We derive general formulae for the first order variation of the ADM mass, angular momentum for linear perturbations of a stationary background in Einstein-Maxwell axion-dilaton gravity being the low-energy limit of the heterotic string theory. All these variations were expressed in terms of the perturbed matter energy momentum tensor and the perturbed charge current density. Combining these expressions we reached to the form of the physical process version of the first law of black hole dynamics for the stationary black holes in the considered theory being the strong support for the cosmic censorship hypothesis.

I. INTRODUCTION

Black hole thermodynamics has played a key role in analyzing the character of gravity in general and quantum gravity in particular. The first law of black hole mechanics as derived by Bardeen et al. [1] considered a linear perturbations of a stationary, electrovac black hole to another stationary electrovac black hole. It bounded small changes in the mass of a stationary, axisymmetric black hole to small changes in its horizon area, angular momentum and the properties of a stationary perfect fluid surrounding it.

For an arbitrary diffeomorphism invariant Lagrangian theory with metric and matter fields possessing stationary and axisymmetric black hole solutions which were asymptotically flat and had bifurcate Killing horizon the problem was considered by Wald and collaborators in [2–4]. It was revealed that the first law of black hole mechanics might be depicted in a form only involving surface integrals on the sphere at spatial infinity and the bifurcation sphere of the black hole horizon. Iyer [5] applied the aforementioned analysis to the problem of stationary, axisymmetric black hole surrounded by a perfect fluid.

It seems that the ultimate nature of quantum gravity should of the form of Lagrangian that describes the dynamics of classical background fields for sufficiently weak fields at sufficiently large distances. This low-energy effective action may also contain higher curvature terms and higher derivative terms in the metric and all other matter fields. Jacobson et al. [6] computed black hole entropy in a generally covariant theories including arbitrary higher derivative interactions. In [7] they examined the zeroth and second law of black hole thermodynamics within context of effective generalized gravitational actions including higher curvature interactions. They showed that entropy can never decrease for quasi-stationary process when black hole acquired positive energy matter. This is independent of the details of the gravitational actions. Koga and Maeda [8] considered black hole thermodynamics in a generalized theory of gravity which admitted Lagrangian being an arbitrary function of the metric, Ricci tensor and a scalar field. They showed
that all thermodynamical variables defined in [2,3] are the same in original frame and in the Einstein frame, under the assumptions that spacetimes in both frames were asymptotically flat, regular and possessed event horizons with non-zero temperature.

Nowadays there has been an active period of constructing black hole solutions in the string theories (see [9] and references therein). The low-energy limit of the heterotic string theory compactified on a six-dimensional torus consists of the pure $N=4$, $d=4$ supergravity coupled to $N=4$ super Yang-Mills. The bosonic sector of the low-energy limit of the heterotic string theory compactified on a six-torus is called Einstein-Maxwell axion-dilaton (EMAD) gravity. This theory provides a simple framework for studying classical solutions which can be considered as solutions of the full effective string theory. In [10] the first law of black hole mechanics in EMAD gravity was derived. This derivation was true for arbitrary perturbation of a stationary black hole in this theory.

It is also possible to consider a physical process when matter is thrown into initially stationary black hole. The tantalizing question is whether the black hole settles down to a final stationary state. As was pointed out if this would be violated than this event provided the strong evidence against cosmic censorship. However if one gets the proof of the a physical process version of the first law of thermodynamics and it will be supporting evidence for the cosmic censorship. The physical version of the first law of black hole thermodynamics was proved in [11] and in [12] it was generalized to the case of charged black holes in Einstein-Maxwell (EM) theory.

In this paper we shall try to provide some continuity with our previous work [10] and find the physical process version of the first law of black hole dynamics in EMAD gravity. In section II we find the first order variation for the Arnowitt-Deser-Misner (ADM) mass and angular momentum for the linear perturbations of a stationary background in EMAD gravity. Then we established the physical process version of the first law of black hole dynamics for stationary, charged black hole solutions in the considered theory. We found that the form of it is the same as derived in Ref. [10], provided in this way the strong support for the cosmic censorship hypothesis. In section III we concluded our results. Our notation and conventions follow that which was used in [13].

