Curved Branes in AdS Einstein-Maxwell Gravity and Killing Spinors

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Abstract

We determine the Killing spinors for a class of magnetic brane solutions with Minkowski worldvolume of the theory of AdS Einstein Maxwell theories in $d$ dimensions. We also obtain curved magnetic brane solutions with Ricci-flat worldvolumes. If we demand that the curved brane solution admits Killing spinors, then its worldvolume must admit parallel spinors. Classes of Ricci-flat worldvolumes admitting parallel spinors are discussed.

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1 Introduction

The conjectured equivalence between string theory on anti-de Sitter (AdS) space and certain superconformal gauge theories living on the boundary of this space [?] has led to a lot of interest in the study of AdS gravitational configurations. In part, this interest has also been motivated by the suggestion that four dimensional Einstein gravity can be recovered on a domain wall embedded in AdS space [?]. Classical AdS solutions may provide the possibility to study the nonperturbative structure of the field theories living on the boundary.

Black hole solutions to Einstein’s equations with a negative cosmological constant were constructed many years ago in [?]. Some activity has been recently devoted to the study of solutions with various horizon topologies given by planar, toroidal or arbitrary genus Riemann surface [?,?,?,?,?,?,?,?,?]. In the context of gauged supergravity models, black hole and brane solutions were discussed in many places (see for example [?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?,?]).

In recent years certain supersymmetric curved p-brane and domain wall solutions were constructed [?,?,?]. A typical p-brane solution is a warped product of two spacetimes, one describing a (p + 1)-dimensional Minkowski worldvolume and the other being the transverse space to the brane. For the solutions discussed in [?,?,?], the Einstein’s equations of motion were shown to be satisfied also when the worldvolume metric is Ricci-flat. The curved solution obtained admits Killing spinors provided that its Ricci-flat worldvolume admits parallel spinors. The classification of possible static brane worldvolumes is related to the known classification of Riemannian holonomy groups [?]. Static metrics admitting parallel spinors are automatically Ricci-flat. However, if one allows for indecomposable Ricci-flat Lorentzian worldvolumes with parallel spinors, more curved solutions with Killing spinors can be obtained [?]. Lorentzian manifolds admitting parallel spinors need not be Ricci-flat. This means that restricting the holonomy of a given manifold (for it to have parallel spinors) is not enough for its metric to satisfy Einstein’s equations of motion; more restrictions must be imposed.

In this work we are mainly interested in the study of a class of magnetic brane solutions with various horizon topologies in d-dimensional gravity with a negative cosmological constant coupled to an abelian gauge field. In [?] d-dimensional magnetic brane solutions with flat, hyperbolic and spherical transverse space and with Minkowski worldvolume were derived. Motivated by the results of [?,?], we show that these solutions can be obtained by searching for magnetically charged configurations with Killing spinors. We determine the projection conditions on the spinors and their explicit forms. In the context of gauged supergravity theories, solutions with Killing spinors are supersymmetric. We also generalize the solutions of [?] and discover new d-dimensional magnetic brane solutions with curved Ricci-flat worldvolumes.
2 Magnetic Branes with Killing Spinors

In this section we obtain magnetic brane solutions of the theory of Einstein-Maxwell theory with a negative cosmological constant in dimensions \( d \geq 5 \). This is done by searching for magnetically charged brane configurations admitting Killing spinors. The Lagrangian of our theory can be given by

\[
e^{-1}L = R - \Lambda - (d - 2)(d - 3)F_{\mu \nu}F^{\mu \nu}
\]  

(2.1)

where the cosmological constant is \( \Lambda = -(d-1)(d-2)l^2 \), \( R \) is the scalar curvature and \( F_{\mu \nu} \) is the field strength of an abelian gauge field \( A_\mu \). Motivated by the equations of gauged supergravity, we write as a Killing spinor equation in \( d \) dimensions

\[
[D_\mu + i4(d - 3) (\gamma_\mu^\nu - 2(d - 3)\delta_\mu^\nu \gamma^\rho) F_{\nu \rho} - i (d - 2) l^2 A_\mu + l^2 \gamma_\mu] \epsilon = 0.
\]  

(2.2)

Here \( D_\mu \) is the covariant derivative given by \( D_\mu = \partial_\mu + 14\omega_{\mu ab} \gamma^a \), where \( \omega_{\mu ab} \) is the spin connection and \( \gamma_a \) are Dirac matrices. In this work, we use the metric \( \eta^{ab} = (-, +, +, +, +, ....) \) and \( \{\gamma^a, \gamma^b\} = 2\eta^{ab} \). In supergravity theories, the strategy for finding supersymmetric bosonic solutions is as follows. The fermi fields are set to zero leading automatically to the vanishing of the supersymmetry variations of the bosonic fields of the theory. The conditions for the existence of solutions preserving some supersymmetry are then obtained from the vanishing of the supersymmetry variation of the fermi fields. AdS Einstein-Maxwell theories with spacetime dimensions \( d < 6 \) can be obtained as consistent truncations of gauged supergravity models and (2.2) comes from the vanishing of the gravitini supersymmetry transformation in a bosonic background. As in [?], we consider a general \((d - 4)\)-magnetic brane solution with metric given in the form

\[
ds^2 = e^{2V(r)} (-dt^2 + dz^\alpha dz^{\alpha}) + e^{2U(r)} dr^2 + N^2 d\Omega^2.
\]  

(2.3)

In what follows we will only consider either \( N(r) = r \) or \( N(r) = N = \text{constant} \). Moreover, \( d\Omega^2 \) denotes the metric of a two-manifold \( S \) of constant Gaussian curvature \( k \) and we choose

\[
d\Omega^2 = d\theta^2 + f^2 d\phi^2,
\]  

(2.4)

where

\[
f = \begin{cases} 
\sin \theta & , \quad k = 1, \\
1 & , \quad k = 0, \\
\sinh \theta & , \quad k = -1.
\end{cases}
\]  

(2.5)