Multiplicity Distribution Studies of $e^+e^-$-annihilation at 50-61.4 and at 172-189 GeV by Two Stage Model hadronization 1

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Abstract

The multiplicity distributions at high energy $e^+e^-$ annihilation are described well within the Two Stage Model in the region from 14 to 189 GeV. Energy dependence of parameters of this model gives the dynamic picture of the parton stage and the stage of hadronization. It is shown that oscillations in sign of the ratio of factorial cumulant moments over factorial moments of increasing order can be confirmed by this model.

1 Introduction

Multiparticle production (MP) is one of the most important topics in high energy physics. Using MP we can get more information about the nature of strong interactions and understand deeper the structure of matter. Over the last few years many thorough reviews devoted to MP have been done [1]. Modern accelerators have made it possible to study MP more extensively and in detail. Developing theory of high energy physics quantum chromodynamics (QCD) [2] and a lot of phenomenologic models are tested by the process of MP.

Multiparticle processes begin at high energy. Among all producing particles we can observe a lot of hadrons. On one hand we want to know high energy physics, but on the other hand the increase of the inelastic channels makes it difficult to describe this process with customary methods. The situation concerning the history of thermodynamics developing and statistical physics is much the same. Analysis of MP process is carried out using of statistical methods because the number of secondaries in $e^+e^-$ annihilation is large (more than 60) [3]. The consideration of MP begins from the behavior of charged multiplicity.

As it is generally known the multiplicity is the number of secondaries $n$ in process of MP: $A + B \rightarrow a_1 + a_2 + \ldots + a_n$. The multiplicity distribution (MD) $P_n$ is the ratio of cross-sections $\sigma_n$ to $\sigma = \sum_n \sigma_n$: $P_n = \sigma_n/\sigma$. This quantity has the following meaning:

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the probability of producing $n$ charged particles in this process. We can also construct quantities such as mean values, moments of MD, can study correlations and so on.

Investigation of MP has led to discovery of jets. Jets phenomena can be studied in all processes, where energetic partons are produced. The most common ones are in $e^+e^-$ annihilation, deep inelastic scattering of $e, \mu$ or $\nu$ on nucleons and hadron- hadron scattering, involving high-$p_T$ particles in final state. Let us consider $e^+e^-\text{-annihilation at high energy. This process is one of the most suitable for the study of MP. In accordance with QCD it can be realized through the production of } \gamma \text{ or } Z^0\text{-boson into two quarks:}

$$e^+e^- \rightarrow (Z^0/\gamma) \rightarrow q\bar{q} \quad (1)$$

Perturbative QCD can describe the process of fission partons (quarks and gluons) at high energy, because the strong coupling $\alpha_s$ is small at that energy. This stage can be called as the stage of cascade. After hard fission, when partons have not high energy, they must be changed into hadrons, which we can observe. On this stage we shouldn’t apply perturbative QCD. Therefore phenomenological models are used for description of hadronization (transformation of quarks and gluons into hadrons) in this case.

The description of the stage quark-gluon cascade by means of perturbative QCD was applied in [4, 5]. Certain features of the predictions at the parton level are expected to be insensitive to details of the hadronization mechanism. They were tested directly by using hadron distributions [6].

The $e^+e^-\text{-reaction is simple for analysis, as the produced state is pure } q\bar{q}. \text{ It is usually difficult to determine the quark species on event-by-event basis. The experimental results are averaged over the quark type. Because of confinement the produced quark and gluons fragment into jets of observable hadrons.}

The hadronization models are more phenomenological and are built on the experience gained from the study of low–$p_T$ hadron collisions. It is usually considered that the producing of hadrons from partons is universal process.

2 Two Stage Model

Parton spectra in QCD quark and gluon jets were studied by Konishi K., Ukawa A. and Veneciano G.[4]. Working at the leading logarithm approximation and avoiding IR divergences by considering finite $x$, the probabilistic nature of the problem has been established [4].

