On the tachyon inflation

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Abstract

Although the formulas of tachyon inflation correspond to those of the inflation driven by the ordinary scalar field, there is obvious difference between them, which cannot be neglected. We calculate the scalar and tensor perturbation of the string theory inspired tachyon inflation, which has been widely studied recently. We also show, through the Hamilton-Jacobi approach, that the rolling tachyon can essentially produce enough inflation. An exact solution with the inverse squared potential of tachyon field has been proposed and its power spectra has been analyzed.

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1 Introduction

The idea of inflation is legitimately regarded as an great advancement of modern cosmology: it solves the horizon, flatness and monopole problem, and it provides a mechanism for the generation of density perturbations needed to seed the formation of structures in the universe [1]. In standard inflationary models [2], the physics lies in the inflation potential. The underlying dynamics is simply that of a single scalar field rolling in its potential. This scenario is generically referred to as chaotic inflation in reference to its choice of initial conditions. This picture is widely favored because of its simplicity and has received by far the most attention to date. Some potentials that give the correct inflationary properties have been proposed [3] in the past two decades. However, most of these investigations were not a stickler for the fundamental physics, such as the standard model or grand unified theories in particle physics. The above-mentioned conventional physics does not yield an inflation potential that agrees with observations, such as enough number of e-folds, the amplitude of the density perturbation and etc.

Recently, pioneered by Sen [4], the study of non-BPS objects such as non-BPS branes, brane-antibrane configurations or space-like branes [5] has attracted physical interests in string theory. Sen showed that classical decay of unstable D-brane in string theories produces pressureless gas with non-zero energy density [6]. Gibbons took into account the gravitational coupling by adding an Einstein-Hilbert term to the effective action of the tachyon on a brane, and initiated a study of "tachyon cosmology" [7]. The basic idea is that the usual open string vacuum is unstable but there exists a stable vacuum with zero energy density. There is evidence that this state is associated with the condensation of electric flux tubes of closed string [6]. These flux tubes are successfully described by using an effective Born-Infeld action [8]. The string theory motivated tachyon inflation has been discussed in Ref.[9, 10]. These investigation are based on the slow-roll approximation so that they are rather incomplete. Slow-roll is not the only possibility for successfully realistic models of inflation, and solutions outside the slow-roll approximation have been found in particular situations [11]. The elegant review of Hamilton-Jacobi formalism in inflationary cosmology with ordinary scalar field can be found in Ref.[12].

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In this paper, we discuss the generic properties of tachyon inflation with the Born-Infeld action. Although the formulas of tachyon inflation are correspond to those of the inflation driven by the ordinary scalar field, there is obvious difference between them. We also show an exact solution and a string motivated model.

2 Rolling tachyon dynamics

2.1 The general formalism

We consider spatially flat FRW line element given by:

\[ ds^2 = dt^2 - a^2(t)\left(dx^2 + dy^2 + dz^2\right) = a^2(\tau)\left[d\tau^2 - (dx^2 + dy^2 + dz^2)\right] \]  

where \( \tau \) is the conformal time, with \( dt = ad\tau \). As shown by Sen [6], a rolling tachyon condensate in either bosonic or supersymmetric string theory can be described by a fluid which, in the homogeneous limit, has energy density and pressure as follows

\[ \rho = \frac{V(T)}{\sqrt{1 - \dot{T}^2}}, \quad p = -V(T)\sqrt{1 - \dot{T}^2} \]  

where \( T \) and \( V(T) \) are the tachyon field and potential, and an overdot denotes a derivative with respect to the coordinate time \( t \). When taking the gravitational field into account, the effective Lagrangian density in the Born-Infeld-type is [7]

\[ L = \sqrt{-g}\left(\frac{R}{2\kappa} - V(T)\sqrt{1 - g^{\mu\nu}\partial_\mu T\partial_\nu T}\right) \]  

where \( \kappa = 8\pi G = M^{-2} \). For a spatially homogenous tachyon field \( T \), we have the equation of motion

\[ \ddot{T} + 3H\dot{T}(1 - \dot{T}^2) + \frac{V'}{V}(1 - \dot{T}^2) = 0 \]  

