum (increasing outwards) (Daigne & Mochkovitch 1997; Abramowicz et al. 1998).

In a recent paper (Font & Daigne 2002) we presented the first time-dependent, hydrodynamical simulations in general relativity of the runaway instability of constant angular momentum thick disks around a Schwarzschild black hole. The self-gravity of the disk was neglected and the evolution of the central black hole was assumed to follow a sequence of Schwarzschild black holes of increasing mass. We found that by allowing the mass of the black hole to grow the runaway instability appears on a dynamical timescale, in agreement with previous estimates from stationary models. For a black hole of 2.5 $M_\odot$ and disk-to-hole mass ratios between 1 and 0.05, our simulations showed that the timescale of the instability never exceeds 1 s for a large range of mass fluxes and it is typically a few 10 ms. Similar results for different initial data have been recently reported by Zanotti et al. (2002).

In this Letter we extend our study to the most interesting case of non-constant angular momentum disks. The main motivation of our work is to check through time-dependent simulations in general relativity whether such distributions have indeed the stabilizing effect previously found in non self-gravitating stationary models (Daigne & Mochkovitch 1997; Abramowicz et al. 1998; Lu et al. 2000).
2. Numerical framework

A given initial state of a thick disk orbiting a black hole is determined by five parameters: the black hole mass $M_{BH}$, the specific angular momentum in the equatorial plane $l$, the potential barrier at the inner edge of the disk $\Delta W_{in} = W_{in} - W_{cusp}$, the polytropic constant $\kappa$ and the adiabatic index $\gamma$ of the equation of state (EoS). We assume a specific law of rotation of the disk in which the angular momentum in the equatorial plane increases outwards as a positive power law $l \propto r^\alpha$. The specific values we have considered for the exponent $\alpha$ are listed in Table 1. The initial equilibrium configurations are built in various steps (see Daigne & Font (2002) for details): first, from the coefficients of the Schwarzschild metric and the radial profile of $l$ in the equatorial plane, the angular momentum distribution outside the equatorial plane is computed by solving the equations describing the constant $l$ surfaces, i.e. the von Zeipel cylinders. Then, the total specific energy $u_t$ is obtained from the distribution of $l$ via the normalization of the 4-velocity, $u_\mu u^\mu = -1$. The integral form of the relativistic Euler equation allows then to compute the quantity $W = - \int_0^P dP/w$, where $P$ is the pressure and $w$ the enthalpy. The Newtonian limit of $W$ is the total (gravitational plus centrifugal) potential. The density and the pressure can be easily computed from $W$ using the EoS. The geometrical structure of the equipotential surfaces is comparable to that of the Roche lobes of a binary system. In particular, there is a cusp where mass transfer from the disk to the black hole is possible.

As we did in our previous investigation (Font & Daigne 2002) we focus on models which are expected to exist at the central engine of gamma-ray bursts (GRBs), formed either after the coalescence of a compact binary system (composed of two neutron stars or a neutron star and a stellar-mass black hole), or after the gravitational collapse of a massive star (see e.g. Woosley (2001)). Taking into account the various results from numerical simulations of those systems (Ruffert & Janka 1999; Kluzniak & Lee 1998; MacFadyen & Woosley 1999; Shibata & Uryū 2000; Aloy et al. 2000) we fix the mass of the black hole to $M_{BH} = 2.5 M_\odot$. The angular momentum $l$ is adjusted to yield a disk-to-hole mass ratio of 1 (models A and

