The $^4He$ $d$ elastic scattering at the momentum of 19.8 GeV/c is analyzed in the framework of the Glauber theory. The scattering amplitude was evaluated using different sets of values of the nucleon-nucleon amplitude parameters and the $^4He$ density function as a superposition of the Gaussian functions. It is shown that it is impossible to describe simultaneously the $p\ ^4He$ and $d\ ^4He$ elastic scattering cross sections using the same set of the $NN$-amplitude parameters. Inclusion of the twelve quark bag admixture to the ground state of the $^4He$ nucleus in the calculations allows one to reproduce the experimental data quite well. It is shown that the admixture manifests itself in the $d\ ^4He$ elastic scattering in all region of the momentum transfer. At small $t$ the effect can be at the level of $\sim 10\%$. At large $t$ it can be $\sim 30\%$. 

---

$^1$Math. & Theor. Phys. Dept., NRC, AEA, Cairo, Egypt.
Introduction

Understanding how the nuclei are react with other nuclei is one of the fundamental goals of nuclear physics. Elastic and inelastic nucleus-nucleus scattering has been studied for years to extract information on the nuclear structure and different aspects of the nuclear reactions. The other interesting topic is a manifestation of the quarks in the reactions. In high energy interactions quarks and gluons received high transverse momentum are observed as jets of hadrons. At lower energies the jet production cross section becomes extraordinary small, and the jets have not been registered at experimental studies until now. At the same time it is known that reactions with participation of the $^4\text{He}$ nucleus can not be described quite well within the framework of standard nuclear physics. In paper by L.G.Dakno and N.N.Nikolaev \cite{1} it was assumed and shown that 12\% admixture of twelve quark bag configuration in the ground state wave function of the $^4\text{He}$ nucleus allows one to understand the irregularities of proton - $^4\text{He}$ elastic scattering at high energies. We returned to the hypothesis in our previous paper \cite{2}, and have shown that it really gives an opportunity to describe $p\ ^4\text{He}$-scattering in a wide energy range. In present paper we continue our study, and consider $d\ ^4\text{He}$ elastic scattering.

Experimental data on the $d\ ^4\text{He}$ elastic scattering at the laboratory momentum of 19.8 GeV/c were presented in Ref. \cite{3}. They were analyzed in the framework of the Glauber theory \cite{4, 5}. The key quantities of the theory are characteristics of $NN$ elastic scattering amplitude and parametrization of the ground state wave function of the $^4\text{He}$ nucleus. In Ref. \cite{3} it was chosen the simplest gaussian parametrization of the wave function. The $NN$ characteristics were considered as the fitting parameters. As a result, all of these hid a big discrepancy between the experimental data and calculations with real parameters. We show it in the next Sec. where the main technical details of our calculations are given. Inclusion of the twelve quark bag state of the $^4\text{He}$ nucleus in the calculation scheme and manifestation of the state in the elastic scattering is considered in Sec. 2. In the last Sec. we summarize our results.

1 Calculation of $d\ ^4\text{He}$ elastic cross section

The Glauber amplitude of nucleus-nucleus scattering has a form \cite{6, 7, 8}:

$$F_{AB}(\vec{q}) = \frac{ip}{2\pi} \int d^2 b \ e^{i\vec{q}\cdot\vec{b}} \Gamma(\vec{b}), \quad (1)$$

$$\Gamma(\vec{b}) = \langle \psi_f^A \psi_f^B | 1 - \prod_{j=1}^{A} \prod_{k=1}^{B} (1 - \gamma(\vec{b} - \vec{s}_j + \vec{\tau}_k)) | \psi_i^A \psi_i^B \rangle, \quad (2)$$

where $\vec{b}$ is the impact parameter, $p$ is the momentum of the projectile nucleus, $\psi_i^A, \psi_i^B$ and $\psi_f^A, \psi_f^B$ are the initial and final states wave functions of the projectile and the target nucleus, respectively, $\gamma$ is the $NN$ elastic scattering amplitude in the impact parameter representation. In high energy physics it is often parametrized as:

$$\gamma(\vec{b}) = \beta \ e^{-\vec{b}^2/2B_{NN}}, \quad (3)$$

where $\beta = \sigma_{NN}^{tot} / (4\pi B_{NN})$, $\sigma_{NN}^{tot}$ is the $NN$ total cross section, $B_{NN}$ is the slope parameter of the $NN$ differential elastic cross section at zero momentum transfer,
\( \alpha_{NN} \) – the ratio of the real to imaginary parts of the \( NN \) elastic scattering amplitude at zero momentum transfer.