At the studying of MP at high energy we used idea of A. Giovannini [7] for description of quark-gluon jets as Markov branching processes. Giovannini proposed to interpret the natural QCD evolution parameter $Y$

$$Y = \frac{1}{2\pi b} \ln[1 + \alpha_s b \ln(\frac{Q^2}{\mu^2})], \quad (2)$$

where $2\pi b = \frac{1}{6}(11N_C - 2N_f)$ for a theory with $N_C$ colours and $N_f$ flavours, as the thickness of the jets and their development as Markov process.

Three elementary processes contribute into QCD jets:

1. gluon fission;
2. quark bremsstrahlung;
3. quark pair creation.
Let $A \Delta Y$ be the probability that gluon in the infinitesimal interval $\Delta Y$ will convert into two gluons, $\tilde{A} \Delta Y$ be the probability that quark will radiate a gluon, and $B \Delta Y$ be the probability that a quark-antiquark pair will be created from a gluon. $A, \tilde{A}, B$ are assumed to be $Y$-independent constants and each individual parton acts independently from others, always with the same infinitesimal probability.

Let us define the probability that parton will be transformed into $m$ gluons over a jet of $Y$ in thickness and call it $P^P_m(Y)$. The probability generating function for a parton jet will be

$$Q^P(z; Y) = \sum_{m=0}^{\infty} P^P_m(Y) z^m. \quad (3)$$

A.Giovannini constructed system of differential equations and obtained explicit solutions of MD for a parton jet in particular case $B = 0$ (process of quark pair creation is absent). In the common case $B \neq 0$ MD are similar to particular one [7].

For quark jet explicit solutions are given [7]

$$P_0(Y) = e^{-\tilde{A}Y},$$

$$P_m(Y) = \mu(\mu+1)\ldots(\mu+m-1) \frac{1}{m!} e^{-\tilde{A}Y} (1 - e^{-AY})^m, \quad (4)$$

where $\mu = \frac{\tilde{A}}{A}$. Further the average gluon multiplicity is $\overline{m} = \mu(e^{AY} - 1)$ and the normalized exclusive cross section for producing $m$ gluons from quark is

$$\frac{\sigma^q_m}{\sigma_{tot}} = P_m(Y) = \frac{\mu(\mu+1)\ldots(\mu+m-1)}{m!} \left[ \frac{\overline{m}}{\overline{m} + \mu} \right]^m \left[ \frac{\mu}{\overline{m} + \mu} \right]^\mu. \quad (5)$$

The generating function (3) will be given by

$$Q^q(z, Y) = \sum_{m=0}^{\infty} z^m P_m(Y) = \left[ \frac{e^{-AY}}{1 - z(1 - e^{-AY})} \right]^\mu. \quad (6)$$

Eq.(4) is Polya-Egenberger distribution, where $\mu$ is non-integer.

In Two Stage Model [8] we took (4) for description of cascade stage and added supernarrow binomial distribution for hadronization stage. We chose it based ourselves on analysis of experimental data in $e^+ e^-$ annihilation lower 9 GeV. Second correlation moments were negative at this energy. The choice of such distributions was the only one could describe experiment.

We suppose that hypothesis of soft colourless is right. We add stage of hadronization to parton stage with aid of it’s factorization. MD in this process can be written

$$P_n(s) = \sum_m P^P_m P^H_n(m, s), \quad (7)$$

where $P^P_m$ is MD for partons (4), $P^H_n(m, s)$ - MD for hadrons produced from $m$ partons on the stage of hadronization. Further we will use instead of parameter $Y$ CM(center of masses) energy $\sqrt{s}$. 

3
In accordance with TSM the stage of hard fission of partons is described by negative binomial distribution (NBD) for quark jet

\[ P_m^P(s) = \frac{k_p(k_p + 1) \ldots (k_p + m - 1)}{m!} \left( \frac{m}{m + k_p} \right)^m \left( \frac{k_p}{k_p + m} \right)^{k_p}, \quad (8) \]

where \( k_p = \tilde{A}/A, \quad m = \sum_{m} m P_m^P. \) We neglect process (3) quark pair production \((B = 0)\).

