QUANTUM COHERENCE, CORRELATED NOISE AND PARRONDO GAMES

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Abstract

We discuss the effect of correlated noise on the robustness of quantum coherent phenomena. First we consider a simple, toy model to illustrate the effect of such correlations on the decoherence process. Then we show how decoherence rates can be suppressed using a Parrondo-like effect. Finally, we report the results of many-body calculations in which an experimentally-measurable quantum coherence phenomenon is significantly enhanced by non-Markovian dynamics arising from the noise source.

1 Introduction

Decoherence is a uniquely quantum phenomenon which results in the decay of the off-diagonal elements of the system’s density matrix. This implies that quantum superposition states decay into a probabilistic mixture. Decoherence, and in particular the control of decoherence, is crucial to the success of quantum information processing and quantum computation [1]. Decoherence can be considered as arising due to the presence of some kind of ‘noise’. Regardless of its origin, such ‘noise’ is typically assumed to be a
statistically independent stochastic process, i.e. lacking any temporal correlations. The off-diagonal elements of the density matrix then tend to decay in time as a simple exponential, with the decay constant denoting a ‘relaxation time’. However, recent work has shown that such approximations are highly questionable - or even plainly wrong - in the ultrafast optical regime being explored for nanostructure-based quantum information processing [2, 3].

In this Letter we investigate the effect of correlated noise on quantum coherent phenomena. First we consider a simple, toy model of a two-level system in order to illustrate the effect that such correlations can have on decoherence. Then we show how decoherence rates can be suppressed via a Parrondo-like effect [4]. Finally, we report the results of detailed many-body calculations in which an experimentally-measurable quantum coherent phenomenon is significantly enhanced by non-Markovian dynamics arising from the noise source. Our results show that, in addition to yielding non-trivial dynamics, correlated noise can play an important role in determining the dynamics of the decoherence process.

2 Toy Model of Decoherence

We start by considering decoherence of a two-level system under the physical assumptions that the channel is isolated (i.e. no entanglement between the system and the environment) and non-dissipative (i.e. only phase damping). We further assume discrete time-evolution for simplicity. Our model is a generalization of Nielsen and Chuang’s treatment of a single phase-kick [1].

Suppose we have a qubit $|\Psi\rangle_0 = a_0|0\rangle + b_0|1\rangle$ at timestep $t = 0$. A rotation $R_z(\theta_1)$ is applied at timestep $t = 1$, to model a phase-kick. The random angle $\theta_1$ is drawn from a probability distribution $P_1(\theta_1)$. The output state is given by the density matrix obtained from averaging over $\theta_1$:

$$\rho_1 = \int_{-\pi}^{\pi} R_z(\theta_1) \rho_0 R_z^\dagger(\theta_1) P_1(\theta_1) d\theta_1 .$$

The density matrix after $n$ timesteps is given by:

$$\rho_n = \int_{-\pi}^{\pi} \cdots \int_{-\pi}^{\pi} R_z(\theta_n) \cdots R_z(\theta_2) R_z(\theta_1) \rho_0 \cdots$$

$$R_z^\dagger(\theta_1) R_z^\dagger(\theta_2) \cdots R_z^\dagger(\theta_n) P(\theta_n, \theta_{n-1}, \ldots, \theta_2, \theta_1) d\theta_1 d\theta_2 \ldots d\theta_n$$

(2)

where Bayes’ rule yields

$$P(\theta_n, \theta_{n-1}, \ldots, \theta_2, \theta_1) = \prod_{i=1}^{n} P(\theta_i | \theta_{i-1}, \theta_{i-2}, \ldots, \theta_2, \theta_1) .$$

(3)
$R_z(\theta_n)R_z(\theta_{n-1})\ldots R_z(\theta_2)R_z(\theta_1)$ can be rewritten exactly as a single net rotation $R_z(\Theta)$ where $\Theta = \sum_{i=1}^{n} \theta_i$. In general the stochastic process producing the kicks may have temporal correlations, and may not be stationary - hence the kicks may be dependent and/or have a time-dependent probability distribution (e.g. if the noise-source were changing in time).