The elastic nucleus-nucleus differential cross section is determined as

\[
\frac{d\sigma}{d\Omega} = |F_{AB}|^2.
\]  

(4)

To take into account all terms of the expansion of the product in Eq. (2), one can represent each term like that shown in the Fig. 1, where the circles correspond to the interacting nuclei, the black and white points – to the nucleons, the solid lines – to the interactions between nucleons.

![Figure 1: Graphical representation of the multiple scattering terms](image)

Using the diagrams, one can calculate how many the diagrams of whatever type are. It is pure combinatorial problem which can be solved with a help of graph theory. In the graph theory the diagrams of the Fig. 1 are called bi-colored labelled graphs. This graphs can be represented with the help of an adjacency matrix. The adjacency matrix \( D = [d_{ij}] \) of a labelled graph \( G \) is a matrix of order \( A \times B \) in which \( d_{ij} = 1 \) if points \( i \) and \( j \) are adjacent (are connected with a line) and \( d_{ij} = 0 \) in other case. By the other way, the graph can be represented by the set of crossing points of \( A \) horizontal and \( B \) vertical lines with dark circles in the places corresponding to the elements \( d_{ij} = 1 \). This
representation is called net graph representation. We will refer to the net graphs as the scattering diagrams.

Each term in the expansion of Eq. (2) has a form

\[-\langle \psi_f^A \bar{\psi}_f^B \prod_{(j,k)} (-\gamma (\vec{b} - \vec{s}_j + \vec{\tau}_k)) | \psi_i^A \bar{\psi}_i^B \rangle,\]  

(5)

\((j, k) \in M \subset \{ I_A \} \otimes \{ I_B \}\)

\[\{ I_A \} = (1, 2, 3, ..., A), \quad \{ I_B \} = (1, 2, 3, ..., B).\]

Because it can be represented by a graph \(G\) or by the corresponding matrix \(D\) we will consider the term as a graph function \(g(D)\). The scattering amplitude \(\Gamma_{AB}\) now can be re-written as

\[\Gamma(\vec{b}) = \sum_{\mu} H_\mu \cdot g \left( \frac{D_\mu}{S(G_\mu)} \right),\]  

(6)

where summation runs on the set of all nonisomorphic graphs. From the graph theory we have that the combinatorial coefficient at the function of graph \(G_\mu\) with \(l\) components, \(k_1\) belonging to one class of isomorphism, \(k_2\) to another class, etc., \((l = k_1 + k_2 + \cdots + k_j)\) is equal to

\[H_\mu = \frac{A!}{(m_1!)^{k_1}(m_2!)^{k_2} \cdots (m_j!)^{k_j}(A - \sum_{i=1}^{j} m_i k_i)!} \times \frac{B!}{(n_1!)^{k_1}(n_2!)^{k_2} \cdots (n_j!)^{k_j}(B - \sum_{i=1}^{j} n_i k_i)!} \prod_{i=1}^{j} \left( \frac{m_i! n_i!}{S(G_i)!} \right),\]  

(7)

where \(m_i\) and \(n_i\) are the numbers of the points of the sets \(A\) and \(B\), respectively, in the component belonging to the \(i\)-th class of isomorphism, and \(S(G_i)\) is the number of symmetries of this component.