Two quarks fracture to partons independently of one another. Total MD of two quarks is equal to \((7)\) too. Parameters \( k_p \) and \( m \) of MD for two joint quark-antiguark jets are doubled, but we use that designations.

\( P_m^P \) and generating function for MD \( Q^P(s, z) \) are

\[ P_m^P = \frac{1}{m!} \frac{\partial^m}{\partial z^m} Q^P(s, z) \bigg|_{z=0}, \quad (9) \]

\[ Q^P_m(s, z) = \left[ 1 + \frac{m}{k_p}(1 - z) \right]^{-k_p}. \quad (10) \]

MD of hadrons formed from parton are described in form \([8]\)

\[ P_n^H = C_{N_p}^m \left( \frac{\overline{\pi}_h}{N_p} \right)^n \left( 1 - \frac{\overline{\pi}_h}{N_p} \right)^{N_p-n}, \quad (11) \]

\((C_{N_p}^m \text{- binomial coefficient})\) with generating function

\[ Q^H_p = \left[ 1 + \frac{\overline{\pi}_h}{N_p}(z-1) \right]^{N_p}, \quad (12) \]

where \( \overline{\pi}_h \) and \( N_p \) \((p = q, g)\) have meaning of average multiplicity and maximum secondaries of hadrons are formed from parton on the stage of hadronization. MD of hadrons in \( e^+e^- \) annihilation are determined by convolution of two stages

\[ P_n(s) = \sum_{m=0}^{\infty} P_m^P \frac{\partial^n}{\partial z^n} (Q^H)^{2+m}|_{z=0}, \quad (13) \]

where \( 2 + m \) is total number of partons (two quarks and \( m \) gluons).

Further we do the following simplification for the second stage: \( \frac{\overline{\pi}_h}{N_q} \approx \frac{\overline{\pi}_h}{N_g} \), considering that probabilities of formation of hadron from quark or gluon are equal. We introduce parameter \( \alpha = \frac{N_g}{N_q} \) for distinguishing between hadron jets, created from quark or gluon on the second stage. We also make simplification for designation \( N = N_q, \overline{\pi}_h = \overline{\pi}_q \). Then we get

\[ Q^H_q = \left( 1 + \frac{\overline{\pi}_h}{N}(z-1) \right)^N, \]

\[ Q^H_g = \left( 1 + \frac{\overline{\pi}_h}{N}(z-1) \right)^{\alpha N}. \]
Introducing in (13) expressions (8), (12) and differentiating on $z$ we obtain MD of hadrons in the process of $e^+e^-$ annihilation in TSM

$$P_n(s) = \sum_{m=0}^{M_g} P_m^P C^m_{(2+\alpha m)N} \left( \frac{\pi^h}{N} \right)^n \left( 1 - \frac{\pi^h}{N} \right)^{(2+\alpha m)N-n}. \quad (14)$$

For comparing with experimental data the normalized factor $\Omega$ was introduced into (13) and a number of gluons in the sum was restricted by $M_g$ - maximal number of possible gluons created on the first stage

$$P_n(s) = \Omega \sum_{m=0}^{M_g} P_m^P C^m_{(2+\alpha m)N} \left( \frac{\pi^h}{N} \right)^n \left( 1 - \frac{\pi^h}{N} \right)^{(2+\alpha m)N-n}. \quad (15)$$

The results of comparison of model expression (15) with experimental data [12] are represented in Table 1 (parameters of two stages) and on Figures (1)-(18). We can see that MD in TSM (solid curve) are describing well the experimental $e^+e^-$-data (black square ■) from 14 to 189 GeV. Summing up is limited to $M_g$ equal 20 – 22 for energies to 61.4 GeV and 33 – 41 above, because further increase does not change $\chi^2$.