We start by considering *independent, identically distributed* (i.i.d.) kicks. Hence $P(\theta_i|\theta_{i-1},\ldots) = P_i(\theta_i)$ with $P_i(\ldots) = P(\ldots)$ for all $i$. Thus

$$P(\theta_n,\theta_{n-1},\ldots,\theta_2,\theta_1) = \prod_{i=1}^{n} P(\theta_i)$$  \hspace{1cm} (4)

If we also assume that $P(\theta)$ is Gaussian, then $P(\theta) = \frac{1}{\sqrt{4\pi\lambda}} e^{-\frac{\theta^2}{4\lambda}}$ where the mean is 0 and the variance is $2\lambda$. With the assumption that the Gaussian distributions have narrow peaks, we can allow the lower (upper) limits of the integrals to go to minus infinity (plus infinity). This yields $\rho_n$ as a matrix with off-diagonal elements $a_0 b_0^* e^{-n\lambda}$. This exponential linear dependence on $n$ is basically just the effect of $n$ independent Gaussian integrals. Equivalently we can think of the net rotation $\Theta = \sum_{i=1}^{n} \theta_i$ as having a variance $2\lambda_n$ given by the sum of the variances $2\lambda_n = 2\lambda n$. For non-Gaussian $P(\theta)$, it can be shown that an exponential decay also arises.

We now consider *dependent*, but identically distributed (d.i.d.) kicks. As an illustration, we consider the simple case of the angles being so strongly correlated that $\theta_2 = \theta_1$, $\theta_3 = \theta_2$, etc. This is equivalent to saying that all the conditional probabilities $P(\theta_i|\theta_{i-1},\theta_{i-2},\ldots,\theta_2,\theta_1) = \delta(\theta_i - \theta_1)$. Assuming that $\theta_1$ has a Gaussian distribution, then $P(\theta) = \frac{1}{\sqrt{4\pi\lambda}} e^{-\frac{\theta^2}{4\lambda}}$. Now $\rho_n$ has off-diagonal elements $a_0 b_0^* e^{-n^2\lambda}$. This exponential quadratic dependence on $n$ is basically just the effect of $n$ dependent integrals. Equivalently we can think of the net rotation $\Theta = \sum_{i=1}^{n} \theta_i = n\theta_1$ as having a variance $2\lambda_n$ given by $2\lambda_n = 2\lambda n^2$. Hence the noise correlations have significantly affected the decoherence process.

### 3 Decoherence Control using Parrondo Effect

We now proceed to investigate the counter-intuitive effect whereby noise correlations might be exploited in an active way, in order to reduce decoherence. The Parrondo effect [4] is a remarkable result whereby two losing ‘games’, when combined, become winning. Pioneered by J.M.R. Parrondo, and by D. Abbott and collaborators, the most striking feature is arguably the fact
that this combination can be random, i.e. random switching between two losing games A and B can produce a winning game C. Several realizations of the Parrondo effect have recently been suggested in the quantum regime: in particular, quantum games [5], quantum lattice gases [6] and quantum algorithms [7]. But couldn’t the same idea be applied to quantum decoherence? In particular, could two decoherence sources (two ‘private baths’) be combined to produce a single decoherence source (a ‘public bath’) with a longer coherence time?

For independent kicks (either identically or non-identically distributed) we have been able to show that it is not possible to produce such a Parrondo effect [7]. Remarkably, however, we can produce a Parrondo-like effect if we allow for correlated kicks, i.e. correlated noise. We now illustrate this, using a specific example motivated by the classical vector-rotating game of Ref. [7]. Consider two probability distributions \( P_A \) and \( P_B \) corresponding to two ‘private baths’ for phase-damping kicks. These distributions are such that the kick rotation angle \( \theta_2 \) is correlated to the previous rotation angle \( \theta_1 \) in the following manner:

\[
P_A(\theta_2|\theta_1) = \begin{cases} \frac{1}{3}(\delta(\theta_2 - \frac{\pi}{2}) + \delta(\theta_2 + \frac{\pi}{2}) + \delta(\theta_2 - \frac{\pi}{2})) & , \theta_1 \in \{-\frac{\pi}{2}, 0, \frac{\pi}{2}\} \\ \delta(\theta_2) & , \text{otherwise} \end{cases}
\]