which is equivalent to the entropy conservation equation. Here, the Hubble parameter \( H \) is defined as \( H \equiv \left(\frac{\dot{a}}{a}\right) \), and \( V' = dV/dT \). If the stress-energy of the universe is dominated by the tachyon field \( T \), the Einstein field equations for the evolution of the background metric, \( G_{\mu\nu} = \kappa T_{\mu\nu} \), can be written as

\[ H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa}{3} \sqrt{\frac{V(T)}{1 - \dot{T}^2}} \]  

and

\[ \left(\frac{\ddot{a}}{a}\right) = H^2 + \dot{H} = \frac{\kappa}{3} \sqrt{\frac{V(T)}{1 - \dot{T}^2}} \left(1 - \frac{3}{2} \dot{T}^2\right) \]  

Eqs.(4)-(6) form a coupled set of evolution equations of the universe. The fundamental quantities to be calculated are \( T(t) \) and \( a(t) \), and the potential \( V(T) \) is given when the model is specified. The period of accelerated expansion corresponds to \( \dot{T}^2 < \frac{3}{2} \) and decelerate otherwise. In the limit case \( \dot{T} = 0 \), there is no difference in meaning of the expansion of universe between tachyon inflation and ordinary inflation driven by inflaton. However, the case of \( \dot{T} \neq 0 \) forms a sharp contrast. Although the formulas of tachyon inflation are correspond to those of the inflation driven by ordinary scalar field, there is obvious difference between them which can not be neglected. From Eqs.(4)-(6), we have two first-order equations

\[ \dot{T} = -\frac{2}{3} \frac{H'(T)}{H^2(T)} \]  

\[ [H'(T)]^2 - \frac{9}{4} H^4(T) = -\frac{\kappa^2}{4} V^2(T) \]
These equations are wholly equivalent to the second-order equation of motion (4).

Analogous to the inflation driven by ordinary scalar field [12], we define the "slow-roll" parameters as follows

\[ \epsilon(T) \equiv \frac{2}{3} \left( \frac{H'(T)}{H^2(T)} \right)^2 \]  

\[ \eta(T) \equiv \frac{1}{3} \left( \frac{H''(T)}{H(T)} \right) \]  

and

\[ \xi(T) \equiv \frac{2}{3} \left( \frac{H'(T)H'''(T)}{H^6(T)} \right)^{1/2} \]

where the sign ambiguity is the result of the convention that \( \sqrt{\epsilon} \) is always taken to be positive. Clearly, the definitions of the parameters Eqs. (9)-(11) are quite different from those defined in ordinary inflation. This is very natural for the Born-Infeld action is sharply different from that of the ordinary scalar field. In term of \( \epsilon \) parameter, Eq.(8) can be reexpressed as

\[ H^4(T)[1 - \frac{1}{3}\epsilon(T)] = \frac{k^2}{9}V^2(T) \]

which is referred to as the Hamilton-Jacobi equation of tachyon inflation. The number of e-folds of the inflation produced when the tachyon field rolls from a particular value \( T \) to the end point \( T_e \) is

\[ N(T, T_e) \equiv \int_{t}^{t_e} H(t) dt = \int_{T}^{T_e} \frac{H}{T} dT \]

Therefore, we have

\[ a(T) = a_e \exp[-N(T)] \]

where \( a_e \) is the value of the scale factor at the end of inflation. To match the observed degree of flatness and homogeneity in the universe, one requires many e-folds of inflation, typically \( N \approx 50 \).

During inflation, the tachyon and graviton fields underwent quantum fluctuations. The most important observational deductions of the inflationary scenario is that inflation explains not only the high degree of large-scale isotropy in the universe, but also the underlying mechanism for the observed anisotropy. The quantum fluctuations on small scales are quickly redshifted to scales much larger than the horizon, where they are "congealed" as perturbations in the background metric during inflation epoch.