In the case of the elastic \(d^4He\) scattering the \(D\)'s matrices will be

\[
\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 \\
H_1 = 8 & H_2 = 12 & H_3 = 4 & H_4 = 12 & H_5 = 8 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 \\
H_6 = 24 & H_7 = 24 & H_8 = 2 & H_9 = 24 & H_{10} = 8 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 \\
H_{11} = 6 & H_{12} = 24 & H_{13} = 6 & H_{14} = 8 & H_{15} = 24 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 \\
H_{16} = 24 & H_{17} = 12 & H_{18} = 4 & H_{19} = 12 & H_{20} = 8 \\
1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 \\
H_{21} = 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
H_{22} = 1 \\
\end{array}
\]

At \(A = 2\) and \(B = 4\) the amplitude \(F_{AB}\) given by Eq. (1) is

\[F_{24}(\vec{q}) = \frac{i p}{2\pi} \int d^2 b \ e^{i \vec{q} \cdot \vec{b}} \Gamma (\vec{b}),\]  

(8)
\[ \Gamma (\vec{b}) = \langle \psi_d^i \psi_{He}^i | 1 - \prod_{j=1}^{2} \prod_{k=1}^{4} \left( 1 - \gamma (\vec{b} - \vec{s}^j + \vec{\tau}_k) \right) | \psi_d^i \psi_{He}^i \rangle. \] (9)

Introducing the distance between the nucleons in the deuteron, \( \vec{r} = (z, \vec{s}) \), and \( \vec{r}_1 = -\vec{r}_2 = \vec{r}/2 \) we have

\[ \Gamma (\vec{b}) = \langle \psi_d^i \psi_{He}^i | 1 - \prod_{k=1}^{4} \left( 1 - \gamma (\vec{b} - \vec{s}/2 + \vec{\tau}_k) \right) \gamma (\vec{b} + \vec{s}/2 + \vec{\tau}_k) \right) | \psi_d^i \psi_{He}^i \rangle, \] (10)

and

\[ F_{24}(\vec{q}) = \frac{i p}{2\pi} \int d^2 b \ e^{i \vec{q} \cdot \vec{b}} \left[ 1 - \prod_{k=1}^{4} \left( 1 - \gamma (\vec{b} - \vec{s}/2 + \vec{\tau}_k) \right) \left( 1 - \gamma (\vec{b} + \vec{s}/2 + \vec{\tau}_k) \right) \right]. \] (11)

For the square module of \( \psi_d \) we use the following parametrization [11],

\[ |\psi_d(\vec{r})|^2 = \sum_{i=1}^{3} W_i e^{-\frac{\vec{r}_i^2}{\rho_i^2}}, \] (12)

\[ \gamma_1 = 225(\text{GeV}/c)^2, \quad W_1 = 0.178/(4\pi \gamma_1)^{3/2}, \]
\[ \gamma_2 = 45(\text{GeV}/c)^2, \quad W_2 = 0.287/(4\pi \gamma_2)^{3/2}, \]
\[ \gamma_3 = 25(\text{GeV}/c)^2, \quad W_2 = 0.535/(4\pi \gamma_3)^{3/2}. \]

The square module of \( \psi_{He} \) was taken as [1].

\[ |\psi(\vec{r}_1, \ldots, \vec{r}_4)|^2 = (2\pi)^3 \rho \delta \left( \sum_{i=1}^{4} \vec{r}_i \right) \prod_{i=1}^{4} |\varphi(\vec{r}_i)|. \] (13)

We accept the two following parametrizations of \( \varphi(\vec{r}) \)

\[ (I) \quad \varphi(\vec{r}) = \exp[-\vec{r}^2/R_1^2], \]
\[ (II) \quad \varphi(\vec{r}) = \exp[-\vec{r}^2/R_1^2] + D_1 \exp[-\vec{r}^2/R_2^2] - (1 + D_1 - D_2) \exp[-\vec{r}^2/R_3^2]. \]

The parameters are given in Table 1.