<table>
<thead>
<tr>
<th>Model</th>
<th>$M_{BH}$ [M_\odot]</th>
<th>$M_D/M_{BH}$</th>
<th>$\alpha$</th>
<th>$\Delta W_{in}/c^2$</th>
<th>$\dot{m}<em>{\text{stat}}$ [M</em>\odot/s]</th>
<th>$r_{\text{cusp}}$</th>
<th>$r_{\text{center}}$</th>
<th>$t_{\text{orb}}$ [ms]</th>
<th>$t_{\text{run}}/t_{\text{orb}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.5</td>
<td>1.0</td>
<td>0.0</td>
<td>$3.81 \times 10^{-2}$</td>
<td>$\sim 25$</td>
<td>4.896</td>
<td>7.592</td>
<td>1.62</td>
<td>$\sim 4$</td>
</tr>
<tr>
<td>B</td>
<td>2.5</td>
<td>1.0</td>
<td>0.1</td>
<td>$2.26 \times 10^{-2}$</td>
<td>$\sim 2.5$</td>
<td>5.225</td>
<td>9.982</td>
<td>2.44</td>
<td>stable</td>
</tr>
<tr>
<td>C</td>
<td>2.5</td>
<td>0.1</td>
<td>0.0</td>
<td>$2.60 \times 10^{-2}$</td>
<td>$\sim 5.9$</td>
<td>4.962</td>
<td>7.459</td>
<td>1.58</td>
<td>$\sim 8$</td>
</tr>
<tr>
<td>D</td>
<td>2.5</td>
<td>0.1</td>
<td>0.075</td>
<td>$1.87 \times 10^{-2}$</td>
<td>$\sim 1.3$</td>
<td>5.243</td>
<td>8.964</td>
<td>2.08</td>
<td>stable</td>
</tr>
</tbody>
</table>
B) or of 0.1 (models C and D). The latter case is more realistic, being the parameters closer to those inferred from simulations of compact binary mergers. Furthermore, neglecting the disk self-gravity is also much better justified for \( M_D/M_{\text{BH}} = 0.1 \). An even more realistic model would assume an initially rotating black hole, as both in collapsars and mergers the hole is expected to form with an initial spin parameter \( \gtrsim 0.5 \). Such models will be considered in a forthcoming study.

The EOS adiabatic index and polytropic constant are, respectively, \( \gamma = 4/3 \) and \( \kappa = 4.76 \times 10^{14} \) cgs. This corresponds to an EOS dominated by the contribution of relativistic degenerate electrons (the typical density in the disk is \( \sim 10^{11-10^{12}} \text{g/cm}^3 \)). The gravitational potential at the inner edge of the disk is fixed to \( \Delta W_{\text{in}} = 0.75|W_{\text{cusp}}| \) in models A and B, and \( 0.5|W_{\text{cusp}}| \) in models C and D, which corresponds to a finite disk (\( W_{\text{in}} < 0 \)), initially overflowing its Roche lobe (\( \Delta W_{\text{in}} > 0 \)) so that mass transfer is possible at the cusp. The parameters describing the models are summarized in Table 1.

The equilibrium initial data are evolved in time using the same relativistic hydrodynamics code employed in Font & Daigne (2002). This code integrates the non-linear hyperbolic system of the general relativistic hydrodynamic equations in a Kerr spacetime, using Boyer-Lindquist \((t, r, \theta, \phi)\) coordinates and under the restriction of axisymmetry, \( \partial_\phi = 0 \). The numerical scheme, based on an approximate Riemann solver, is second-order accurate in both space and time due to the use of a piecewise linear reconstruction algorithm and a conservative, two-step Runge-Kutta scheme for the time update. The simulations use a numerical grid of \( 400 \times 100 \) zones in \( r \) and \( \theta \), respectively. The radial grid is logarithmically spaced to account for sufficient resolution near the horizon (\( r_{\text{min}} = 2.12M_{\text{BH}}, \Delta r_{\text{min}} = 1.99 \times 10^{-2}M_{\text{BH}}; G = c = 1 \)) and includes the whole disk.