II. PHYSICAL PROCESS VERSION OF THE FIRST LAW OF BLACK HOLE MECHANICS

In this section we shall consider the effective Lagrangian of the low-energy heterotic string theory compactified on a six-dimensional torus. The bosonic sector of this theory with a simple vector field is called EMAD gravity. This theory provides a non-trivial generalization of the ordinary EM gravity. It consists of a coupled system containing a metric $g_{\mu\nu}$, $U(1)$ vector fields $A_\mu$, a dilaton field $\phi$ and three-index antisymmetric tensor field, namely [14]

$$
L = \epsilon \left( R - 2(\nabla \phi)^2 - \frac{1}{3} e^{-4\phi} H_{\alpha\beta\gamma} H^{\alpha\beta\gamma} - e^{-2\phi} F_{\alpha\beta} F^{\alpha\beta} \right),
$$

where by $\epsilon$ we denoted the volume element, $F_{\mu\nu} = 2\nabla_{[\mu} A_{\nu]}$ and $H_{\alpha\beta\gamma}$ stands for three-index antisymmetric tensor field defined by

$$
H_{\alpha\beta\gamma} = \nabla_\alpha B_{\beta\gamma} - A_\alpha F_{\beta\gamma} + \text{cyclic}.
$$

We proceed now to finding the first order variation of the conserved quantities in the theory under consideration. Our main task will be to obtain the explicit formulae for the variation of mass and angular momentum in EMAD gravity.
Thus, calculating the first order variation of the Lagrangian (1), one finds

$$\delta L = \epsilon \left( G_{\mu \nu} - T_{\mu \nu}(\phi, F, H) \right) \delta g^{\mu \nu} + 4 \left( \nabla_{\mu}(e^{-2\phi} F^{\mu \nu}) + e^{-4\phi} H^{\alpha \beta \nu} F_{\alpha \beta} \right) \delta A_\nu$$

where the energy momentum tensor is given by

$$T_{\mu \nu}(\phi, F, H) = 2\nabla_{\mu} \phi \nabla_{\nu} \phi - g_{\mu \nu} \left( \nabla^2 \phi \right) + e^{-2\phi} \left( 2F_{\mu \gamma} F_{\nu \gamma} - \frac{1}{2} g_{\mu \nu} F^2 \right) + e^{-4\phi} \left( H_{\alpha \beta} H^{\nu \alpha \beta} - \frac{1}{6} g_{\mu \nu} H^2 \right).$$

The totally divergent term in (3) is a functional of the field variables \((A_\mu, B_{\mu \nu}, \phi)\) and their variations \((\delta A_\mu, \delta B_{\mu \nu}, \delta \phi)\), which for simplicity we denote respectively by \(\psi\) and \(\delta \psi\). From equation (3) we have the following expression for the symplectic three-form \(\Theta_{\alpha \beta \gamma}[\psi, \delta \psi]\)

$$\Theta_{\alpha \beta \gamma}[\psi, \delta \psi] = \epsilon_{\alpha \beta \gamma} \nu^\mu,$$

where

$$\nu^\mu = \omega^\mu - 4e^{-2\phi} F^{\mu \nu} \delta A_\nu - 2e^{-4\phi} H^{\alpha \beta} \delta B_{\alpha \beta} + 4e^{-4\phi} H^{\mu \alpha \beta} \delta A_\gamma \delta A_\beta - 4\nabla^\mu \phi \delta \phi,$$

and

$$\omega^\mu = \nabla^\alpha \delta g_{\alpha \mu} - \nabla_\mu \delta g^{\beta \beta}.$$