It is not difficult to prove that the gravitational and curvature perturbations in the tachyon inflation take similar form as those in the conventional inflation, except the definitions of the parameters. Therefore, we will carry out our analysis in analogy with the discussions for ordinary inflation [12, 13, 14, 15, 16]. The spectrum of curvature perturbation \( P_R(k) \) as function of wavenumber \( k \) could be expressed as [15]

\[ P_R^{1/2}(k) = \sqrt{\frac{k^3}{2\pi^2}} \left| \frac{u_k}{z} \right| \]

where mode function \( u_k \) satisfies following equation [14]

\[ \frac{d^2u_k}{dt^2} + \left( k^2 - \frac{1}{z} \frac{d^2z}{dt^2} \right) u_k = 0 \]

and the quantity \( z \) is defined as

\[ z \equiv \frac{aT}{H} \]

and

\[ \frac{1}{z} \frac{d^2z}{dt^2} = 2a^2H^2(1 + 4\epsilon - 3\eta + 9\epsilon^2 - 14\epsilon\eta + 2\eta^2 + \frac{1}{2}\xi^2) \]
Clearly, the above expression Eq. (18) is different from that of the ordinary scalar field because the coupling of curvature perturbations to the stress-energy of tachyon field is in a very different manner.

Similarly, the spectrum of gravitational wave \( P_g(k) \) is defined by

\[
\frac{d^2 v_k}{d \tau^2} + \left( k^2 - \frac{a_{\tau\tau}}{a} \right) v_k = 0
\]

where

\[
a_{\tau\tau} = 2a^2 H^2 (1 - \epsilon)
\]

The above expression Eq. (21) is the same as that in ordinary scalar field inflation, because the tensor perturbations only describe the propagation of gravitational waves and do not couple to the matter term.

### 2.2 Exact solution for power-law inflation of tachyon

In this subsection, we will analyze an exactly solvable model in tachyon inflation. The potential in this model is the inverse square potential

\[
V(T) = 2\kappa \left( n^2 - \frac{n}{3} \right) \frac{1}{2} (T - T_0)^{-2}
\]

which played an important role in the inflationary cosmology of tachyon. The spectrum equations (16) and (20) with the inverse square potential can be solved exactly. This potential corresponds to the well known Power-law inflation, in which the scale factor expands as \( a(t) \propto t^n \). In this case, the parameters take the particularly simple forms:

\[
H(T) = \frac{\left( \frac{n}{2} \right) \sqrt{2}}{T - T_0}
\]

and

\[
\epsilon = \frac{1}{n}, \quad \eta = \frac{1}{n}, \quad \xi = \frac{\sqrt{6}}{n}
\]

According to Eq. (23), the spectrum equations for curvature perturbations and gravitational perturbations are reduced to the following Bessel equations respectively,

\[
\left[ \frac{d^2}{d \tau^2} + k^2 - \frac{\mu^2 - \frac{1}{2}}{\tau^2} \right] u_k = 0
\]

\[
\left[ \frac{d^2}{d \tau^2} + k^2 - \frac{\nu^2 - \frac{1}{2}}{\tau^2} \right] v_k = 0
\]

where \( \mu = \frac{n}{2} + \frac{2}{n+1} \) and \( \nu = \frac{n}{2} + \frac{1}{n+1} \). It is worth noting that the mode functions with \( \mu \neq \nu \) in the tachyon inflation is quite different from that in the power-law inflation driven by ordinary scalar field, in which \( \mu = \nu \). In the long wavelength limit, \( k/aH \rightarrow 0 \), the asymptotic forms of the power spectrum are

\[
P_R^{\frac{1}{2}}(k) = 2^{\nu + \frac{1}{2}} \frac{\Gamma(\mu)}{\Gamma(3/2)} \left( \mu - \frac{1}{2} \right)^{\frac{1}{2} - \mu} \frac{1}{m_{pl}} \frac{H^2}{|H'|} |_{k=aH}
\]

\[
P_g^{\frac{1}{2}}(k) = 2^{\nu - \frac{3}{2}} \frac{\Gamma(\nu)}{\Gamma(3/2)} \left( \nu - \frac{1}{2} \right)^{\frac{1}{2} - \nu} \frac{H}{m_{pl}} |_{k=aH}
\]

The property \( \mu \neq \nu \) in tachyon power-law inflation will help us to distinguish it from that driven by ordinary scalar field.
2.3 The slow-roll approximation of tachyon inflation

The slow-roll approximation has played an important role in the inflationary cosmology. In the tachyon inflation, the situation is much the same. The slow-roll approximation is the assumption that the field evolution is dominated by drag from the expansion in small parameters \(\epsilon, |\eta| \ll 1\) about the de Sitter limit \(\epsilon = 0\).

