Generalized Cardassian Expansion: a Model in which the Universe is Flat, Matter Dominated, and Accelerating

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\textsuperscript{a}[The Cardassian universe is a proposed modification to the Friedmann Robertson Walker equation (FRW) in which the universe is flat, matter dominated, and accelerating. In this presentation, we generalize the original Cardassian proposal to include additional variants on the FRW equation; specific examples are presented.

In the ordinary FRW equation, the right hand side is a linear function of the energy density, \( H^2 \sim \rho \). Here, instead, the right hand side of the FRW equation is a different function of the energy density, \( H^2 \sim g(\rho) \). This function returns to ordinary FRW at early times, but modifies the expansion at a late epoch of the universe. The only ingredients in this universe are matter and radiation: in particular, there is NO vacuum contribution. Currently the modification of the FRW equation is such that the universe accelerates; we call this period of acceleration the Cardassian era. The universe can be flat and yet consist of only matter and radiation, and still be compatible with observations. The energy density required to close the universe is much smaller than in a standard cosmology, so that matter can be sufficient to provide a flat geometry. The new term required may arise, e.g., as a consequence of our observable universe living as a 3-dimensional brane in a higher dimensional universe. The Cardassian model survives several observational tests, including the cosmic background radiation, the age of the universe, the cluster baryon fraction, and structure formation. As will be shown in future work, he predictions for observational tests of the generalized Cardassian models can be very different from generic quintessence models, whether the equation of state is constant or time dependent.

Recent observations of Type Ia Supernovae [1,2] as well as concordance with other observations (including the microwave background and galaxy power spectra) indicate that the universe is accelerating. Many authors have explored a cosmological constant, a decaying vacuum energy [3,4], and quintessence [5–7] as possible explanations for such an acceleration.

Recently we proposed Cardassian expansion [8] (hereafter Paper I)\textsuperscript{1} as an explanation for acceleration which invokes no vacuum energy whatsoever. In our model the universe is flat and accelerating, and yet consists only of matter and radiation. Previously we considered the addition of a new term to the right hand side of the FRW equation:

\[
H^2 = A\rho + B\rho^n
\]

where energy density \( \rho \) contains only matter and radiation (no vacuum) and \( n \) is a time independent number with

\[
n < 2/3.
\]

Here \( H = \dot{R}/R \) is the Hubble constant (as a function of time) and \( R \) is the scale factor of the universe. In the usual FRW equation \( B = 0 \). To be consistent with the usual FRW result, we take \( A = \frac{8\pi G}{3m_p} \). The new term is initially negligible, and only comes to dominate at redshift \( z \sim O(1) \).

Once it dominates, it causes the universe to accelerate, as discussed further below.

\textsuperscript{1}The name Cardassian refers to a humanoid race in Star Trek whose goal is accelerated expansion of their evil empire. This race looks foreign to us and yet is made entirely of matter.
1. Generalized Cardassian Models

Here we wish to generalize this proposal to other functions on the right hand side of the FRW equation. Pure matter (or radiation) alone can drive an accelerated expansion if the Friedmann Robertson Walker (FRW) equation is modified to become

\[ H^2 = g(\rho), \]  

(3)

We take \( g(\rho) \) to be a function of \( \rho \) that returns simply to \( \rho \) at early epochs, but that can drive an accelerated expansion in the recent past of the universe at \( z < \mathcal{O}(1) \). We take the usual energy conservation:

\[ \dot{\rho} + 3H(\rho + p) = 0, \]  

(4)

which gives the evolution of matter:

\[ \rho_M = \rho_{M,0}(R/R_0)^{-3}. \]  

(5)

Here subscript 0 refers to today. Eqs.(3) and (4) contain the complete information of the two Friedmann equations.

We note here that the geometry is flat, as required by measurements of the cosmic background radiation [9], so that there are no curvature terms in the equation. There is no vacuum term in the equation. This paper does not address the cosmological constant (\( \Lambda \)) problem; we simply set \( \Lambda = 0 \).

The simplest example of this type of behavior is the sum of two terms:

\[ H^2 = \rho + f(\rho) \]  

(6)

where \( f(\rho) \) is a different function of \( \rho \).

As mentioned above, in Paper I, the specific form of \( f(\rho) \) that we considered was \( H^2 = A\rho + B\rho^n \) with \( n < 2/3 \) and \( n \) constant in time. Another way to write this equation is

\[ H^2 = A\rho[1 + (\frac{\rho}{\rho_{\text{car}}})^{n-1}]. \]  

(7)

The first term inside the bracket dominates initially but the second term takes over once the energy density has dropped to the value \( \rho_{\text{car}} \). Here, \( \rho_{\text{car}} \) is the energy density at which the two terms are equal: the ordinary energy density term on the right hand side of the FRW equation is equal in magnitude to the new term. Hence there are two parameters in the model: one can take them to be \( B \) and \( n \), or equivalently, \( \rho_{\text{car}} \) and \( n \), or equivalently, \( z_{\text{car}} \) and \( n \).

