Comparion of Analytical Methods of $E1$
Strength Calculations in Middle and Heavy
Nuclei

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Simple analytical models for $E1$ strength function calculations of
the $\gamma$-decay are investigated. The MLO and GFL models ([1]-
[4]) are recommended as the best models for $E1$ gamma-decay
strength function calculations.

1 Introduction

Gamma-emission is one of the most universal channel of the nuclear de-
excitation processes which can accompany any nuclear reaction. Gamma-
decay as well as photoabsorption can be described through the use of the
radiative strength (RS) functions [5, 6]. The electron- positron decay is
also depended on shape of these functions. There are radiative strength
functions of two types. The photoexcitation strength function is connected
with cross-section of the gamma-ray absorption. The average radiative width
$\Gamma_\lambda$ is determined by a gamma- decay (downward) strength function $\bar{f}_\lambda$, $\bar{f}_\lambda \equiv \Gamma_\lambda(\epsilon_\gamma)\rho(U)/3\epsilon_\gamma^3\rho(U - \epsilon_\gamma)$, where $\rho(U)$ is total density of the initial
states in heated nuclei at initial excitation energy $U$ and $\epsilon_\gamma$ is the gamma-ray
energy. Therefore the RS function is important constituent of the compound
nucleus model calculations of capture cross sections, gamma-ray production
spectra, isomeric state populations and of competition between gamma-ray
and particle emission. The gamma-ray strengths are ingredient of time-
consuming calculations of different nuclear processes, for these reasons simple
closed-form expressions are preferred for strength functions. The approaches
based on recent achievements in nuclear studies are also preferable to improve
the reliability of the RS function expressions.
Dipole electric gamma-transitions are dominant, when they occur simultaneously with transitions of other multipolarities and types. According to the Brink hypothesis [7, 8], the Lorentzian line shape with the energy-independent width (SLO model) is used widely for calculations of the dipole RS function. This approach is probably the most appropriate simple method for a description of the photoabsorption data in medium-weight and heavy nuclei [6, 9, 10]. In the case of the gamma-emission, the SLO model strongly underestimates [11] the gamma-ray spectra at low energies $\epsilon_\gamma \lesssim 1\text{MeV}$. A global description of the gamma-spectra by the Lorentzian can be obtained in the range $1 \lesssim \epsilon_\gamma \lesssim 8\text{MeV}$ only when the giant dipole resonance (GDR) parameters are inconsistent with that ones for photoabsorption data. On the whole, the SLO approach overestimates the integral experimental data like the capture cross sections and the average radiative widths in heavy nuclei ([6, 12]-[16]).

The first model with correct description of the $E1$ strengths at the energies $\epsilon_\gamma$ near zero was proposed in Ref.[17]. Thereafter an enhanced generalized Lorentzian model (EGLO) was developed and analyzed in Refs.[15, 16] for a unified description of the low-energetic and integral data. The EGLO radiative strength function consists of two components for spherical nuclei: the Lorentzian with the energy and temperature dependent empirical width and additional term [17] corresponding to zero value of $\gamma$-ray energy. The EGLO method reproduces the experimental data on gamma-decay rather well in the mass region $A = 50 \div 200$. It should be noted that the EGLO and SLO expressions for the gamma-decay strength function of heated nuclei are in fact the parameterizations of the experimental data. They are in contradiction with some aspect of the recent theoretical studies, specifically:

1) the shapes of the radiative strengths within these approaches do not consistent with general relation between the RS function for gamma-decay of heated nuclei and the imaginary part of the nuclear response function on electromagnetic field [18, 19];

2) the damping width of the EGLO model is proportional to collisional component of zero sound damping width in the infinite Fermi-liquid when the collisional (two-body) relaxation is taken into account only. However the important contribution to the total width in nuclei is also given by the fragmentation (one-body) width arising from nucleon motion in self-consistent mean field [20]. This width is almost independent of the nuclear temperature and is not included in EGLO model. Note that the SLO-model width is independent of energy; that is, it has properties of the fragmentation width but it is not allowing for collisional damping.