The description of experimental MD [12] by (15) gives $\chi^2 \sim 1$ for the most energies with the expection of 34.8 GeV and 55 GeV (more high $\chi^2$), $N_{DF} = 14$. Calculated MD are giving small deviations from experimental data at 34.8 GeV, but for almost all multiplicities (Fig.3). Probably it is connected with small statistics of events.

At 55 GeV they give big deviations, but only to the right side of central region (Fig.7). It can be described as energy suppression of formation of hard gluons connected with appearance of heavy $q\bar{q}$ pair or heavy hadrons. Energy behavior of total cross section of $e^+e^- \rightarrow$ hadrons [9] is changed to a sharp rise from here. It also may be connected with the reason of narrowing of distribution.
Table 1. Parameters of TSM.

<table>
<thead>
<tr>
<th>√s GeV</th>
<th>m</th>
<th>k_p</th>
<th>N</th>
<th>m^4</th>
<th>α</th>
<th>Ω</th>
<th>χ^2</th>
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<td>14</td>
<td>0.08</td>
<td>2.4 \times 10^8</td>
<td>27.7</td>
<td>2.87</td>
<td>0.97</td>
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<td>2.</td>
<td>2.</td>
<td>1.29</td>
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<td>34.8</td>
<td>6.58</td>
<td>6.96</td>
<td>12.5</td>
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<td>43.6</td>
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<td>48.3</td>
<td>5.16</td>
<td>2.31</td>
<td>0.44</td>
<td>2.</td>
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<td>1.3</td>
<td>24.6</td>
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<td>2.09</td>
<td>1.97</td>
</tr>
<tr>
<td>52</td>
<td>11.5</td>
<td>1</td>
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<td>0.104</td>
<td>2.46</td>
<td>2.51</td>
</tr>
<tr>
<td>55</td>
<td>8.6</td>
<td>6. \times 10^{-4}</td>
<td>17.</td>
<td>4.</td>
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<td>14.4</td>
<td>4</td>
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<td>2.</td>
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<td>2.97</td>
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<tr>
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<td>9.17</td>
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<td>0.195</td>
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<tr>
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<td>11.6</td>
<td>5.15</td>
<td>0.215</td>
<td>2.01</td>
<td>2.37</td>
</tr>
</tbody>
</table>

3 Dynamics of multiparticle production

We will analyse dynamics of MP which corresponds to values of parameters of TSM (Table 1). We will begin from a cascade stage. This stage is described by two parameters: m and k_p.

The average multiplicity of gluons m formed on fission stage has tendency to rise. It is changed from ∼0.1 at 14 GeV to ∼20 at 183 GeV. But we can see certain insignificant deviation from this direction at √s=50–61.4 GeV, at 183 GeV. It follows from QCD that in particular case (B = 0) the parameter k_p that equal to ratio 2 \bar{A}/A → 1. Values k_p are changed insignificantly. They are remained ∼10 at almost all energies. There is some physical senses of this parameter. One of the most interesting from them is temperature T [10]:

\[ k_p^{-1} = T_0 + 1/cE, \]

where T_0 is the temperature of system before interaction, c - thermal capacity, E - energy spented on creationg new particles [10]. In this sense we can make assumptions: temperature of parton system with developed cascad are lowest at 14 and 55 GeV than at the others. The highest temperatures are reached on first stage at 50, 52, 61.4 and 183 GeV.

The interesting picture of hadronization is discovered in conformity with parameters of second stage N_q, m_q^4 and α. The parameter N_q determines maximum number of hadrons, which can be formed from quark on this stage. In TSM (Table 1) it takes different values from 4 to 55. We can’t reveal steady energy rise or fall for it. Big N_q point to predominance of hadrons formed from quark jets, small N_q point to essential contribution gluon jets in the hadron multiplicity. More probably that this parameter remaines constant and ∼16 with small deviations.