The new term in the equation (the second term on the right hand side) is initially negligible. It only comes to dominate recently, at the redshift \( z_{\text{car}} \sim \mathcal{O}(1) \) indicated by the supernovae observations. Once the second term dominates, it causes the universe to accelerate. When the new term is so large that the ordinary first term can be neglected, we find

\[ R \propto t^{\frac{2}{3n}}, \]  

(8)

so that the expansion is superluminal (accelerated) for \( n < 2/3 \). As examples, for \( n = 2/3 \) we have \( R \sim t^2 \); for \( n = 1/3 \) we have \( R \sim t^3 \); and for \( n = 1/6 \) we have \( R \sim t^4 \). The case of \( n = 2/3 \) produces a term in the FRW equation \( H^2 \propto R^{-2} \); such a term looks similar to a curvature term but is generated here by matter in a universe with a flat geometry. Note that for \( n = 1/3 \) the acceleration is constant, for \( n > 1/3 \) the acceleration is diminishing in time, while for \( n < 1/3 \) the acceleration is increasing (the cosmic jerk).

Note that the parameter \( B \) here is chosen to make the second term kick in at the right time to explain the observations. As yet we have no explanation of the coincidence problem; i.e., we have no explanation for the timing of \( z_{\text{car}} \). Such an explanation would arise if we had a reason for the required mass scale of \( B \); such an explanation may arise in the context of extra dimensions.

We were motivated to study an equation of this form by the work of Chung and Freese [13] who showed that terms of the form \( \rho^n \) can arise as a consequence of embedding our observable universe as a brane in extra dimensions.

2. Examples of Alternative FRW Equations

We wish to mention here some alternative forms of \( g(\rho) \) in Eq.(3). Wang, Freese, Frieman, and Gondolo [11] are studying three Cardassian alternatives:
1) A simple generalization of Eq.(1) is:

\[ H^2 = A\rho [1 + (\rho/\rho_{car})^{g(n-1)}]^{1/q}. \] (9)

Here, \( q > 0 \). As before, we require \( n < 2/3 \).

The right hand side returns to \( A\rho \) (the ordinary FRW equation) at early times, but becomes \( \rho^n \) at late times, just as in Eq.(7). However, the cross over time period during which the two terms are roughly comparable is different here.

2) Another possibility is

\[ H^2 = D[1 + (\rho/\rho_{car})^{n}]^{1/q}. \] (10)

This example can have a particularly interesting equation of state. Gondolo and Freese [12] are considering treating the right hand side of Eq.(10) as a single fluid. Then this fluid behaves as a polytrope of negative index:

\[ p \propto -\left( \frac{\rho}{\rho_{car}} \right)^{1-q}, \] (11)

which corresponds to a polytrope \( p = K\rho^{1+1/N} \) with negative index \( N = -1/q \) and negative pressure \( (K < 0) \).

3) A third possibility modifies the simplest Cardassian proposal with a logarithm:

\[ H^2 = A\rho + B\rho^n \log^q \rho. \] (12)

Many other possibilities for the function \( g(\rho) \) in Eq.(3) exist.

As will be shown in future work [11], the predictions for observational tests of these models can be very different from generic quintessence models whether the equation of state is constant or time dependent.

3. The simplest Cardassian Model: FRW with additional \( \rho^n \) term

For the rest of this presentation, we study specifically the case where \( g(\rho) = A\rho + B\rho^n \) for constant \( n < 2/3 \). This is the case that was studied in Paper I. We use it to illustrate the basic properties of a Cardassian model.

3.1. What is the Current Energy Density of the Universe?

Observations of the cosmic background radiation show that the geometry of the universe is flat with \( \Omega_0 = 1 \). In the Cardassian model we need to revisit the question of what value of energy density today, \( \rho_0 \), corresponds to a flat geometry. We will show that the energy density required to close the universe is much smaller than in a standard cosmology, so that matter can be sufficient to provide a flat geometry.

From evaluating Eq.(1) today, we have

\[ H_0^2 = A\rho_0 + B\rho_0^n. \] (13)

The energy density \( \rho_0 \) that satisfies Eq.(13) is, by definition, the critical density. We can solve Eq.(13) to find that the critical density \( \rho_c \) has been modified from its usual value, i.e., the number has changed. We find

\[ \rho_c = \rho_{c,old} \times F(n) \] (14)

where

\[ F(n) = [1 + (1 + z_{car})^{3(1-n)}]^{-1} \] (15)

and

\[ \rho_{c,old} = 1.88 \times 10^{-29} h_0^2 \text{gm/cm}^{-3} \] (16)

and \( h_0 \) is the Hubble constant today in units of 100 km/s/Mpc.

In the (simplest) Cardassian model with new \( \rho^n \), the value of the critical density can be much lower than previously estimated. Since \( \Omega_0 = 1 \) today, we have today’s energy density as \( \rho_0 = \rho_c \) as given above\(^2\).