Recently new closed-form models for the $E1$ strength were proposed in Refs. ([1]-[4]) which avoid some of these defects at least in approximate way.
Specifically, the approach considered in Refs.([1]-[3]) is in line with detailed balance principle [21]. This method was named previously as the thermodynamic pole approximation. It is denoted below as the modified Lorentzian (MLO) approach, because a resulting expression for the dipole RS function has a Lorentzian shape scaled by an enhanced factor. An expression for energy-dependent damping width of the MLO model includes also component corresponding to fragmentation contribution in some simplified way. Damping width appearing in the generalized Fermi liquid (GFL) model [4] of the E1 strength has fragmentation component too; it is taken as resulted from the dipole-quadrupole interaction. Below we investigate these new approaches to compare their with EGLO and SLO models.

2 Simple closed-form models for radiative strength function calculations

The expression for dipole radiative strength function has the following form in SLO model

\[ \langle f \rangle_{SLO}(\epsilon_\gamma) = 8.674 \cdot 10^{-8} \sigma_r \Gamma_r \frac{\epsilon_\gamma \Gamma_r}{(\epsilon_\gamma^2 - E_r^2)^2 + (\Gamma_r \epsilon_\gamma)^2}, \]  

where \( \sigma_r \) (in mb), \( \Gamma_r \) and \( E_r \) (in MeV) are GDR parameters.

The expression for gamma-decay dipole RS function within EGLO model [17] is the following,

\[ \langle f \rangle_{EGLO}(\epsilon_\gamma) = 8.674 \cdot 10^{-8} \sigma_r \Gamma_r \times \]

\[ \times \left[ \frac{\epsilon_\gamma \Gamma_k(\epsilon_\gamma, T_f)}{(\epsilon_\gamma^2 - E_r^2)^2 + (\epsilon_\gamma \Gamma_k(\epsilon_\gamma, T_f))^2} + 0.7 \Gamma_k(\epsilon_\gamma = 0, T_f) \right], \]  

where \( T_f \) is the temperature of the final state. The energy-dependent width \( \Gamma_k(\epsilon_\gamma, T_f) \) is taken proportionally to the collisional damping width in Fermi-liquid scaled by an empirical [17] function \( K(\epsilon_\gamma) \):

\[ \Gamma_k(\epsilon_\gamma, T) = K(\epsilon_\gamma) \Gamma_r \left[ \epsilon_\gamma^2 + (2\pi T)^2 \right] / E_r^2, \]

\[ K(\epsilon_\gamma) = \kappa + (1 - \kappa)(\epsilon_\gamma - \epsilon_0)/(E_r - \epsilon_0). \]  

Dipole strength of the GFL model [4], \( \langle f \rangle_{E1} \equiv \langle f \rangle_{GFL} \), can be given in the following form for spherical nuclei

\[ \langle f \rangle_{GFL}(\epsilon_\gamma) = 8.674 \cdot 10^{-8} \sigma_r \Gamma_r K_{GFL} \epsilon_\gamma \Gamma_m(\epsilon_\gamma, T_f)(\epsilon_\gamma^2 - E_r^2)^2 + K_{GFL}(\Gamma_m(\epsilon_\gamma, T_f)\epsilon_\gamma)^2), \]  

\[ K_{GFL} = \sqrt{E_r/E_0} = (1 + F_1'/3)^{1/2}/(1 + F_0')^{1/2} = 0.63, \]  

\[ \frac{\Gamma}{\Gamma} \]
where $F'_0$ and $F'_1$ are the Landau parameters of the quasiparticle interaction in isovector channel of the Fermi system; $F'_0 = 1.49$ and $F'_1 = -0.04$; $E_0$ is the average energy of one-particle one-hole states forming GDR. The Eq.(4) is an extension of original expression of the GFL model [4] for wide range of the gamma-ray energies: term $K_{GFL}(\Gamma_m \varepsilon_{\gamma})^2$ is added to the denominator of the Eq.(4) to avoid singularity of the GFL approach near GDR-energy in a way similar to the other models for E1 strength but with additional factor $K_{GFL}$ in order to keep standard relationship between value of the RS function at the GDR energy and peak value $\sigma_r$ of the photoabsorption cross-section.

