Inhomogeneous Big Bang Cosmology

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Abstract
In this letter, we outline an inhomogeneous model of the Big Bang cosmology. For the inhomogeneous spacetime used here, the universe originates in the infinite past as the one dominated by vacuum energy and ends in the infinite future as the one consisting of “hot and relativistic” matter. The spatial distribution of matter in the considered inhomogeneous spacetime is arbitrary. Hence, observed structures can arise in this cosmology from suitable “initial” density contrast. Different problems of the standard model of Big Bang cosmology are also resolved in the present inhomogeneous model. This inhomogeneous model of the Big Bang Cosmology predicts “hot death” for the universe.

Introduction
In the 3+1 formulation mtw of General Relativity, source, matter or energy, data can be specified on some suitable “initial” spacelike hyper-surface and, that data can be evolved using the Einstein field equations. Different four-dimensional spacetime geometries are obtainable for different initial source data.

When matter or energy sources are present over all of the initial hyper-surface, we obtain a “cosmological” situation or a “cosmological” spacetime. A famous is the case of maximally symmetric, homogeneous and isotropic, Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime of the Big Bang Cosmology (BBC) mtw, sussp, stdbooks.

Now, to fix ideas, recall that the Newtonian Law of Gravitation is a statement of the force of attraction between two mass points in space. The presence of other mass-points does not alter the statement of this Law of Gravitation. Moreover, re-distribution of mass-points does not affect the statement of this Law of Gravitation.

General Relativity prescribes a spacetime geometry for a Law of Gravitation. In analogy with the Newtonian case, we therefore seek a spacetime which “does not change” its geometrical characters when the source distribution in it is altered in some way. Moreover, the requirement that the addition of “more” sources does not change the geometrical characters of this spacetime means that it is also a “cosmological” spacetime.

But, we note that the FLRW spacetime does change its geometrical characters when matter is differently distributed, say, in some inhomogeneous way, in this spacetime. Therefore, it does not satisfy our above criteria.

Hence, we consider here a Petrov-type D, cosmological spacetime, of (genhsp), which “does not change” its geometrical characters under redistribution of matter in it and explore its cosmology, broadly the nature of the dynamics of the “cosmological spacetime” under consideration.

We recall that the, homogeneous and isotropic, FLRW metric displays mtw, sussp, stdbooks different dynamical behavior for differing source constituents in it. Similarly, the inhomogeneous spacetime of (genhsp) displays different dynamical behaviour for differing source contents in it. In this letter, we focus on the inhomogeneous realization of the big bang cosmology using the spacetime of (genhsp).

The Cosmological Spacetime spacetime
Consider the spacetime metric: eqnarray

\[ ds^2 = -X^2Y^2Z^2dt^2 \]

Singularity of the first type, \( X = 0 \), vanishing of temporal function, corresponds to a singular hyper-surface of (genhsp). On the other hand, singularities of the second type constitute a part of the initial data, singular spatial data, for (genhsp).

The locations for which the spatial derivatives vanish are, however, coordinate singularities. The curvature invariants of (genhsp) do not blow up at such locations. Therefore, before such coordinate singularities are reached, we may transform coordinates to other suitable ones.

There are also obvious degenerate metric situations when any of the spatial functions is infinite for some range of the coordinates.

In what follows, we shall assume that there are no singular spatial initial data and that there are no spatially degenerate situations for the cosmological spacetime of the metric (genhsp).
*Other features of the metric (genhsp) The metric (genhsp) admits three hyper-surface orthogonal spatial homothetic Killing vectors (HKVs) eqnarray\[ X = (0, X\gamma X^\prime, 0, 0) \]

Now, the existence of three HKVs is equivalent to two Killing vectors (KVs) and one HKV. Then, the metric (genhsp) can be expressed in a form that displays the existence of KVs explicitly. However, we will use the form (genhsp) in this letter.

The spacetime of (genhsp) is required, by definition, to be locally flat at all of its points. In general, this will require some conditions on the metric functions \( X, Y, Z \).

Also, the non-vanishing components of the Weyl tensor for (genhsp) are: eqnarray\[ C_{txtx} = B^2\gamma^2 X^2Y^2Z^26F(t) \]

Now, for (genhsp), it can be verified that the non-vanishing Newman-Penrose (NP) complex scalars \( \Psi_0 = \Psi_4 \) and \( \Psi_2 \).

Note that \( \Psi_4 \neq 0 \). Now, consider a NP-tetrad rotation of Class II, with complex parameter \( b \), that leaves the NP-vector \( n \) unchanged, and demand that the new value, \( \Psi_0^{(1)} \), of \( \Psi_0 \) is zero. Then, we have: eqnarray\[ \Psi_4 b^4 + 6\Psi_2 b^2 + \Psi_4 = 0 \]