

String breaking with Wilson loops?

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A convincing, uncontroversial observation of string breaking, when the static potential is extracted from Wilson loops only, is still missing. This failure can be understood if the overlap of the Wilson loop with the broken string is exponentially small. In that case, the broken string ground state will only be seen if the Wilson loop is long enough. Our preliminary results show string breaking in the context of the $3d$ $SU(2)$ adjoint static potential, using the Lüscher-Weisz exponential variance reduction approach. As a by-product, we measure the fundamental $SU(2)$ static potential with improved accuracy and see clear deviations from Casimir scaling.

1. INTRODUCTION

The breaking of a long flux tube between two quarks into a matter-antimatter pair is one of the most fundamental phenomena in QCD. Three approaches have been used to measure the static potential: (i) Correlation of Polyakov Loops, at finite T [1]. (ii) Variational Ansatz using two types of operators: One for the string-like state and one for the broken string state [2,3]. (iii) Wilson loops [4,5].

String breaking has been seen using the first two methods, but no clear signal using the third one has been observed. The failure of the Wilson loop method seems to be mainly due to the poor overlap of the operators with the broken state. The strong coupling model [6] uses the heavy quark expansion to show that there is an exponentially small overlap. Based on a topological argument, [7] suggests that there may be no overlap at all. We show that there is a small overlap but not nearly as small as predicted.

2. STATIC POTENTIAL

The static potential helps to characterize confining forces. Assume that a static charge and a static anticharge are separated by a distance

$R < R_b$, where R_b is the string breaking distance. If R is increased to a value larger than R_b , a matter-antimatter pair will be created which screens the static charges. A further increase of R has no effect on the static potential - it remains \approx constant, this indicates string breaking. In our case, we deal with adjoint static charges which are screened by the gluons of the gauge field. The object adjoint_charge-gluon is called a gluelump.

In a Hamiltonian formulation, a static charge and a static anticharge are created at Euclidean time 0, forming a state $|\phi\rangle$, and annihilated at time T . This can be expressed as the Wilson loop $W(R, T)$:

$$\langle\phi|e^{-TH}|\phi\rangle = W(R, T) \quad (1)$$

On the other hand, one can expand the left side in (1) in the eigenbasis $\Psi^{(n)}$ of the Hamiltonian. At large T , only the ground state survives:

$$\begin{aligned} \langle\phi|e^{-TH}|\phi\rangle &= \sum_{n=0}^{\infty} |\langle\Psi^{(n)}|\phi\rangle|^2 e^{-TE_n} \\ &\underset{T\rightarrow\infty}{\sim} |\langle\Psi^{(0)}|\phi\rangle|^2 e^{-TV(R)} \end{aligned} \quad (2)$$

By using (1) and (2), the static potential $V(R)$, which is defined as the energy of the ground state E_0 , can be connected with the Wilson loop:

$$V(R) = -\lim_{T\rightarrow\infty} \frac{1}{T} \log W(R, T) \quad (3)$$

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This holds as long as the overlap $\langle \phi | \Psi^{(0)} \rangle$ does not vanish.

In section 4, we fit the energy of the unbroken case using the Ansatz:

$$V(R) \sim V_0 - \frac{c}{R} + \sigma R \quad (4)$$

where σ is the string tension.

3. METHOD

We are using a $3d$ -lattice with extent $48^2 \times 64$ at $\beta = \frac{4}{ag^2} = 6.0$. The gauge group is $SU(2)$. We measure adjoint static charges in order to avoid costly dynamical quarks. The price to pay is that the signal decays much faster because $V_{adj} \approx \frac{8}{3} V_{fund}$ (Casimir scaling).

We make use of Lüscher-Weisz exponential variance reduction [8]. This method generalizes Multihit [9] from single links to link-link-correlators $\mathbf{T}(na)$. The possibility to use a hierarchical scheme allows us to measure long Wilson loops up to the desired accuracy. Using the notation introduced in [8], we actually implement the following scheme:

$$W(R, T) = \mathbf{L}(0)_{\alpha\gamma} \left\{ \cdots \left[\left[\mathbf{T}(na) \right] \left[\mathbf{T}((n+1)a) \right] \right] \right. \\ \left. \left[\left[\mathbf{T}((n+2)a) \right] \left[\mathbf{T}((n+3)a) \right] \right] \right\} \cdots \}_{\alpha\beta\gamma\delta} \mathbf{L}(T)_{\beta\delta}^* \quad (5)$$

The parameters are the following: The innermost average $[\mathbf{T}(na)]$ was calculated from 10 sets of time-like links, each obtained after 10 updates (1HB/4OR). The second-level average $[[\mathbf{T}(na)][\mathbf{T}((n+1)a)]]$ was calculated from 160 averages of $[\mathbf{T}(na)]$ separated by 200 updates of the spatial links on the time-slice $[(n+1)a]$. Finally, the third-level average was calculated from 39 second-level averages separated by 200 updates of the spatial links on the time-slice $[(n+2)a]$.