In this approximation, it is consistent to take \(\epsilon\) and \(\eta\) to be approximately constant, and the solutions are again Hankel function of the first kind with

\[\mu = \frac{3}{2} + 4\epsilon - 2\eta\]  \(\text{(29)}\)

and

\[\nu = \frac{3}{2} + \epsilon\]  \(\text{(30)}\)

to first order in the small parameters \(\epsilon, \eta\). These give the final result of lowest-order

\[P_R^{1/2}(k) = \left[1 - (4C + 2)\epsilon + 2C\eta\right] \frac{2H^2}{m_{\text{pl}}^2 |H'|}_{k=aH}\]  \(\text{(31)}\)

and

\[P_g^{1/2}(k) = \left[1 - (C + 1)\epsilon\right] \frac{4}{\sqrt{\pi} m_{\text{pl}}} \frac{H}{m_{\text{pl}}} |k=aH|\]  \(\text{(32)}\)

where \(C = -2 + \ln 2 + \gamma \approx -0.73\) is a constant, \(\gamma\) being the Euler constant originated in the expansion of the \(\Gamma\)-function.

From above discussion, we can find that the expressions of tachyon inflation in the slow-roll approximation have obvious distinction from the inflation driven by ordinary scalar field. It is not surprising for the Born-Infeld action is essentially different from the that of ordinary scalar field. Therefore, we shall obtain different observable values for current astrophysical interest in the tachyon case.

3 Towards a realistic tachyon inflation

So far, we have discussed the generic procedures for dealing with the tachyon inflation. In this section, we will analyze a realistic tachyon model. The form of the tachyon potential \(V(T)\) may depend on the underlying (bosonic or supersymmetric) string field theory. Recently, Kutasov, Mariño and Moore [17] have given the tachyon potential around the maximum in the bosonic theory,

\[V(T) = \frac{m_s^4}{2\pi} g_s \left(1 + \frac{T}{l_s}\right) \exp\left(-\frac{T}{l_s}\right)\]  \(\text{(33)}\)

where \(g_s\) is the string coupling, and \(l_s\) are the fundamental string length and mass scales \([18]\). It is worth noting that in the above expression (Eq.(33)), we recover the dimension of the tachyon field in the units of \(m_s\).

The 4D Planck mass can be rewritten by

\[m_{\text{pl}}^2 = \frac{m_s^{d+2} r^d}{\pi g_s}\]  \(\text{(34)}\)

where \(r\) is a radius of the compactification, and \(d\) is the numbers of compactified dimensions. Customarily, we can assume \(r \gg l_s\) in order to be able to use the effective 4D field theory. However, following Sen[6], the potential should be exponential at large \(T\)

\[V(T) = e^{-\frac{T}{l_s}}\]  \(\text{(35)}\)

For definiteness one should assume that the potential \(V(T)\) in the Born-Infeld type action is a smooth function interpolating between two asymptotic expressions, (33) at maximum and (35) at infinity.
Kofman and Linde [10] have shown that tachyon will acquire immediately a matter dominated equation of state when it rolls to its ground state at \( T \to \infty \). The energy density of matter decreases as \( a^{-3} \), whereas the density of radiation decreases as \( a^{-4} \). In addition, all matter that appears after inflation should be produced in the reheating process. In most of reheating theory, creation of particles occurs only when the inflation field oscillates near the minimum of its effective potential. Since, the effective potential (33) does not have any minimum at finite \( T \), this mechanism does not work. Even though a small fraction of tachyon energy is released into radiation, it will be quickly redshifted away. Therefore if the tachyon originally dominated the energy density of the universe, then it never becomes radiation dominated, in contradiction with the theory of nucleosynthesis. Summarily, Kofman and Linde argue that above fact rules out inflationary models where tachyon rolls to the minimum of the tachyon potential (33) at \( T \to \infty \) [9].