The procedure we follow to account for the spacetime dynamics is equivalent to the one employed in Font & Daigne (2002), with the important difference that now, both the black hole mass and its angular momentum are allowed to increase. This leads to additional metric coefficients – the \( \phi \) component of the shift vector – to be non zero, making the whole numerical treatment appreciably more difficult than in the case of the Schwarzschild metric. The spacetime metric is hence approximated at each time step by a stationary (Kerr) black hole metric of varying mass \( M_{\text{BH}} \) and angular momentum \( J_{\text{BH}} \). We note that both quantities increase very slowly during the evolution. Details of our procedure as well as code tests for rotating black holes will be reported in Daigne & Font (2002).
3. Results

Each of our four initial models is evolved three times. In the first series of runs $M_{\text{BH}}$ and $J_{\text{BH}}$ are kept constant, hence the spacetime is held fixed. In the second series of runs only $M_{\text{BH}}$ is allowed to increase while in the third series of runs both, $M_{\text{BH}}$ and $J_{\text{BH}}$ are allowed to increase. The growth rate is monitored through the mass and angular momentum transfer from the disk to the black hole across the innermost grid zone. Since in realistic disks angular momentum is transported outwards by dissipative processes or removed from the system by gravitational radiation, we adopt a conservative value for the efficiency of the angular momentum transfer, assuming that only 20% of the angular momentum of the accreted material is transferred to the black hole. Results for other values of the efficiency are reported in Daigne & Font (2002).

The evolution of the mass flux $\dot{m} = 2\pi \int_0^\pi \sqrt{-g} D v^r d\theta$ for all twelve simulations is plotted in Fig. 1. In this expression $g$ is the determinant of the Kerr metric, $v^r$ is the radial component of the 3-velocity and $D = \rho \Gamma$ is the relativistic mass density, $\rho$ being the rest-mass density and $\Gamma$ the Lorentz factor. At early times the mass flux evolution for all three series in each model is exactly the same, irrespective of the increase of the black hole mass and spin being taken into account or not. For the first series of runs the mass flux reaches a stationary regime where $\dot{m} \propto \Delta W_{\text{in}}^4$ (Font & Daigne 2002), which is the theoretical expectation for $\gamma = 4/3$ (Kozlowski et al. 1978). Since we consider only models with fixed ratios $M_D/M_{\text{BH}}$ and $\Delta W_{\text{in}}/W_{\text{cusp}}$, the stationary mass flux is different for each model. However our results are not modified when all models have the same initial $\dot{m}$ (see Daigne & Font (2002)).

For constant angular momentum disks (models A and C; $\alpha = 0$) the time evolution of the system changes dramatically when the spacetime dynamics is taken into account, i.e. when either $M_{\text{BH}}$ only or $M_{\text{BH}}$ and $J_{\text{BH}}$ increase. In either case the runaway instability, reflected in the rapid growth of the mass accretion rate, appears on a dynamical timescale of $\sim 4 \, t_{\text{orb}} \sim 6.5$ ms for model A ($\sim 8 \, t_{\text{orb}} \sim 12$ ms for model C). The time derivative of the mass flux also increases, which implies the rapid divergence of $\dot{m}$. Comparing the case where only $M_{\text{BH}}$ changes with the case where both $M_{\text{BH}}$ and $J_{\text{BH}}$ change, the instability appears slightly later in the latter case (dotted line in Fig. 1), the qualitative behaviour of the mass flux being, however, identical.