\[
P_B(\theta_2|\theta_1) = \begin{cases} \frac{1}{3}(\delta(\theta_2 - \epsilon) + \delta(\theta_2 + \frac{3\pi}{4}) + \delta(\theta_2 - \frac{3\pi}{4})) & , \theta_1 \in \{-\frac{3\pi}{4}, \epsilon, \frac{\pi}{4}\} \\ \delta(\theta_2 - \epsilon) & , \text{otherwise} \end{cases}
\]

with similar conditions holding for all subsequent pairs \( \theta_i \) and \( \theta_{i-1} \). We note that the specific choice of angles is designed to make analytical calculations possible, and may be generalized. The parameter \( \epsilon \) is supposed to be small—its presence serves to ‘memorize’ which probability distribution was selected in the previous step. If \( P_A \) represents the only noise-source applied to the system, and assuming the initial angle of rotation is 0 (i.e. \( \theta_1 = 0 \)) then we have

\[
P_A(\theta_n, \ldots, \theta_1) = \prod_{i=2}^{n} P_A(\theta_i|\theta_{i-1}) = \left(\frac{1}{3}\right)^{n-1}
\]

since the angles always lie in the set \( \{-\pi/2, 0, \pi/2\} \). If the initial density matrix is

\[
\rho_1 := \begin{pmatrix} a & b \\ b^* & d \end{pmatrix},
\]

then the density matrix after \( n \) time steps is:

\[
\rho_n := \begin{pmatrix} a & b \gamma^n e^{-in\phi} \\ b^* \gamma^n e^{in\phi} & d \end{pmatrix},
\]
where
\[ \gamma e^{\pm i\phi} := \frac{1}{3} \] (10)
assuming that the system is under the influence of \( P_A \) alone. Similar arguments hold if \( P_B \) is the only noise-source applied to the system and if we assume \( \theta_1 = \epsilon \). In this case,
\[ \gamma e^{\pm i\phi} := \frac{1}{3} e^{\pm i\epsilon}. \] (11)

Combining the two noise-sources (i.e. probability distributions) at random gives
\[
P(\theta_2|\theta_1) = \begin{cases} 
\frac{1}{2}\delta(\theta_2 - \epsilon) + \frac{1}{6}[\delta(\theta_2 - 0) + \delta(\theta_2 + \frac{\pi}{2}) + \delta(\theta_2 - \frac{\pi}{2})] & , \theta_1 \in \{-\frac{\pi}{2}, 0, \frac{\pi}{2}\} \\
\frac{1}{2}\delta(\theta_2 - 0) + \frac{1}{6}[\delta(\theta_2 - \epsilon) + \delta(\theta_2 + \frac{3\pi}{4}) + \delta(\theta_2 - \frac{\pi}{4})] & , \theta_1 \in \{-\frac{3\pi}{4}, \epsilon, \frac{\pi}{4}\} \\
\frac{1}{2}[\delta(\theta_2) + \delta(\theta_2 - \epsilon)] & , \text{otherwise.}
\end{cases}
\] (12)

The density matrix \( \rho_n \) is given by
\[
\int R_z(\theta_1) \cdots \int R_z(\theta_n) \rho_0 R_z^{\dagger}(\theta_n) P(\theta_n|\theta_{n-1}) d\theta_n \cdots R_z^{\dagger}(\theta_1) P(\theta_1) d\theta_1
\] (13)
where \( P \) only has one-step correlations. We define the following functions recursively:
\[
f_1(\theta) := \int e^{i\phi} P(\phi|\theta) d\phi \tag{14}
\]
\[
f_{k+1}(\theta) := \int e^{i\phi} f_k(\phi) P(\phi|\theta) d\phi \tag{15}
\]
for \( 1 \leq k \leq n \). Assuming the initial angle is 0, we have
\[
\rho_n := \begin{pmatrix}
    a & b[f_n(0)]^* \\
    b^* f_n(0) & d
\end{pmatrix}. \tag{16}
\]