<table>
<thead>
<tr>
<th>( R_1^2 ) ( (\text{GeV}/c)^{-2} )</th>
<th>( R_2^2 ) ( (\text{GeV}/c)^{-2} )</th>
<th>( R_3^2 ) ( (\text{GeV}/c)^{-2} )</th>
<th>( D_1 )</th>
<th>( D_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I )</td>
<td>51.01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( II )</td>
<td>62.06</td>
<td>19.0</td>
<td>10.13</td>
<td>3.79</td>
</tr>
</tbody>
</table>

We will use a general form for the function \( \varphi \) as

\[ \varphi(\vec{r}) = \sum_{i=1}^{N} C_i e^{-\vec{r}^2/R_i^2}. \] (14)
In the Eq. (13) $\rho_c$ is the normalization constant given in the Ref. [2]. Substituting the Eqs. (12), (13) and (14) in the Eq. (11) we have

$$F_{24}(q) = \frac{i p}{2 \pi} \rho_c \int d^2 b \, e^{i q \cdot b} \left[ 1 - \prod_{j=1}^{4} \left( 1 - \gamma(b - \frac{s}{2} + \bar{r}_j) \right) \right]$$

$$\left( \sum_{j=1}^{3} W_j e^{i \phi_j} \right) d^3 r \prod_{j=1}^{4} \left( \sum_{k=1}^{N} C_k e^{i \phi_j/R_k^2 - i \alpha_j} \right) d^3 r_j$$

$$= \frac{i p \rho_c}{2 \pi} \sum_{i_1=1}^{3} \sum_{j_1=2-5}^{N} W_{i_1} C_{i_2} C_{i_3} C_{i_4} C_{i_5} \int d^2 b \, e^{i q \cdot b} \left[ 1 - \prod_{k=1}^{4} \left( 1 - \gamma(b - \frac{s}{2} + \bar{r}_k) \right) \right]$$

$$\left( 1 - \gamma(b + \frac{s}{2} + \bar{r}_k) \right)$$

$$e^{-\frac{r^2}{\alpha_1}} d^3 r \prod_{k=1}^{4} e^{-\frac{r^2}{R_k^2} - i \alpha_j} d^3 r_k$$

$$\left( \frac{1}{R_{i_1}^2} + \frac{d_{i_1} + d_{i_1}}{2B_{NN}} \right) 0 0 0 0 \frac{-d_{i_1} + d_{i_1}}{2B_{NN}} 1$$

$$0 0 \frac{1}{R_{i_2}^2} + \frac{d_{i_2} + d_{i_2}}{2B_{NN}} 0 0 0 0 \frac{-d_{i_2} + d_{i_2}}{2B_{NN}} 1$$

$$0 0 0 \frac{1}{R_{i_3}^2} + \frac{d_{i_3} + d_{i_3}}{2B_{NN}} 0 0 0 0 \frac{-d_{i_3} + d_{i_3}}{2B_{NN}} 1$$

$$-\frac{d_{i_1} + d_{i_1}}{2B_{NN}} -\frac{d_{i_1} + d_{i_1}}{2B_{NN}} -\frac{d_{i_1} + d_{i_1}}{2B_{NN}} -\frac{d_{i_1} + d_{i_1}}{2B_{NN}} 1 0 0 0 0$$

$$1 1 1 1 1 1 1 1 1$$

where the integration on $\bar{r}$ is performed in order to account the $\delta$ function in the Eq. (13) (see for details the Ref. [2]).

Using the graph functions, the Eq. (15) can be written in the form

$$F_{24}(q) = \frac{i p \rho_c}{2 \pi} \sum_{n=1}^{21} \sum_{i_1=1}^{3} \sum_{j_1=2-5}^{N} W_{i_1} C_{i_2} C_{i_3} C_{i_4} C_{i_5} (-1)^{n+1} \beta^n \frac{\pi}{\rho_c(A)} e^{\frac{-q^2}{4 \rho_c(A)}}$$

$$\left( \frac{1}{R_{i_1}^2} + \frac{d_{i_1} + d_{i_1}}{2B_{NN}} \right) 0 0 0 0 \frac{-d_{i_1} + d_{i_1}}{2B_{NN}} 1$$