The main differences appear in the case of non-constant angular momentum disks (models B and D). In Fig. 1 it becomes apparent that a slight increase of the specific angular momentum outwards (well below the Keplerian limit $\alpha = 0.5$) strongly stabilizes the disk and completely suppresses the runaway instability. During the accretion process, the material at the inner edge of the disk is replaced by new material with a higher angular momentum. Therefore, the centrifugal barrier which acts against accretion is much more efficient. We
note that the long-term behaviour of the mass flux evolution simply reflects the fact that the initial accretion rate is non zero (highly super-Eddington: \( \sim 2.5 \, M_\odot/\text{s} \) for model B, \( \sim 1.3 \, M_\odot/\text{s} \) for model D). Hence, by integrating \( \dot{m} \) along the entire time of the simulation, it is possible to check that the total mass transferred becomes, asymptotically, of the order of the initial mass of the disk, which results in the end of the accretion process. This effect is more pronounced for smaller disk-to-hole mass ratios (model D). Notice that the mass flux decreases during the long-term evolution. This is due to the fact that for non-constant angular momentum disks (models B and D), the accretion process makes the cusp move faster towards the black hole than the inner radius of the disk (which is precisely why the instability is physically suppressed). Therefore, the potential at the inner edge \( \Delta W_{\text{in}} \) decreases with time and, consequently, also the mass flux. For this reason, accretion becomes increasingly slower, which makes numerically difficult to follow the evolution until the entire disk has been fully accreted.

In Fig. 2 we plot the time evolution of the mass of the disk for all twelve simulations. The unstable behaviour of models A and C, with \( \alpha = 0 \), is characterized by the sudden decrease of \( M_D \) in less than \( \sim 6 - 10 \) orbital periods. By the end of the simulation the disk and black hole masses are \( M_D \sim 0.13 \, M_\odot \) and \( M_{\text{BH}} \sim 4.88 \, M_\odot \) for model A (\( M_D \sim 0.013 \, M_\odot \) and \( M_{\text{BH}} \sim 2.74 \, M_\odot \) for model C), which means that 95\% of the disk mass has already been accreted. However, when the spacetime is held fixed (solid line), the mass of the disk for model A at \( t \sim 200 \, t_{\text{orb}} \) is still \( M_D \sim 0.1 \, M_\odot \) (correspondingly, \( M_D \sim 0.015 \, M_\odot \) at \( t \sim 100 \, t_{\text{orb}} \) for model C). This means that the accretion timescale is at least 30 and 10 times longer, for models A and C respectively, than the runaway (dynamical) timescale \( t_{\text{run}} \). On the other hand, for models B (\( \alpha = 0.1 \)) and D (\( \alpha = 0.075 \)), the stability properties of the non-constant distribution of angular momentum are dramatically reflected in the small loss of the mass of the disk throughout the evolution. At the end of the simulation the mass loss is only \( \sim 2\% \) for model B and \( \sim 14\% \) for model D (the lines are almost indistinguishable). The final masses of the black holes are \( M_{\text{BH}} \sim 2.55 \, M_\odot \) and \( \sim 2.54 \, M_\odot \) for models B and D, respectively.

The transfer of mass and angular momentum from the disk to the black hole leads to the gradual increase of the rotation law of the initially non-rotating black hole. In Fig. 3 we plot the time evolution of the black hole spin \( a = J_{\text{BH}}/M_{\text{BH}}^2 \) for both the stable and unstable cases. The stable disks, models B and D, only transfer a very small fraction of their angular momentum to the black hole. At the end of the simulation the black hole spin is \( a = 0.014 \) for model B and \( a \sim 0.005 \) for model D. On the contrary, for the unstable disks the transfer of angular momentum is considerably higher, accelerating rapidly during the runaway instability. Therefore, in \( \sim 6 - 10 \) orbital periods the initial Schwarzschild black hole turns into a mildly (slowly) rotating Kerr black hole with \( a \sim 0.4 \) (\( a \sim 0.05 \)) for model
4. Discussion