For the past ten years, a multitude of observations has pointed towards a value of the matter density \( \rho_0 \sim 0.3\rho_{c,old} \). The cluster baryon fraction [14,15] as well as the observed galaxy power spectrum are best fit if the matter density is 0.3 of the old critical density. Recent results from

\(^2\)An alternate possible definition would be to keep the standard value of \( \rho_c \) and discuss the contribution to it from the two terms on the right hand side of the modified FRW equation. Then there would be contribution to \( \Omega \) from the \( \rho \) term and another contribution from the \( \rho^n \) term with the two terms adding to 1. This is the approach taken when one discusses a cosmological constant in lieu of our second term. However, the situation here is different in that we have only matter in the equation. The disadvantage of this second choice of definitions would be that the value of the energy density today equal to \( \rho_c \) equal to \( \rho_c \) according to this second definition would not correspond to a flat geometry.
the CMB [9,10] also obtain this value. In the standard cosmology this result implied that matter could not provide the entire closure density. Here, on the other hand, the value of the critical density can be much lower than previously estimated. Hence the cluster motivated value for $\rho_o$ is now compatible with a closure density of matter, $\Omega_o = 1$, all in the form of matter.

For example, if $n = 0.6$ with $z_{car} = 1$, or if $n = 0.2$ with $z_{car} = 0.4$, a critical density of matter corresponds to $\rho_o \sim 0.3\rho_{c,old}$, as required by the cluster baryon fraction and other data. If we assume that the value $\rho_o = 0.3\rho_{c,old}$ is correct, for a given value of $n$ (that is constant in time) we can compute the value of $z_{car}$ for our model from Eq.(15). Henceforth we shall use these combinations of parameters.

3.2. Other observational tests

As discussed in Paper I, the simplest Cardassian model with an additional term $\rho^a$ satisfies many observational constraints: the universe is somewhat older, the first Doppler peak in the microwave background is slightly shifted, early structure formation ($z > 1$) is unaffected, but structure will stop growing sooner. In addition the modifications to the Poisson equation will affect cluster abundances and the ISW affect in the CMB.

3.3. Comparing to Quintessence

We note that, with regard to observational tests, one can make a correspondence between the $\rho^a$ Cardassian and Quintessence models for constant $n$; we stress, however, that the two models are entirely different. Quintessence requires a dark energy component with a specific equation of state ($p = w\rho$), whereas the only ingredients in the Cardassian model are ordinary matter ($p = 0$) and radiation ($p = 1/3$). However, as far as any observation that involves only $R(t)$, or equivalently $H(z)$, the two models predict the same effects on the observation. Regarding such observations, we can make the following identifications between the Cardassian and quintessence models: $n \Rightarrow w + 1$, $F \Rightarrow \Omega_m$, and $1 - F \Rightarrow \Omega_Q$, where $w$ is the quintessence equation of state parameter, $\Omega_m = \rho_m/\rho_{c,old}$ is the ratio of matter density to the (old) critical density in the standard FRW cosmology appropriate to quintessence, $\Omega_Q = \rho_Q/\rho_{c,old}$ is the ratio of quintessence energy density to the (old) critical density, and $F$ is given by Eq.(15). In this way, the Cardassian model with $\rho^a$ can make contact with quintessence with regard to observational tests.

3.4. Best Fit of Parameters to Current Data

We can find the best fit of the Cardassian parameters $n$ and $z_{car}$ to current CMB and Supernova data. The current best fit is obtained for $\rho_o = 0.3\rho_{c,old}$ (as we have discussed above) and $n < 0.4$ (equivalently, $w < -0.6$) [16,17]. In Table I one can see the values of $z_{car}$ compatible with this bound, as well as the resultant age of the universe. As an example, for $n = 0.2$ (equivalently, $w = -0.8$), we find that $z_{car} = 0.42$. Then the position of the first Doppler peak is shifted by a factor of 1.12. The age of the universe is 13 Gyr. The cutoff energy density is $\rho_{cutoff} = 2.7\rho_c$, so that the new term is important only for $\rho < 2.7\rho_c$. Hence, the Cardassian term won’t affect the physics of the Earth or solar system in any way.

4. Discussion

We have presented $H^2 = g(\rho)$ as a modification to the FRW equations in order to suggest an explanation of the recent acceleration of the universe. In the Cardassian model, the universe can be flat and yet matter dominated. We have found that the new Cardassian modifications can dominate the expansion of the universe after $z_{car} = O(1)$ and can drive an acceleration. We have found that matter alone can be responsible for this behavior. The current value of the energy density of the universe is then smaller than in the standard model and yet is at the critical value for a flat geometry. We reported on results for the simplest Cardassian case of Eq.(1): Structure formation is unaffected before $z_{car}$. The age of the universe is somewhat longer. The first Doppler peak of the cosmic background radiation is shifted only slightly and remains consistent with experimental results. Such a modified FRW equation
may result from the existence of extra dimensions. Further work is required to find a simple fundamental theory responsible for Eq.(1). In this presentation, generalized cardassian models were discussed. As will be shown in future work, the predictions for observational tests of these models can be very different from generic quintessence models, whether the equation of state is constant or time dependent.

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