The width $\Gamma_m$ in (4) is taken as a sum of a collisional damping width, $\Gamma_C$, and a term, $\Gamma_{dq}$, which simulates the fragmentation width,

$$\Gamma_m(\varepsilon_{\gamma}, T) = \Gamma_C(\varepsilon_{\gamma}, T) + \Gamma_{dq}(\varepsilon_{\gamma}), \quad \Gamma_C \equiv C_{coll} \left( \varepsilon_{\gamma}^2 + 4 \pi^2 T^2 \right), \quad (5)$$

$C_{coll} = 16 m \cdot \sigma(np)/4\pi^2 \hbar^2$. Here, a magnitude of the in-medium cross section $\sigma(np)$ is taken proportional to the value of the free space cross section $\sigma_f(np) = 5 f m^2$ (near Fermi surface) with a factor $F$. The form of collisional component corresponds to damping width in infinite Fermi-liquid and $\Gamma_{dq}$ is considered as resulted from spreading GDR over surface quadrupole vibrations due to dipole-quadrupole interaction:

$$\Gamma_{dq}(\varepsilon_{\gamma}) = C_{dq} \varepsilon_{\gamma} \left| \tilde{\beta}_2 \right| \sqrt{1 + E_2 \varepsilon_{\gamma}} = C_{dq} \sqrt{\varepsilon_{\gamma}^2 \tilde{\beta}_2^2 + \varepsilon_{\gamma} s_2}, \quad s_2 = E_2 \tilde{\beta}_2^2. \quad (6)$$

Here, $C_{dq} = (5 \ln 2/\pi)^{1/2} = 1.05$; $E_2$ is energy of the first excited vibrational $2^+$ state; $\tilde{\beta}_2$ is effective deformation parameter of nuclear surface. It is determined by reduced electric photoabsorption transition rate for the ground state to $2^+$ state transition [22]. The global parameterization [22] is adopted for $s_2$: $s_2 \equiv E_2 \tilde{\beta}_2^2 = 217.16/A^2$, when experimental data for even-even nuclei and in the case of odd and odd-odd nuclei.

It should be noted that there is not consistency of GFL model with detailed balance principle in systems with constant temperature and the collisional components of the damping width of the GFL model can have negative values in some deformed nuclei if magnitude $C_{coll}$ in Eq.(5) is found from fitting width $\Gamma_m$ to experimental value of GDR width.

Expression for dipole RS within MLO model, $\tilde{f}_{E1} \equiv \tilde{f}_{MLO}$, is obtained by calculating the average radiative width in nuclei with microcanonically distributed initial states ([1]-[3, 19]). In spherical nuclei the dipole radiative strength function is proportional to the strength function for nuclear response on dipole field with frequency $\omega = \varepsilon_{\gamma}/\hbar$. An analytical semiclassical expression for nuclear response function in cold and heated nuclei with excitation of GDR and theory-supported expressions for damping are used. As the result
the radiative strength function within MLO model has the following general form (in MeV)

$$f_{MLO}(\epsilon, T_f) = 8.674 \times 10^{-8} \sigma_r \Gamma(\epsilon, T_f) \frac{\epsilon_r \Gamma(\epsilon, T_f)}{(\epsilon^2 - E_r^2)^2 + (\Gamma(\epsilon, T_f)\epsilon)^2}, \quad (7)$$

where scaling factor $L(\epsilon, T_f) = 1/[1 - \exp(-\epsilon/T_f)]$ is essential for low-energy radiation and leads to consistency of the Eq. (7) with detailed balance principle in systems with constant temperature [18, 19].