The inspection of (3) enables one to read off the source-free EMAD Eqs. of motion. They yield

$$G_{\mu \nu} - T_{\mu \nu}(\phi, F, H) = 0,$$

$$\nabla_{\nu}(e^{-2\phi} F^{\mu \nu}) + e^{-4\phi} H^{\alpha \beta \mu} F_{\alpha \beta} = 0,$$

$$\nabla_{\mu}(e^{-4\phi} H^{\mu \alpha \beta}) = 0,$$

$$\nabla^2 \phi + \frac{1}{3} e^{-4\phi} H^2 + \frac{1}{2} e^{-2\phi} F^2 = 0.$$

When one identifies the variations of the fields \(\delta \psi\) with a general coordinate transformations \(\mathcal{L}_{\xi} \psi\) induced by an arbitrary Killing vector \(\xi_\alpha\), the Noether three-form with respect to this Killing vector \(\xi_\alpha\) implies \([3,4]\)

$$\mathcal{J}_{\alpha \beta \gamma} = \epsilon_{\rho \alpha \beta \gamma} \mathcal{J}^\rho[\psi, \mathcal{L}_{\xi} \psi],$$

where the vector field \(\mathcal{J}^\rho[\psi, \mathcal{L}_{\xi} \psi]\) is given by

$$\mathcal{J}^\delta[\psi, \mathcal{L}_{\xi} \psi] = \Theta^\delta[\psi, \mathcal{L}_{\xi} \psi] - \xi^\delta L.$$

From relation (13) we can establish the resultant expression for the Noether current three-form with respect to the Killing vector field \(\xi_\alpha\), namely

$$\mathcal{J}_{\alpha \beta \gamma} = dQ_{\alpha \beta \gamma}^R + 2\epsilon_{\delta \alpha \beta \gamma} \left( G^\delta_\eta - T^\delta_\eta(\phi, F, H) \right) \xi^\eta - 4\epsilon_{\delta \alpha \beta \gamma} \left[ \nabla_{\mu}(e^{-2\phi} F^{\delta \mu}) + e^{-4\phi} H^{\delta \mu \nu} F_{\nu \mu} \right] \xi^\eta A_\rho$$

$$- 4\epsilon_{\delta \alpha \beta \gamma} \nabla_{\mu}(e^{-4\phi} H^{\delta \mu \nu}) B_{\nu \eta} \xi^\eta - 4\epsilon_{\delta \alpha \beta \gamma} \nabla_{\mu}(e^{-4\phi} H^{\delta \mu \nu}) A_\rho \xi^\eta A_\eta$$

$$- 4\epsilon_{\delta \alpha \beta \gamma} \nabla_{\mu} \left( e^{-2\phi} F^{\delta \mu} \xi^\rho A_\rho \right) + 4\epsilon_{\delta \alpha \beta \gamma} \nabla_{\mu} \left( e^{-4\phi} H^{\delta \mu \nu} \xi^\rho B_{\nu \rho} \right)$$

$$+ 4\epsilon_{\delta \alpha \beta \gamma} \nabla_{\mu} \left( e^{-4\phi} H^{\delta \rho \mu} A_\rho \xi^\eta A_\eta \right).$$
where we have denoted by $Q^{GR}_{\alpha\beta\gamma} = -\epsilon_{\alpha\beta\gamma\delta} \nabla^\gamma \xi^\delta$. Using differentiation by parts we obtain the following expression for equation (14):

$$J_{\alpha\beta\gamma} = dQ_{\alpha\beta\gamma} + 2\epsilon_{\delta\alpha\beta\gamma} \left( G^\delta_{\mu} - T^\delta_{\mu}(\phi, H, F) \right) \xi^\eta - 4\epsilon_{\delta\alpha\beta\gamma} \left[ \nabla_\mu (e^{-2\phi} F^{\delta\mu}) + e^{-4\phi} H^{\delta\mu\nu} F_{\mu\nu} \right] \xi^\rho A_\rho$$

(15)