By means of time-dependent hydrodynamical simulations in general relativity we have shown that the runaway instability of (non self-gravitating) thick disks around (rotating) black holes (Abramowicz et al. 1983) is strongly suppressed when the distribution of angular momentum in the disk is non constant. A small increase outwards of the specific angular momentum distribution, well below the Keplerian limit, could result in a strong stabilizing effect. In particular, we have demonstrated that for disks whose angular momentum follows a power-law, \( l \propto r^\alpha \), the case \( \alpha = 0 \) is unstable for both, \( M_D/M_{BH} = 1 \) and 0.1, while disks with small non-zero values of \( \alpha \) are stable (\( \alpha = 0.1 \) for \( M_D/M_{BH} = 1 \) and \( \alpha = 0.075 \) for \( M_D/M_{BH} = 0.1 \)). This result is in complete agreement with previous studies based on stationary sequences of equilibrium configurations, either using a pseudo-Newtonian potential (Daigne & Mochkovitch 1997) or in general relativity (Abramowicz et al. 1998). For the first time we have shown this effect with a time-dependent relativistic calculation, and we have been able to estimate the lifetime of a thick disk orbiting a black hole in both, the unstable and stable cases.

According to our results the runaway instability is most likely avoided in non rigidly rotating disks formed in the coalescence of a binary neutron star system or in the gravitational collapse of a massive star. If the growth of non-axisymmetric modes in the disk by the Papaloizou-Pringle instability is suppressed by the accretion process itself, as suggested by Blaes (1987), systems consisting of a Kerr black hole surrounded by a high density torus may then be long lived. The lifetime is probably controlled by the viscous timescale (a few seconds) rather than by the dynamical one, which may provide enough time for any plausible magneto-hydrodynamical process to efficiently transfer part of the energy reservoir of the system to a relativistic outflow. Therefore, the most favoured current GRB models (see Woosley (2001)) can indeed be based on such central engines. This is especially important in the context of the so-called internal shock model for the prompt gamma-ray emission (Rees & Meszaros 1994), where the observed lightcurve reflects the activity of the central engine, with no modification of the timescales other than the time dilation due to the redshift. In particular, the central engine has to survive for a duration at least comparable with the observed duration of the GRB.

We note that our study does not include the self-gravity of the disk, despite its relevance when \( M_D/M_{BH} \sim 1 \). Successful attempts to long-term stable simulations of thick disks orbiting black holes by solving the coupled system of Einstein and general relativistic hydro-
dynamic equations seem, however, out of the scope of present day numerical relativity codes (see, e.g. Font (2000) and references there in). According to previous studies (Khanna & Chakrabarti 1992; Nishida et al. 1996; Masuda et al. 1998), based on either time-dependent simulations with pseudo-potentials or stationary models in general relativity, self-gravity has a strong destabilizing effect. However, this effect has to decrease for small disk-to-hole mass ratios. Therefore, we expect that for $M_D/M_{BH} \lesssim 0.1$ the stabilizing effect of non-constant angular momentum distribution demonstrated in this study could overcome the destabilizing effect of the much less relevant disk’s self-gravity.

A comprehensive study of a broader sample of initial models accounting for different disk-to-hole mass ratios and initial accretion rates will be presented elsewhere (Daigne & Font 2002). We further plan to perform a careful study of the critical value of $\alpha$ separating stable and unstable disks and to find its dependence on the disk-to-hole mass ratio and on the black hole spin.

We are grateful to L. Rezzolla and O. Zanotti for interesting comments. J.A.F. acknowledges financial support from a EU Marie Curie fellowship (HPMF-CT-2001-01172) and from the Spanish Ministerio de Ciencia y Tecnología (grant AYA 2001-3490-C02-01). F.D. acknowledges financial support from a postdoctoral fellowship from the French Spatial Agency (CNES).

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Fig. 1.— Time evolution of the mass flux. The solid lines correspond to evolutions where the spacetime is held fixed, while the dashed and dotted lines correspond to the cases where only the growth of $M_{\text{BH}}$ is allowed, and where both $M_{\text{BH}}$ and $J_{\text{BH}}$ increase, respectively.
Fig. 2.— Time evolution of the mass of the disk. The line style follows the same convention as in Fig. 1. The non-constant angular momentum disks, models B and D, remain stable throughout the whole simulation.
Fig. 3.— Time evolution of the spin of the black hole for the simulations in which the transfer of mass and angular momentum from the disk is allowed.