Three variants of the modified Lorentzian model (MLO1, MLO2 and MLO3) are considered below. They include different expressions for damping width $\Gamma(\epsilon, T_f)$. The expression for MLO1 approach has the following form ([1]-[3])

$$\Gamma_{MLO1}(\epsilon, T_f) \approx \beta \gamma_c(\epsilon, T_f) \frac{E_r^2 + E_0^2}{(E_r^2 - E_0^2)^2 + (\gamma_c(\epsilon, T_f)\epsilon)^2}, \quad (8)$$

$$\beta = \left(1 + \frac{E_r^2}{E_0^2}\right)^2 \frac{E_0^2}{2}, \quad \gamma_c(\epsilon, T) = \frac{2\hbar}{\tau_c(\epsilon, T)}.$$ 

The energy $E_0$ of particle-hole state is taken as equal the harmonic oscillator energy $\hbar \omega_0 = 41/A^{1/3},$ MeV. In this model the relaxation time within doorway state mechanism [23] is used

$$\frac{\hbar}{\tau_c(\epsilon, T)} = b(\epsilon + U), \quad b = \frac{E_r F}{4\pi \alpha}, \quad \alpha = \frac{9h/16m}{\sigma_{\text{free}}(np)} , \quad F = \frac{\sigma(n_{p})}{\sigma_{\text{free}}(np)}. \quad (9)$$

The damping widths in MLO2 and MLO3 models are taken in approximation of independent sources of dissipation [24] as a sum of the collisional damping width $\Gamma_C$ and a term which simulate the fragmentation component of the width: $\Gamma(\epsilon, T_f) = \Gamma_C(\epsilon, T_f) + \Gamma_F(\epsilon)$. The component $\Gamma_C$ is inversely proportional to the collision relaxation time $\tau$ in isovector channel at dipole distortion of the Fermi surface [23]. The magnitude of $\Gamma_F$ is taken proportionally to the wall formula value [25] $\Gamma_w$ with a scaling factor $k_s$: $\Gamma_F(\epsilon) = k_s(\epsilon) \Gamma_w, \Gamma_w = 36.43 \cdot A^{-1/3} (MeV)$. As a result, the expression for the damping width has the following form,

$$\Gamma_{MLO2,3}(\epsilon, T_f) = \hbar/\tau_c(\epsilon, T_f) + k_s\Gamma_w. \quad (10)$$

The energy-dependent power approximation is adopted for simplicity for factor $k_s$ : $k_s(\epsilon) = k_r + (k_0 - k_r)(\epsilon - E_r)/E_r^{\alpha_s}$, when $\epsilon < 2E_r$, and $k_s(\epsilon) = k_0$ if $\epsilon \geq 2E_r$, where the quantities $k_0 \equiv k_s(\epsilon = 0), k_r \equiv k_s(\epsilon = E_r)$ determine the contribution of the "wall" component to the width at zero
energy and GDR-energy, respectively. The value of the \( k_r \) is obtained from fitting the GDR width \( \Gamma_r \) at zero temperature by the expression (10) with \( \epsilon_\gamma = E_r \). The quantities \( k_0 \) and \( n_s \) are some parameters which are obtained below from condition of correct description of a general behaviour of the experimental gamma-decay strengths. The collisional damping width \( \Gamma_C \) is taken in form given by Eq.(9) in the case of MLO2 model. The expression for relaxation time according to the Fermi-liquid approach is used for MLO3 model:

\[
\frac{\hbar}{\tau_c(\epsilon_\gamma, T)} \equiv \frac{F}{\alpha} \left[ \left( \frac{\epsilon_\gamma}{2\pi} \right)^2 + T^2 \right].
\] (11)

3 Analysis of the dipole strength function calculations

The RS functions were calculated on following nuclei: \(^{198}\text{Au}, ^{168}\text{Er}, ^{156}\text{Gd}, ^{158}\text{Gd}, ^{146}\text{Nd}, ^{144}\text{Nd}, ^{196}\text{Pt}, ^{148}\text{Sm}, ^{182}\text{Ta}, ^{174}\text{Yb}, ^{172}\text{Yb}, ^{90}\text{Zr}, ^{139}\text{Ba}, ^{137}\text{Ba}, ^{120}\text{Sn}, ^{106}\text{Pd} \). The shape of the RSF within all models was investigated and compared with experimental data; the \( \chi^2 \)-criteria was used. In Fig.1 the shape of the RSF in \(^{146}\text{Nd} \) is given as an example. Here, only the MLO2 model is shown, because of similarity of the RSF for different MLO approaches. The following global set of parameters was used \( F = 1, k_s = 0, n_s = 0.1 \).