Having in mind [4] that $J[\xi] = dQ[\xi] + \xi^\alpha C_\alpha$, where $C_\alpha$ is an $(n - 1)$ form locally constructed from the dynamical fields we may identify $Q_{\alpha\beta}$ as the Noether charge. The result yields

$$Q_{\alpha\beta} = Q^{GR}_{\alpha\beta} + Q_{\alpha\beta}(F) + Q_{\alpha\beta}(H) + Q_{\alpha\beta}(A - H)$$

(16)

and also $C_{\alpha\beta\gamma\mu}$ may be read off the above expression. It is given by the following formula:

$$C_{\alpha\beta\gamma\mu} = 2\epsilon_{\delta\alpha\beta\gamma} \left( G^\delta_{\mu} - T^\delta_{\mu}(\phi, H, F) \right) - 4\epsilon_{\delta\alpha\beta\gamma} \left[ \nabla_\mu (e^{-2\phi} F^{\delta\rho}) + e^{-4\phi} H^{\delta\nu\gamma} F_{\gamma\nu} \right] \xi^\rho A_\mu$$

(17)

When the quantity $C_\alpha = 0$, we have equations of motion fulfilled. On the other hand, in the case when it does not hold, one gets

$$G_{\mu\nu} - T_{\mu\nu}(\phi, H, F) = T_{\mu\nu}(\text{matter}),$$

(18)

$$\nabla_\nu (e^{-2\phi} F^{\mu\nu}) + e^{-4\phi} H^{\alpha\beta\mu} F_{\alpha\beta} = -j^\mu(\text{matter}),$$

(19)

$$\nabla_\nu (e^{-4\phi} H^{\nu\alpha\beta}) = 0,$$

(20)

$$\nabla^2 \phi + \frac{1}{3} e^{-4\phi} H^2 + \frac{1}{2} e^{-2\phi} F^2 = 0.$$

(21)

Let us have that $(g_{\mu\nu}, A_\mu, B_{\mu\nu}, \phi)$ be the solution of the source-free EMAD equations of motion and let us assume that $(\delta g_{\mu\nu}, \delta A_\mu, \delta B_{\mu\nu}, \delta \phi)$ be linearized perturbations fulfilling EMAD gravity equations with sources $\delta T_{\mu\nu}(\text{matter})$ and $\delta j_\mu(\text{matter})$. It implies that for a perturbed $\delta C_{\alpha\beta\gamma\mu}$ quantity we arrive at the following relation:

$$\delta C_{\alpha\beta\gamma\mu} = \epsilon_{\delta\alpha\beta\gamma} \left[ 2\delta T^\delta_{\mu}(\text{matter}) + 4A_\mu \delta j^\delta(\text{matter}) \right].$$

(22)

Equation (22) enables us to obtain the explicit formula for the variation of the conserved quantity $\delta H_\xi$ associated with the Killing vector field [15]. It satisfies the following:

$$\delta H_\xi = \int_\Sigma \left[ 2\xi^\mu \delta T^\delta_{\mu}(\text{matter}) + 4\xi^\mu A_\mu \delta j^\delta(\text{matter}) \right] + \int_{\partial\Sigma} \left[ \delta Q(\xi) - \xi \cdot \Theta \right].$$

(23)

Choosing $\xi^\alpha$ to be an asymptotic time translation $t^\alpha$ we write $M = H_t$ and get the desired variation of the ADM mass

$$\delta M = \int_\Sigma \left[ 2\xi^\mu \delta T^\delta_{\mu}(\text{matter}) + 4t^\mu A_\mu \delta j^\delta(\text{matter}) \right] + \int_{\partial\Sigma} \left[ \delta Q(t) - t \cdot \Theta \right].$$

(24)

On the other hand, when one chooses $\xi^\alpha$ to be an asymptotic rotation $\varphi^\alpha$ we have the variation of angular momentum
\[
\delta J = \int_{\Sigma} \left[ 2\varphi^\mu \delta T^\delta_{\mu} \text{(matter)} + 4\varphi^\mu A_\mu \delta j^\delta \text{(matter)} \right] + \int_{\partial \Sigma} \left[ \delta Q(\varphi) - \varphi \cdot \Theta \right].
\] (25)