To reduce contributions from excited states $\Psi^{(n \neq 0)}$, we replace the simple spatial transporters $\mathbf{L}(0)_{\alpha\gamma}$ and $\mathbf{L}(T)_{\beta\delta}^*$ with "staples", which are constructed in the following way: (i) After each calculation of second level averages, \square , we form the smeared adjoint spatial links, --- , at the

time-slice $(n+2)a$. (ii) We multiply them with the second level averages to obtain the staples \square . (iii) This procedure is repeated 39 times to provide an error reduction in the new spatial transporters.

After processing 40 configurations as above, we obtain the following preliminary results.

4. RESULTS

We measure both the adjoint and the fundamental static potential (Fig. 1). The spatial separation R starts at 2 and is increased up to 12. In the adjoint case, the Ansatz (4) works well for the energy $V'(R)$ of the unbroken string in the whole range of R . However, at $R \geq 10$ the minimal energy remains constant, $V(R \geq 10) = 1.95(10)$. Therefore, we observe string breaking at a distance $R_b \approx 10$.

The dotted line at 2.06(1) represents twice the energy of a gluelump using a direct measurement [4] on our lattice. This value is close to the groundstate potential for $R > R_b$ (See Fig.1). Both $R_b \approx 10$ [3], as the string breaking distance, and $2E(Qg)$ [3,4], as twice the gluelump-energy, are in agreement with the literature.

In the fundamental case, the Ansatz (4) again works very well. An important issue is the fulfilment of the Casimir scaling law. At $R \lesssim 4$, $\frac{V_{adj}}{V_{fund}} \approx \frac{8}{3}$ holds as required by perturbation theory. However at large distances, the slopes of the potentials are in the ratio $2.31(1) < \frac{8}{3}$.

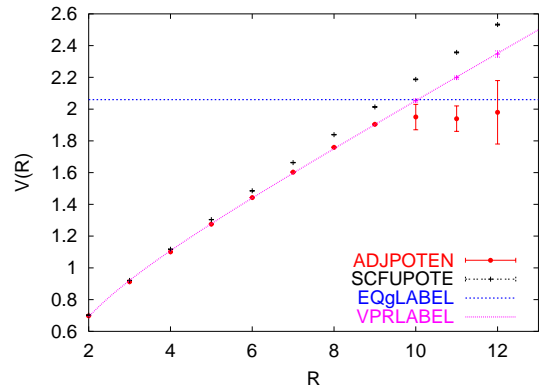


Fig. 1. The adjoint and $\frac{8}{3}$ fundamental static potentials $V(R)$ vs R . The horizontal line at 2.06(1) represents twice

the energy of a gluelump.

The value of the static potential $V(R)$ at a fixed R , e.g. $R = 12$, is determined by the slope of the logarithm of the Wilson loop (3). In Fig. 2 it can be seen that a single exponential fit is not sufficient in the adjoint case. At small temporal extent T we get a larger slope than at large T . This can be explained as follows: At small T the signal is dominated by the unbroken string state. The broken string state can only be observed after T is large. The way to fit all data points is to use the double-exponential fit $\sim c_0 e^{-V'(R)T} + c_1 e^{-2E(Q\bar{g})T}$. $V'(R)$ corresponds to the energy of the unbroken string, $E(Q\bar{g})$ to the energy of the gluelump. The ratio of the amplitudes is $\frac{c_0}{c_1} \sim 10^4$, the turning point is at $T_t \approx 20$.

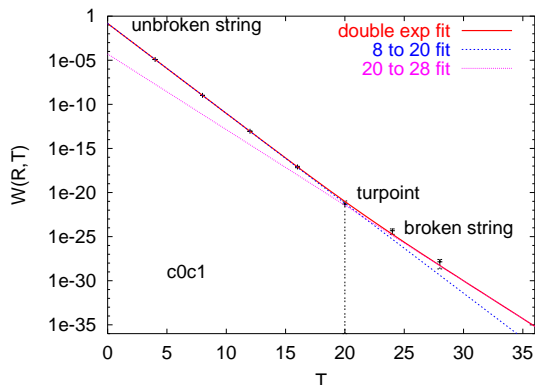


Fig. 2. Adjoint Wilson loop $W(R, T)$ vs T , for $R = 12$. The double-exponential fit matches all data points. Single-exponential fits (dotted lines) do not. The unbroken string state can be fitted by using data from $T = 8$ up to 20, the broken string state by using data from $T = 20$ up to 28. Note how small a signal can be measured.

5. CONCLUSION

In the context of the $3d$ $SU(2)$ adjoint static potential, string breaking has been observed using Wilson loops only.

It is a necessary condition that both R and T have to be larger than the string breaking distance R_b . Note, however, that the variational approach allows to bypass this requirement.

The strong coupling model [6] is too pessimistic. A necessary condition to be able to measure the broken state is that the signal should

be larger than the one from the unbroken state: $c_1 e^{-2M_{Qg}T} \gg c_0 e^{-V(R)T}$. The heavy quark expansion results in the ratio $\frac{c_0}{c_1} \sim e^{M_{Qg}R}$, which would imply $T \gg 43$ at $R = 12$, whereas in our simulation the turning point was $T_t \approx 20$.

A similar observation using Wilson loops in full QCD should be difficult but feasible [10].

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