$$0 0 \frac{1}{R_{i_2}^2} + \frac{d_{i_2} + d_{i_2}}{2B_{NN}} 0 0 0 0 \frac{-d_{i_2} + d_{i_2}}{2B_{NN}} 1$$

$$0 0 0 \frac{1}{R_{i_3}^2} + \frac{d_{i_3} + d_{i_3}}{2B_{NN}} 0 0 0 0 \frac{-d_{i_3} + d_{i_3}}{2B_{NN}} 1$$

$$-\frac{d_{i_1} + d_{i_1}}{2B_{NN}} -\frac{d_{i_1} + d_{i_1}}{2B_{NN}} -\frac{d_{i_1} + d_{i_1}}{2B_{NN}} -\frac{d_{i_1} + d_{i_1}}{2B_{NN}} 1 0 0 0 0$$

$$1 1 1 1 1 1 1 1 1$$

The result of the complete integration is [8]

$$F_{24}(q) = \frac{i p \rho_c}{2 \pi} \sum_{n=1}^{21} \sum_{i_1=1}^{3} \sum_{j_1=2-5}^{N} W_{i_1} C_{i_2} C_{i_3} C_{i_4} C_{i_5} (-1)^{n+1} \beta^n \frac{\pi}{\rho_c(A)} e^{\frac{-q^2}{4 \rho_c(A)}}$$

where

$$\left| \frac{a}{b} \right| = \left| \frac{a}{b} \right|$$

$$\left( \frac{1}{R_{i_1}^2} + \frac{d_{i_1} + d_{i_1}}{2B_{NN}} \right) 0 0 0 0 \frac{-d_{i_1} + d_{i_1}}{2B_{NN}} 1$$

$$0 0 \frac{1}{R_{i_2}^2} + \frac{d_{i_2} + d_{i_2}}{2B_{NN}} 0 0 0 0 \frac{-d_{i_2} + d_{i_2}}{2B_{NN}} 1$$

$$0 0 0 \frac{1}{R_{i_3}^2} + \frac{d_{i_3} + d_{i_3}}{2B_{NN}} 0 0 0 0 \frac{-d_{i_3} + d_{i_3}}{2B_{NN}} 1$$

$$-\frac{d_{i_1} + d_{i_1}}{2B_{NN}} -\frac{d_{i_1} + d_{i_1}}{2B_{NN}} -\frac{d_{i_1} + d_{i_1}}{2B_{NN}} -\frac{d_{i_1} + d_{i_1}}{2B_{NN}} 1 0 0 0 0$$

$$1 1 1 1 1 1 1 1 1$$

To consider the elastic scattering at small momentum transfer we take into account the Coulomb interaction (see the Ref. [3]) writing

$$\frac{d\sigma}{dt} = \frac{\pi}{p^2} \left| -\frac{2p}{|q|} G(q^2) e^{i\varphi} + F_N(q) \right|^2,$$

(19)
where \( p \) is the laboratory momentum of the \( ^4\text{He} \), \( n = Z_{\text{He}} \cdot Z_d/137\beta \), \( Z_{\text{He}} \), \( Z_d \) are the charge of \( \text{He} \) and \( d \) respectively, \( \beta = \frac{v}{c} \) is the \( ^4\text{He} \) velocity in the laboratory system

\[
\phi = 2n \ln(1.06/a|q|); \ G(q^2) = G_{\text{He}}(q^2) \cdot G_d\left(\frac{q^2}{4}\right).
\]

(20)

\( G_{\text{He}}(t) \) is the \( ^4\text{He} \) form factor, and \( G_d(t) \) is the deutron form factor,

\[
G_{\text{He}}(t) = e^{11.9\cdot t},
\]

\[
G_d\left(\frac{t}{4}\right) = 0.34e^{141.5\cdot t} + 0.58e^{26.1\cdot t} + 0.08e^{15.5\cdot t}.
\]