Now, we proceed to the physical version of the first law of black hole mechanics. We shall consider a classical black hole solution to EMAD gravity theory which satisfies the equations of motion (18-21). The perturbation of the black hole is gained by dropping matter into it. We assume further that the black hole settles down to a stationary final state and its change of mass, angular momentum and changes of currents of the fields in the theory under consideration can be found. Let \((g_{\alpha\beta}, A_\alpha, B_{\alpha\beta}, \phi)\) be a solution of the source-free EMAD gravity equations of motion corresponding to a stationary black hole. The Killing vector field normal to the horizon is given as follows \(\xi_\alpha = t_\alpha + \Omega \varphi_\alpha\), where \(\Omega\) is the angular velocity of the black hole.

As in [12] we suppose that \(\Sigma_0\) is an asymptotically flat hypersurface which terminates on the event horizon. We shall consider the initial data on \(\Sigma_0\) for a linearized perturbations \((\delta g_{\mu\nu}, \delta A_\mu, \delta B_{\mu\nu}, \delta \phi)\) with \(\delta T_{\mu\nu} \text{(matter)}\) and \(\delta j^\mu \text{(matter)}\). We require that \(\delta T_{\mu\nu} \text{(matter)}\) and \(\delta j^\mu \text{(matter)}\) vanish at infinity and the initial data for \((\delta g_{\mu\nu}, \delta A_\mu, \delta B_{\mu\nu}, \delta \phi)\) vanish in the vicinity of the black hole horizon \(\mathcal{H}\) on the hypersurface \(\Sigma_0\). It envisages the fact that for the initial time \(\Sigma_0\), the considered black hole is unperturbed. In our considerations we assume that all the matter, charges and so on will fall into black hole and the black hole will settle down to another stationary black hole solution of the source-free equations of EMAD gravity.

Our main aim is to compute \(\delta M\), \(\delta J\), \(\delta Q\) and \(\delta A\) for the final state of black hole and verify the validity of the first law of black hole mechanics in EMAD gravity. From relations (24) and (25), one has

\[
\delta M - \Omega \delta J = -\int_{\Sigma_0} \epsilon^\delta_{\alpha\beta\gamma} \left[ 2\xi^\mu \delta T^\delta_{\mu} \text{(matter)} + 4\xi^\mu A_\mu \delta j^\delta \text{(matter)} \right] = \int_{\Sigma_0} \alpha^\delta n_\delta \epsilon_{\alpha\beta\gamma},
\] (26)

where \(n_\mu\) is the future directed unit normal vector to the hypersurface \(\Sigma_0\) and \(\epsilon_{\alpha\beta\gamma} = n^\mu \epsilon_{\mu\alpha\beta\gamma}\). The fact that our assumption is that all the matter fall into black hole enables one to replace \(n_\mu\) by \(k_\mu\) which is tangent vector to null geodesic generators of the event horizon \(\mathcal{H}\) of the black hole in equation (26) and \(\mathcal{H}\) for \(\Sigma_0\).

The second term in \(\alpha^\delta\) can be written in the form

\[
ST = -4 \int_{\mathcal{H}} \Phi_{BH} \delta j^\mu k_\mu \epsilon_{\alpha\beta\gamma},
\] (27)

where we have denoted by \(\Phi_{BH} = -(\xi^\mu A_\mu) \big|_{\mathcal{H}}\). Using the Raychanduri’s equation (see, e.g., [13]) and because of the symmetry of the background we have \(\mathcal{L}_\xi A_\mu = 0\), one can show that \(\Phi_{BH}\) is constant on black hole horizon \(\mathcal{H}\). By \(\delta Q \text{(matter)}\) we denote the flux of the charge into the black hole

\[
\delta Q \text{(matter)} = -4 \int_{\mathcal{H}} \delta j^\mu \text{(matter)} k_\mu \epsilon_{\alpha\beta\gamma}.
\] (28)