Alternatively, we here consider the possibility to achieve tachyonic inflation by assuming that the inflation occurs near the top of tachyon potential (33). We assume that the displacement of the field from its stationary value \( T_0 \) is small compared to the Planck scale, \( T - T_0 \ll m_{pl} \), and \( T_0 = 0 \) for tachyon potential (33). We argue that slow-roll approximation is inconsistent in this case. We may prove this argument by contradiction. The slow-roll approximation is an assumption that the tachyon field evolution is dominated by dragging from expansion, \( \ddot{T} \approx 0 \), so we have

\[
\epsilon = (2\pi)^2 \frac{R}{(l_s)}^d \frac{\left(\frac{T}{l_s}\right)^2}{(1 + \frac{T}{l_s})^3} e^{\frac{T}{l_s}}
\]

and

\[
\eta = \pi^2 \frac{R}{(l_s)}^d \frac{[3(\frac{T}{l_s})^2 - 4]}{(1 + \frac{T}{l_s})^3} e^{\frac{T}{l_s}}
\]

where the parameter \( \eta \) becomes large near the top of the tachyon potential, indicating a breakdown of the slow-roll assumption. However, in the region

\[
0 < T \ll (2\pi)^{\frac{3}{2}} g_s \left( \frac{m_{pl}}{m_s^2} \right)
\]

the Hamilton-Jacobi equation (8) can be reduced to

\[
[1 - \frac{1}{3} \epsilon(T)]^\frac{1}{2} = \frac{V(T)}{V(T_0)}
\]

where \( T_0 \) is a stationary point of tachyon field. The parameters \( \epsilon \) and \( \eta \) can be written as

\[
\epsilon(T) = \frac{3}{2} \left[ 1 - (1 + \frac{T}{l_s})^2 \exp\left( -\frac{2T}{l_s} \right) \right]
\]

and

\[
\eta(T) = \epsilon \frac{\epsilon}{4\sqrt{\frac{48}{3} V(T)}} (12 - 8\epsilon) m_{pl} \gamma \left( \frac{3V(T)}{4\epsilon(T)} \right)^\frac{1}{4}
\]

where the second term dominates, which is equivalent to \( |\eta| \gg |\epsilon| \). The number of e-folds \( N \) is given by

\[
N(T, T_e) = \int_T^{T_e} \left[ \frac{4\kappa^2 V^2(T)}{\epsilon^2(T)(81 - 54\epsilon(T))} \right] \frac{dT}{dx} = \int_{x_e}^{x} \frac{dx}{\sqrt{1 - (1 + x)^2 \exp(-2x)}}
\]

where \( x_e = \frac{T_e}{l_s} \approx 1.44 \) and \( x = \frac{T}{l_s} \). In this model, the tachyon field must initially be displaced from \( T_0 = 0 \), and we denote the initial value of \( T \) by \( T_i \). Following Linde’s viewpoint of "chaotic inflation" [19], we envision that the initial distribution of \( T_i \) is "chaotic", with \( T_i \) lacking on different values in different region of the Universe.
The number of e-folds must be sufficiently large to solve the major cosmological problems. For our universe, we usually assume $N(T, T_e) \approx 50$. Therefore, the initial value $T_i$ should be

$$T_i \approx 10^{-5} m_s^{-1} \exp\left(-\frac{300 \sqrt{g_s} \pi^2 m_{pl}}{m_s}\right).$$

(43)

It is extremely close to the stationary point of the tachyon field $T_0$ and depends on the parameters of string theory, $m_s$ and $g_s$. If we take $m_s/\sqrt{g_s} = 10^{-1} m_{pl}$, $m_s/\sqrt{g_s} = 10^{-2} m_{pl}$, and $m_s/\sqrt{g_s} = 10^{-3} m_{pl}$, the value of $\ln T_i$ is $(\frac{1}{2} \ln g_s - 40888)$, $(\frac{1}{2} \ln g_s - 409157)$ and $(\frac{1}{2} \ln g_s - 4091838)$ respectively.