To perform the calculations one needs the \( NN \) elastic scattering amplitude parameters. A large number of publications both theoretical and experimental have been devoted to processes the values of the parameters. We take the values of the parameters – \( \sigma_{\text{tot}}^{\text{NN}} \), \( \alpha_{\text{NN}} \) and \( B_{\text{NN}} \) as follows: \( \sigma_{\text{tot}}^{\text{NN}} \) and \( \alpha_{\text{NN}} \) were taken from the compilations [9] and [10], respectively. \( B_{\text{NN}} \) was estimated [2] using \( \sigma_{\text{tot}}^{\text{NN}} \) and \( \sigma_{\text{el}}^{\text{NN}} \) taken from the compilation [9]. We call the set of the parameters as set \( A \). Other set of the values (set \( B \)) was taken from the paper by V.V.Avdeichikov [3], where the parameters were obtained at fitting the experimental data on \( ^4\text{He} \ d \) elastic scattering.

![Graph](image)

Figure 2: The \( ^4\text{He} \ d \) differential cross sections at \( P_l = 19.8 \text{ GeV/c} \). Points are the experimental data [3], lines are our calculations using the set of values \( A \).

At the begining let us study the role of the different parametrizations of the modules of the wave function of \( ^4\text{He} \). The top figure 2 gives the \( ^4\text{He} \ d \) elastic scattering differential
cross section calculated with the simplest parametrization I and set A. The bottom figure presents the ratio of two calculations with parametrization II and I. As seen, the difference between the calculations is at the level of 3%. It is much smaller than the difference between the experimental data and the calculations. Thus in the following we will use only the parametrization I for simplicity.

The differential cross sections calculated with two sets of the $NN$ amplitude parameters $A$ and $B$ in a comparison with the experimental data [3] are shown in figure 3. As one can see, the set $B$ reproduces the data quite well. At the same time it is failed to describe the other reactions like $p^4He$ scattering. The set $A$ leads to a big disagreement with the experimental data especially at large values of the momentum transfer $t$. So, the main problem of the analysis is a choice of the $NN$ parameters.

Figure 3: The $^4He d$ differential cross sections at $P_l = 19.8\text{ GeV/c}$. Points are the experimental data [3]. Solid and dashed lines are our calculations using the set of values $B$ and $A$, respectively.

In Ref. [11] the deutron - deutron scattering at momentum 8.9 GeV/c was analyzed. The authors used two sets of the NN-parameters presented in the table 2 as C and D sets. Both sets allowed to describe the $dd$-scattering quite well (see Ref. [11]).
Table 2: The \( NN \)–amplitude parameter sets used in the calculations

<table>
<thead>
<tr>
<th></th>
<th>( \sigma_{NN}^i ) ( mb )</th>
<th>( \sigma_{PP}^i ) ( mb )</th>
<th>( \sigma_{NP}^i ) ( mb )</th>
<th>( B_{NN} ) ( (GeV/c)^{-2} )</th>
<th>( \alpha_{NN} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>12</td>
<td>41.67</td>
<td>42.04</td>
<td>41.86</td>
<td>7.39</td>
</tr>
<tr>
<td>( B )</td>
<td></td>
<td>41</td>
<td>7.9</td>
<td></td>
<td>-0.55</td>
</tr>
<tr>
<td>( C )</td>
<td></td>
<td>42.4</td>
<td>6.3</td>
<td></td>
<td>-0.43</td>
</tr>
<tr>
<td>( D )</td>
<td></td>
<td>42.4</td>
<td>7.3</td>
<td></td>
<td>-0.43</td>
</tr>
</tbody>
</table>

As seen from the Fig. 4 both sets give a satisfactory description of the \( ^4He \ p \) elastic scattering except the region of large \( t \).

The differential \( ^4He \ p \) elastic scattering at \( P_L = 17.8 \) GeV/c. Points are the experimental data \[12\]. Lines are our calculations with the different sets of \( NN \)-amplitude parameters.

Figure 4: The differential \( ^4He \ p \) elastic scattering at \( P_L = 17.8 \) GeV/c. Points are the experimental data \[12\]. Lines are our calculations with the different sets of \( NN \)-amplitude parameters.