Consequently having in mind equation (19) and the fact that \(H_{\alpha\beta\gamma} k^\alpha k^\beta = 0\) which implies that \(H_{\alpha\beta\gamma} k^\alpha \propto k^\beta\) and hence the pull-back to \(\mathcal{H}\) of \(H_{\alpha\beta\gamma} k^\alpha\) vanishes, we obtain only the dependence on the dilaton field \(\phi\) and \(F_{\mu\nu}\). Therefore the expression for the flux of the charge implies

\[
\delta Q \text{(matter)} = -4 \int_{\mathcal{H}} \left[ \nabla_\mu (e^{-2\phi} F^{\mu\nu}) k_\nu \epsilon_{\alpha\beta\gamma} \right] = \delta Q(\phi - F).
\] (29)

Substituting it into relation (26) we conclude the following:
\[ \delta M - \Omega \delta J - \Phi_{BH} \delta Q_{(\phi-F)} = 2 \int_{\mathcal{H}} \delta T^{\mu \nu} \xi^\nu k_\mu. \] (30)

In order to find the change in the area of the black hole we use the fact that null generators of the black hole horizon of the perturbed black hole coincide with the null generators of the unperturbed stationary black hole \( \delta k_\mu \propto k_\mu \) and since the expansion \( \theta \) and shear \( \sigma_{\mu\nu} \) vanish in the stationary background, the perturbed Raychaudhury’s equation is of the form

\[ \frac{d(\delta \theta)}{d\lambda} = -\delta \left( T^{\mu \nu}_{(total)} k^\mu k^\nu \right) |_{\mathcal{H}} = -\delta \left( T^{\mu \nu}_{(matter)} k^\mu k^\nu \right) |_{\mathcal{H}} - \delta \left( T^{\mu \phi}_{(F, H)} k^\mu k^\phi \right) |_{\mathcal{H}}, \] (31)

where we exploit the fact that \( T(\phi, F, H)_{\mu\nu} k^\mu k^\nu \big|_{\mathcal{H}} = 0 \) and \( \delta k_\mu \propto k_\mu \) to eliminate terms in the form \( T(\phi, F, H)_{\mu\nu} k^\mu \delta k^\nu \).

But we have left with the terms of the form \( \delta T^{\mu \nu}_{\phi, F, H} k^\mu k^\nu \big|_{\mathcal{H}} \). We shall compute them part by part

\[ \delta T^{\mu \nu}(F)k^\mu k^\nu \big|_{\mathcal{H}} = \left( 4\delta F_{\gamma\lambda} F_{\nu \gamma} - \frac{1}{2} \delta g_{\mu\nu} F^2 - g_{\mu\nu} F_{\alpha\beta} \delta F^{\alpha\beta} \right) e^{-2\phi} k^\mu k^\nu + \delta(e^{-2\phi}) T^{\mu \nu}_{(F)} k^\mu k^\nu, \] (32)

where the last two terms in the first part of Eq. (32) are equal to zero because of the fact that \( k_\mu \) is a null vector in the perturbed as well as in the unperturbed metric. Moreover, we have that \( F_{\mu\nu} k^\nu \propto k_\mu \), thus because of the antisymmetry of \( \delta F_{\mu\nu} \) this term is equal to zero. The second part of equation (32) is also equal to zero as we have seen above.

In the case of the energy momentum tensor \( T^{\mu \nu}_{\phi}(H) \) for the three-index antisymmetric tensor field we arrive at the expression

\[ \delta T^{\mu \nu}_{\phi}(H) k^\mu k^\nu \big|_{\mathcal{H}} = \left( 2\delta H_{\alpha\beta\gamma} H^{\alpha\beta} - \frac{1}{6} \delta g_{\mu\nu} H^2 - \frac{1}{3} g_{\mu\nu} \delta H_{\alpha\beta\gamma} H^{\alpha\beta\gamma} \right) e^{-4\phi} k^\mu k^\nu + \delta(e^{-4\phi}) T^{\mu \nu}_{\phi}(H) k^\mu k^\nu. \] (33)