Next parameter m_q^4 has meaning of mean hadron multiplicity from quark on second stage. We can see the tendency to weak rise with big scatter. Such behavior of parameter
may be connected with the growth of spectrum of mass hadron states (appearance of new mass states with increase of energy). The average value of $\pi_h$ is about $5 - 6$ in the research region.

Parameter $\alpha$ was introduced for comparison quark and gluon jets. It is almost constant and equal to 0.2 with some deviations. If we know $\alpha$ then we can determine analogous parameters $N_g = \alpha N_q$ and $\pi_g^h = \alpha \pi_q^h$ for gluon jet. It is interesting that these parameters remain constant without considerable deviations: $N_g \sim 3$ and $\pi_g^h \sim 1$ (Figures 19 and 20). From this result we can affirm about universality of hadronization.

The fact that $\alpha < 1$ says that hadronization of gluon jets are more soft than quark one. The simplest explanation to this phenomenon is the fact that a quark takes away more considerable energy than gluon.

The ratio $\pi_g/\pi_q$ determines the probability of formation of hadron from parton. It is increased from $\sim 0.1$ to $\sim 0.5$ with rise of energy from 14 to 43.6 GeV, then we have big variations in region $50 - 61.4$ GeV ($0.23 - 0.69$) and the ratio is almost constant at higher energies ($\sim 0.4$) (Figure 21). It should be noted that the small probabilities are realized at 55 GeV (in the region $50 - 61.4$ GeV) and at 183 GeV.

The normalized factor $\Omega$ remains constant and is equal to 2.

## 4 Oscillation of moments in MD

It was shown recently [13] that the ratio of factorial cumulative moments over factorial moments changes sign as a function of order. We can use MD formed in TSM for explanation of this phenomenon.

The factorial moments can be obtained from MD $P_n$ through the relation

$$F_q = \sum_{n=q}^{\infty} n(n-1) \ldots (n-q+1) P_n,$$

and factorial cumulative moments are found from expression

$$K_q = F_q - \sum_{i=1}^{q-1} C_{q-i}^i K_{q-i} F_i. \quad (18)$$

The ratio of their quantities is

$$H_q = K_q/F_q. \quad (19)$$

We can use the generating function for MD of hadrons (14) in $e^+ e^-$ annihilation $G(z)$

$$G(z) = \sum_{m=0}^{\infty} P_{g/m} [Q_g^H(z)]^m Q_q^2(z) =$$

$$= Q^g (Q_g^H(z))^m Q_q^2(z). \quad (20)$$

We are calculating $F_q$ and $K_q$ in TSM using (20)

$$F_q = \left. \frac{1}{\pi^q(s)} \frac{\partial^q G}{\partial z^q} \right|_{z=0} \quad (21)$$

$$K_q = \left. \frac{1}{\pi^q(s)} \frac{\partial^q \ln G}{\partial z^q} \right|_{z=0} . \quad (22)$$
The expression (20) for $G(z)$ after taking a logarithm

$$\ln G(s, z) = -k_p \ln[1 + \frac{m}{k_p}(1 - Q_g^H)] + 2 \ln Q_q^H$$

and the expansion to series in power on $Q_g^H$ will be

$$\ln G(s, z) = k_p \sum_{m=1} \left( \frac{m}{m+k_p} \right)^m \frac{Q_g^m}{m} + 2 \ln Q_q^H. \quad (23)$$

Inserting $Q_g$ into (23)

$$\ln G(s, z) = k_p \sum_{m=0} \left( \frac{m}{m+k_p} \right)^m \frac{1}{m} \left[ 1 + \frac{\pi^h}{N} (z - 1) \right]^{amN} + 2N \ln[1 + \frac{\pi^h}{N} (z - 1)],$$

and using (22) we obtain

$$K_q = \left( k_p \sum_{m=1} \frac{am(am - 1)}{N} \ldots (am - q - 1) \left( \frac{m}{m+k_p} \right)^m \frac{1}{m} \right.$$

$$\left. -2(-1)^q \frac{(q - 1)!}{N^{q-1}} \right) \left( \frac{\pi^h}{\pi(s)} \right)^q \quad (24)$$

where $\pi(s)$ is the average multiplicity hadrons in process (1). It is possible to find $F_q$ using (21)

$$F_q = \sum_{m=0} (2 + am)(2 + am - \frac{1}{N}) \ldots (2 + am - \frac{q - 1}{N}) P_m \left( \frac{\pi^h}{\pi(s)} \right)^q \quad (25)$$

with $P_m$ equal (8).