The differential cross sections of the \( ^4He \ d \) elastic scattering calculated with the sets \( C \) and \( D \) are shown in Fig. 5 in a comparison with the experimental data \[3\]. As seen, we have a big disagreement with the data in the region of the diffraction minimum and also in the region of small momentum transfer \( t \) with the set \( D \). Inverse situation takes place with the set \( C \), the calculation are above the experimental data at small \( t \) and above the data at large \( t \).
Figure 5: The differential $^4He$ $d$ elastic scattering at $P_L = 19.8$ GeV/c. Points are the experimental data [3]. Short dashed and long dashed lines are our calculations using the sets $C$ and $D$, respectively. Solid lines – calculations performed with inclusion of the 12-quark bag state of $^4He$ and with the parameter set C.

Summing up we conclude that it is impossible to describe simultaneously the $p\ ^4He$ and $d\ ^4He$ elastic scattering cross sections using the same set of the $NN$-amplitude parameters. The parameters what are quite well for understanding $dd$-scattering [11] can not be applied in the case of $d\ ^4He$ scattering.

2 Manifestation of the twelve quark bag admixture

Let us neglect all transition amplitudes like that $|4N> \rightarrow |12q>$, $|12q> \rightarrow |4N>$ following papers [1, 2]. In this case the $d\ ^4He$ scattering amplitude will be

$$F_{d\ ^4He} = (1 - w_{12q}) F_{d,4N} + w_{12q} F_{d,12q}$$  \hspace{1cm} (21)

where $F_{d,4N}$ is the Glauber amplitude of the deuteron - four nucleon scattering given by the Eq. (11), $F_{d,12q}$ is the deuteron - $12q$ bag scattering amplitude, and $w_{12q}$ is the weight
of the $12q$ bag quark state in the ground state of the $^4\text{He}$ nucleus. The values of the $12q$ bag weight was estimated in our previous paper [2] as $w_{12q} = 10.5\%$.

$F_{d,12q}$ amplitude can be estimated as $d\, p$ one.

$$F_{d,12q} = \frac{ip}{2\pi} \sum_{j=1}^{3} W_j \int d^2 e^{i\vec{q}\cdot\vec{b}} \left[ \gamma_{N,12q}(\vec{b} - \frac{\vec{s}}{2}) + \gamma_{N,12q}(\vec{b} + \frac{\vec{s}}{2}) - \gamma_{N,12q}(\vec{b} - \frac{\vec{s}}{2}) \gamma_{N,12q}(\vec{b} + \frac{\vec{s}}{2}) \right] e^{-\vec{r}^2/\sigma} d^3r$$

(22)

Using the gaussian parametrization for the nucleon - $12q$ bag scattering amplitude

$$\gamma_{N,12q}(\vec{b}) = \frac{\sigma_{12q}^3}{4\pi C_{12q}} \cdot e^{-\vec{r}^2/\sigma_{12q}}$$

(23)

all integrations in the Eq. (22) can be done exactly. In particular,

$$F_{d,12q}^1 = \int d^2 e^{i\vec{q}\cdot\vec{b}} \gamma_{N,12q}(\vec{b} - \frac{\vec{s}}{2}) e^{-\vec{r}^2/\sigma} d^3r$$

$$= \frac{\sigma_{12q}^4}{4\pi B_{12q}} \int d^2 e^{i\vec{q}\cdot\vec{b}} e^{-\frac{(\vec{s} - \vec{q})^2}{2\sigma_{12q}}} e^{-\frac{\vec{r}^2}{\gamma}} d^2sdz$$

$$= \frac{\sigma_{12q}^4}{4\pi B_{12q}} \left( \frac{8B_{12q}\gamma_j\pi}{2B_{12q} + \gamma_j} \right) \left( 4\gamma_j\pi \right)^{1/2} \int d^2 e^{i\vec{q}\cdot\vec{b}} e^{x} \left[ -\frac{\vec{b}^2}{2B_{12q} + \gamma_j} \right]$$

$$= \frac{\sigma_{12q}^4}{4\pi B_{12q}} \left( \frac{8B_{12q}\gamma_j\pi}{2B_{12q} + \gamma_j} \right) \left( 4\gamma_j\pi \right)^{1/2} \left( 2B_{12q} + \gamma_j \right) \pi \exp \left[ -\frac{\frac{B_{12q} + \gamma_j}{4}}{\sigma_{12q}} \right]$$