The previous arguments can be repeated, i.e., using the antisymmetry of \( \delta H_{\alpha\beta\gamma} \) and the property of the vector field \( k_\mu \) we reach to the conclusion that the right-hand side of (33) is equal to zero. On the other hand for the dilaton field energy momentum tensor we have

\[ \delta T^{\mu \nu}_{\phi}(\phi) k^\mu k^\nu \big|_{\mathcal{H}} = \left( 4\delta(\nabla_\mu \phi)\nabla_\nu \phi - \delta g_{\mu\nu} \left( \nabla_\phi \right)^2 - 2g_{\mu\nu} \delta(\nabla_\gamma \phi) \nabla^\gamma \phi \right) k^\mu k^\nu = 0, \] (34)

where in this case we used the fact that \( L_k \phi = 0 \), as well as the property of \( k_\mu \) vector field. Thus, we finally receive from equation (31) only one term which yields

\[ \frac{d(\delta \theta)}{d\lambda} = -\delta T^{\mu \nu}_{(matter)} k^\mu k^\nu \big|_{\mathcal{H}}, \] (35)

In order to compute Eq. (35) one can repeat the same steps as done in [11]. Now we shall briefly review the key points of the procedure (see also for particulars [13,16]). Namely, one begins with integrating the right-hand side of equation (35) over the horizon of black hole, the changes in the black hole geometry may be neglected. Thus, it is possible to substitute for \( k_\mu \) the following expression

\[ k_\mu = \left( \frac{\partial}{\partial V} \right)_\mu = \frac{1}{\kappa V} \left( t_\mu + \Omega \varphi_\mu \right), \] (36)

where \( \kappa \) is the surface gravity, \( V \) is an affine parameter along the null geodesics tangent to \( \xi_\beta \) which generates the adequate Killing horizon. One should remark [11] that if the function \( v \) (it is called \( \text{Killing parameter time} \)) on the
portion of Killing horizon satisfies $\xi^\beta \nabla_\beta v = 1$, then it is related with $V$ by the expression $V = \exp(\kappa v)$. Next, one multiplies both sides of the resulting equation by $\kappa V$ and integrates over the horizon.

We also recall that $\theta$ measures the local rate of change of the cross-sectional area as the observer moves up the null geodesics, i.e., $\theta = \frac{1}{A} \frac{dA}{d\lambda}$, where $\lambda$ is an affine parameter which parametrized null geodesics generators of the horizon. The left-hand side of equation (35) is evaluated by integration by parts, having in mind that $V = 0$ at the lower limit and $\theta$ has to vanish faster than $1/V$ as the affine parameter tends to infinity. The consequence of the above establishes the result

$$\kappa \delta A = \int \delta T_{\mu \nu}^{(\text{matter})} \xi^\nu k_\mu$$

Then, after rescaling the area we received the *physical process version* of the first law of black hole mechanics in EMAD gravity. It has the form known from [10] being a strong support for the cosmic censorship hypothesis, namely

$$\delta M - \Omega \delta J - \Phi_{\text{BH}} \delta Q(\phi - F) = \frac{\kappa}{8\pi} \delta A.$$ (38)

**III. CONCLUSIONS**

In our work we have studied stationary black hole solution to EMAD gravity being the low-energy limit of the heterotic string theory. We perturbed the considered black hole by dropping into it some matter. Assuming that our black hole will not be destroyed in this process and eventually settles down to a final stationary state we calculate the change of the mass, angular momentum, dilaton-gauge field current and change in the area of the black hole horizon. We obtained in such way the *physical process version* of the first black hole dynamics in EMAD gravity. It happens that it has the same form as the first law of black hole dynamics as derived in [10]. Then, the proof of the *physical process version* of the first law of black hole mechanics in EMAD gravity provides the strong support for the cosmic censorship hypothesis in this theory.

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