This set can be recommended for calculations within MLO models. The optimal parameters for each nuclear can be used for more precise calculation. Calculation within GFL and MLO models are rather similar, and they are in close agreement with experimental data.

The GFL and MLO models can be also used to estimate the M1 strength \( \tilde{f}_{M1}(E_\gamma) \) in a wide interval of \( \gamma \)-ray energies on the base of the ratio \( R = \tilde{f}_{E1}(B_n)/\tilde{f}_{M1}(B_n) \) for the E1 and M1 strength functions at neutron binding energy \( B_n \). Then the M1 strength function is calculated by the following relationship

\[
\tilde{f}_{M1}(E_\gamma) = \tilde{f}_{E1}(B_n) R \phi_{M1}(E_\gamma) \phi_{M1}(B_n),
\] (12)

where \( \phi_{M1}(E_\gamma) \) is a function describing shape of the dipole magnetic radiative strength function. The magnitude of the \( R \) can be obtained from experimental data or systematics [16]: \( R = \tilde{f}_{E1}(B_n)/\tilde{f}_{M1}(B_n) = 0.0588 \cdot A^{0.878}, \quad B_n \approx 7 \text{ MeV} \).

Two models [16] for a function \( \phi_{M1} \) are usually used:
1) $\phi_{M1}(E_\gamma) = \text{const}$ according to single-particle model;

2) $\phi_{M1}(E_\gamma)$ is taken in form of the SLO model(1) corresponding to spin-flip giant resonance mode with the following global values [16] for energy and damping width (in MeV): $E_r = 41 \cdot A^{-1/3}, \Gamma_r = 4$.

The radiative strength functions for $E1 + M1$ transitions were calculated for the following nuclei: $^{200}$Hg, $^{196}$Pt, $^{198}$Au, $^{192}$Ir, $^{190}$Os, $^{183}$W, $^{182}$Ta, $^{181}$Hf, $^{177,176}$Lu, $^{174}$Yb, $^{170}$Tm, $^{168}$Er, $^{166}$Ho, $^{164}$Dy, $^{160}$Tb, $^{158}$Gd, $^{150}$Sm, $^{146}$Nd, $^{140}$La, $^{139,138,137}$Ba, $^{128}$I, $^{125,124}$Te, $^{114}$Cd, $^{90}$Zr, $^{80}$Br. The experimental data were taken from Ref.[26].

In Fig.2 the RS functions in $^{174}$Yb are shown. Here, the global parameter set is used for MLO2 model. It can be seen that MLO and GFL models are also useable in case of the $E1 + M1$ emission.

4 Conclusions

The numerical studies led to the following conclusions on the description of the $E1$ and $E1+M1$ gamma-decay strength functions by the simple analytical models. The calculations by the MLO and GFL models are in more close agreement with experimental data than that ones within the EGLO and SLO methods at the energies $\epsilon_\gamma \gtrsim 4.5 \text{ MeV}$. The overall comparison between calculations within the MLO, EGLO, SLO and GFL models and experimental data showed that the MLO and GFL approaches provide a rather reliable method of the $\gamma$-decay strength description in a relatively wide energy interval ranging from zero gamma-ray energy to values above GDR peak energy. The MLO and GFL methods are not time consuming. They can be applicable for calculations and predictions of the statistical contribution to the dipole strength functions as well as for extraction of the GDR parameters of heated nuclei with small errors with use of the $\gamma$-emission data.

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References


Fig. 1: The E1 gamma-decay strength functions versus gamma-ray energy for $^{146}$Nd; $F = 1$, $k_s = 0.3$, $n_s = 0.5$.

Fig. 2: The $E1 + M1$ gamma-decay strength functions versus gamma-ray energy for $^{174}$Yb.