Next, one can introduce a new variable $y \equiv \kappa aH$ to replace conformal time $\tau$ in mode equation, which corresponds to one of conventional inflation [11]. However, there are obvious difference between them. It is easy to find

$$dy = -k[1 - \frac{2}{3}(\frac{H'}{H^2})^2]d\tau$$

(44)

and the mode equations reduced to

$$y^2(1 - \epsilon)\frac{d^2v_k}{dy^2} + 4y\epsilon(\epsilon - \eta)\frac{dv_k}{dy} + (y^2 - 2 + \epsilon)v_k = 0$$

(45)

and

$$y^2(1 - \epsilon)^2\frac{d^2u_k}{dy^2} + 4y\epsilon(\epsilon - \eta)\frac{du_k}{dy} + [y^2 - F(\epsilon, \eta, \xi)]u_k = 0$$

(46)

where

$$F(\epsilon, \eta, \xi) = 2(1 + 4\epsilon - 3\eta + 9\epsilon^2 - 14\epsilon\eta + 2\eta^2 + \frac{1}{2}\xi^2).$$

(47)

In the approximately de Sitter limit $\epsilon \ll 1, |\eta| \gg \epsilon$ and

$$\eta = -\sqrt{\frac{7}{8}} \left(\frac{2\pi}{g_s}\right)^\frac{1}{2} \left(\frac{r}{l_s}\right)^\frac{1}{2}.$$ 

(48)

Eq.(46) can be expressed as a bessel equation

$$\left[\frac{d^2}{dy^2} + \frac{\mu^2}{y^2}\right]u_k = 0$$

(49)

where $\mu = \frac{4}{3} - 2\eta$. The spectrum of curvature perturbation is

$$P_R^k = 2^{\mu - 1/2} \frac{\Gamma(\mu)}{\Gamma(3/2)} \frac{H}{m_{pl}^2} \sqrt{\epsilon} |_{\kappa = aH},$$

(50)

and the scalar spectral index is

$$n_R - 1 \equiv \frac{d \ln(P_R)}{d \ln(k)} = 2\eta.$$ 

(51)

On the other hand, Eq.(45) can be solved in a fashion similar to the scalar mode equation (46). However, in the $\epsilon \ll |\eta|$ case, tensor fluctuations become negligible relative to scalar fluctuations.

By now, the inflationary universe is generally recognized to be the most likely scenario that explains the origin of the Big Bang. So far, its predictions of the flatness of the universe and the almost scale-invariant power spectrum of the density perturbation that seeds structure formation are in good agreement with the cosmic microwave background (CMB) observations. The key data of CMB are the density perturbation amplitude measured by COBE [20] and its power spectrum index $n$ [21] which leads us to conclude that the string coupling generically should be very strong.
4 Summary and remarks

In this paper, we have shown a general approach to characterizing inflationary theory which is based on the rolling tachyon dynamics. In this approach, the standard slow-rolling approximation corresponds to taking $\epsilon \approx 0$ and the corresponding solution of the Hamilton-Jacobi equation is $\epsilon = \frac{1}{2}\frac{V'(T)}{V(T)}$. However, in the limit $T - T_0 \ll m_{pl}/\sqrt{V(T_0)}$, a general non-slow-roll solution exists, $\epsilon = \frac{3}{4} \left[ 1 - \frac{V^2(T)}{V^2(T_0)} \right]$. We calculate the spectrum of the scalar and tensor perturbation and show an exact solution of the spectrum of perturbation for power-law tachyon inflation. We apply the general formalism beyond the slow-roll approximation to the string motivated tachyon inflation, which allows one to argue that rolling tachyon provides an possibility of the origin of inflation.

On the other hand, the present experimental data does not determine the values of $m_s$ and $g_s$. Therefore, a careful analysis in realistic string models may be helpful for determining the string parameters by using the CMB observations.

The Born-Infeld action is quite different from that of the ordinary scalar field. Therefore, although the expressions of tachyon inflation correspond to those of the inflation driven by the ordinary scalar field, there is obvious difference between them, which can not be neglected.

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