For the combined probability distribution \( P \) above, the angles of rotation can only take on seven values, \( \{-\pi/3, -\pi/2, 0, \epsilon, \pi/3, \pi/2, \pi\} \). We can calculate the \( f_k \)'s as follows:
\[
f_1 := \begin{cases} 
\{-\pi/2, 0, \pi/2\} & \mapsto e^{i\epsilon}/2 + 1/6 \\
\{-3\pi/4, \epsilon, \pi/4\} & \mapsto 1/2 + e^{i\epsilon}/6 
\end{cases} \tag{17}
\]
\[
f_{k+1} := \begin{cases} 
\{-\pi/2, 0, \pi/2\} & \mapsto \frac{e^{i\epsilon}}{2} f_k(\epsilon) + \frac{1}{6} f_k(0) \\
\{-3\pi/4, \epsilon, \pi/4\} & \mapsto \frac{1}{2} f_k(0) + \frac{e^{i\epsilon}}{6} f_k(\epsilon). \tag{18}
\end{cases}
\]
Letting $\epsilon$ go to zero and writing $e^{i\epsilon}$ as $1 + O(\epsilon)$, we see that $f_1(0) = f_1(\epsilon) = 2/3 + O(\epsilon)$, $f_2(0) = 1/3 f_1(\epsilon) + 1/3 f_1(0) = (2/3)(2/3) + O(\epsilon)$, and so on. Indeed, $f_k(0) = (2/3)^{k-1} + O(\epsilon)$ for all $k$. Hence the decay factor $\gamma$ here is $2/3 + O(\epsilon)$, which is an improvement over $1/3$. This reflects the Parrondo effect. Hence our toy model has established that, in principle, it might be possible to change decoherence properties in a favourable way by adding more noise to a quantum system. Our toy model has also demonstrated that temporal correlations can have a significant effect on decoherence properties and hence must be accounted for properly in any realistic calculations.

4 Enhancement of Quantum Coherent Phenomena

The possibility of performing quantum information processing in nanostructure systems, such as semiconductor quantum dots (QD), is of great interest from the perspectives of both fundamental science and future emerging technologies [3]. Significant advances have been made recently in the fabrication of such nanostructures. Apart from quantum dots, there are a wide range of inorganic and organic structures which also qualify as ‘nanostructures’, including microbiological molecular structures such as the photosynthetic complexes in purple bacteria. However the great challenge facing any such information processing in the quantum regime lies in avoiding, controlling or overcoming the effects of decoherence. Typical decoherence times are of the order of picoseconds, and hence impose severe constraints on the timescales within which quantum logic gates need to be performed. For this reason, it is now widely believed that excitons generated optically in such nanostructures, could serve as useful qubits. Their manipulation (i.e. quantum logic gates) could then be achieved by ultrafast femtosecond laser pulses. Remarkably, such manipulation has already been demonstrated experimentally for single quantum dot nanostructures [8].

Given the potential importance of correlated noise on decoherence effects demonstrated in this Letter, we have investigated the effect of non-Markovian dynamics (i.e. correlated noise) in such nanostructure systems. In particular, we performed large-scale, many-body calculations of the ultrafast second-order coherence function of the emitted light from the optically-generated exciton in a single nanostructure (QD). Non-Markovian effects are included for both exciton-photon and exciton-phonon couplings. We find that a strong photon antibunching effect (a purely quantum phenomenon) arises in the resonance fluorescence response at very short times, if and only if the initial ex-
citon state comprises a quantum superposition [2]. More importantly for the present study, we find that correlation effects significantly enhance the antibunching signal, hence demonstrating explicitly that temporal correlations cannot be neglected a priori in such ultrafast regimes. The typical Markov approach, via Master equations with time-independent damping coefficients corresponding to an exponential decay in decoherence (c.f. earlier discussion for i.i.d. kicks), cannot account for the evolution of an open system on very short time scales. In short, Markov approximations (which are valid on long time-scales) overestimate the decay effects at short times. Hence great care must be exercised when treating temporal correlation effects on such short time-scales.

5 Conclusion

We have discussed the effects of correlated noise on quantum coherence, and have shown the possibility of decoherence control through a Parrondo-like effect. We have also reported the crucial role that non-Markovian damping effects (temporal correlations) can play in nanostructure-based quantum information processing.

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References