(24)

$$= 2\pi \gamma_j \sigma_{12q}^4 \left( 4\gamma_j\pi \right)^{1/2} \exp \left[ -\frac{\frac{B_{12q} + \gamma_j}{4}}{\gamma_j} \right]$$

The second is given as

$$F_{d,12q}^2 = \int d^2 e^{i\vec{q}\cdot\vec{b}} \gamma_{N,12q}(\vec{b} - \frac{\vec{s}}{2}) \gamma_{N,12q}(\vec{b} + \frac{\vec{s}}{2}) e^{-\vec{r}^2/\sigma} d^3r$$

$$= \left( \frac{\sigma_{12q}^4}{4\pi B_{12q}} \right)^2 \int d^2 e^{i\vec{q}\cdot\vec{b}} e^{-\frac{(\vec{s} - \vec{q})^2}{2\sigma_{12q}}} e^{-\frac{\vec{r}^2}{\gamma}} d^2sdz$$

$$= \left( \frac{\sigma_{12q}^4}{4\pi B_{12q}} \right)^2 \left( 4\gamma_j\pi \right)^{1/2} \left( \frac{4B_{12q}\gamma_j\pi}{2B_{12q} + \gamma_j} \right) \left( B_{12q}\pi \right) e^{-\frac{B_{12q}q^2}{4}}$$

(25)
The $d$ $12q$ bag amplitude will be

$$F_{d,12Q} = \frac{i\hbar}{2\pi} \sum_{i=1}^{3} W_i (4\pi \gamma_i)^{\frac{1}{2}} \left[ 2\pi \gamma_i \sigma_{12q}^t \left( \frac{2B_{12q} + \gamma_i}{4} \right)^2 - \frac{(\sigma_{12q}^t)^2 \gamma_i}{4(B_{12q} + \gamma_i)} e^{-\frac{B_{12q} q^2}{4}} \right].$$  \hspace{1cm} (26)$$

The values of the nucleon-$12q$ bag amplitude parameters were estimated in the Ref. [2]: $w_{12q} = 10.5\%$, $\sigma_{12q}^t = 34mb$ and $B_{12q} = 23\, (GeV)^{-2}$. These values have been used with the parameter set $C$ for the calculations of the differential cross section with and without the $12q$ bag admixture shown in the Fig. 5. One can see that when the $12q$ bag admixture is included in the calculations it gives a good description of the data at small and large values of $t$. The discrepancy between the calculations and the experimental data in the region $0.09 \leq |t| \leq 0.13\, (GeV/c)^{-2}$ can be erased at taking into account the D-wave of the deuteron [5].

**Conclusion**

1. Within the framework of the Glauber theory it is impossible to describe simultaneously the $p\, ^4He$ and $d\, ^4He$ elastic scattering cross sections using the same set of the $NN$-amplitude parameters.

2. The $12q$ bag admixture to the ground state of the $^4He$ nucleus manifests itself in the $d\, ^4He$ elastic scattering in all region of the momentum transfer. At small $t$ the effect can be at the level of $\sim 10\%$. At large $t$ it can be $\sim 30\%$.

3. To study the effect at an experiment it is needed to measure the $d\, ^4He$ elastic cross section with absolute normalization accuracy better than $10\%$.

V.V. Uzhinskii thanks RFBR (grant No 00-01-00307, 01-02-16431) and INTAS (grant No 00-00366) for their financial support. A.M. Mosallem is thankful to Profs. A.B.B. Kalil and K.M. Hanna for support, and JINR officials for hospitality.

**References**