The sought-for expression for $H_q$ will be

$$H_q = \Omega_1 \sum_{m=1} \frac{k_p am(am - \frac{1}{N}) \ldots (am - \frac{q - 1}{N})(m^m - k_p)^m}{m} \frac{1}{m} - 2(-1)^q \frac{(q - 1)!}{N^{q-1}}$$

$$\sum_{m=0} (2 + am)(2 + am - \frac{1}{N}) \ldots (2 + am - \frac{q - 1}{N}) P_m \quad (26)$$

where $\Omega_1$ is the normalized factor. The comparison with experimental data [13] shows that (26) describes the ratio of factorial moments (Figures 22-39). It is seen minimum at $q=5$.

In the region before $Z^0$ $H_q$ may oscillate in sign only with the period equal 2, changed sign with parity $q$. At more high energies the period is increased to 4 and higher. It can be explained by influence of hadronization. Values $K_q$ (as well $H_q$) may change sign only owing to second summand in (24). More develope cascad of partons with hadronization comes to big period of oscillations of sign.

The immediate calculations $H_q$ based on (17)-(19) with using MD(15) gives very good description of the oscillation value of $H_q$ ($\chi^2 \approx 2$). Significant oscillations start near region producing of $Z^0$ and can be explained by non-integer values of parameters of hadronization $N_q$ and $N_g = \alpha N_q$ or by convolution of wide (for parton jets) and narrow (for hadron jets on second stage) MD.
5 Conclusions

It is shown that TSM does not contradict to the experimental data on MD and the oscillations ratio of factorial moments. TSM offers concreted physical picture of multiplicity production in high energy $e^+e^-$ annihilation.

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Figure 1: MD at 14GeV.

Figure 2: MD at 22GeV.

Figure 3: MD at 34.8GeV

Figure 4: MD at 43.6GeV.

Figure 5: MD at 50GeV.

Figure 6: MD at 52GeV.

Figure 7: MD at 55GeV.

Figure 8: MD at 56GeV.

Figure 9: MD at 57GeV.
Figure 10: MD at 60GeV.

Figure 11: MD at 60.8GeV.

Figure 12: MD at 61.4GeV.

Figure 13: MD at 91.4GeV.

Figure 14: MD at 172GeV.

Figure 15: MD at 183GeV.

Figure 16: MD at 189GeV.
Figure 17: Parameter $N_g = \alpha N_q$.

Figure 18: Parameter $\overline{r}_{g}^{h} = \alpha \overline{r}_{q}^{h}$.

Figure 19: Ratio $\overline{r}_{g}^{h} / N_q$. 
Figure 20: $H_q$ at 14GeV.

Figure 21: $H_q$ at 22GeV.

Figure 22: $H_q$ at 34.8GeV.

Figure 23: $H_q$ at 43.6GeV.

Figure 24: $H_q$ at 50GeV.

Figure 25: $H_q$ at 52GeV.

Figure 26: $H_q$ at 55GeV.

Figure 27: $H_q$ at 56GeV.

Figure 28: $H_q$ at 57GeV.
Figure 29: $H_q$ at 60GeV.

Figure 30: $H_q$ at 60.8GeV.

Figure 31: $H_q$ at 61.4GeV.

Figure 32: $H_q$ at 91.4GeV.

Figure 33: $H_q$ at 172GeV.

Figure 34: $H_q$ at 183GeV.

Figure 35: $H_q$ at 189GeV.