Direct Urca processes on nucleons in cooling neutron stars.

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We use the field theoretical model to perform relativistic calculations of neutrino energy losses caused by the direct Urca processes on nucleons in the degenerate baryon matter. By our analysis, the direct neutron decay in the superdense nuclear matter under beta equilibrium is open only due to the isovector meson fields, which create a large energy gap between protons and neutrons in the medium. Our expression for the neutrino energy losses, obtained in the mean field approximation, incorporates the effects of nucleon recoil, parity violation, weak magnetism, and pseudoscalar interaction. For numerical testing of our formula, we use a self-consistent relativistic model of the multicomponent baryon matter. The relativistic emissivity of the direct Urca reactions is found substantially larger than predicted in the non-relativistic approach. Weak magnetism effects approximately double the neutrino emissivity, while the pseudoscalar interaction slightly suppresses the energy losses, approximately by 10%.

Key words: Neutron star, Neutrino radiation
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1 Introduction

Modern calculations [1] based on relativistic equations of state indicate that the neutron star cores consist of neutrons with the admixture of protons, electrons, muons and some exotic particles (including hyperons, K-mesons, quarks and so on . . . ). The composition is governed by the charge neutrality and equilibrium of the medium under weak processes \( B_1 \rightarrow B_2 + l + \bar{\nu}_l \), \( B_2 + l \rightarrow B_1 + \nu_l \), where \( B_1 \) and \( B_2 \) are baryons (or quarks), and \( l \) is a lepton, either an electron or a muon. These reactions, widely known as the direct Urca processes, are a central point of any modern scenarios of evolution of neutron stars. Neutrino
energy losses caused by the direct Urca processes lead to a rapid cooling of
degenerate neutron star cores [2]. The corresponding reactions on nucleons,
\( n \rightarrow p + l + \bar{\nu}_l \), \( p + l \rightarrow n + \nu_l \), are the most powerful sources of neu-
trinos and antineutrinos in cooling neutron stars. In spite of widely adopted
importance of these reactions, the corresponding neutrino energy losses are
not well investigated yet. A simple formula suggested by Lattimer et al. [3]
more than ten years ago has been derived in a non-relativistic manner. Ac-
tually, however, the superthreshold proton fraction, necessary for the direct
Urca processes to operate in the degenerate nuclear matter, appears at large
densities, where the Fermi momenta of participating nucleons are comparable
with their effective mass. Moreover, according to modern numerical simula-
tions, the central density of the star can be up to eight times larger than the
nuclear saturation density [1]. This implies a substantially relativistic motion
of nucleons in the superdense neutron star core. The appropriate equation of
state for such a matter is actually derived in the relativistic approach, and the
relevant neutrino energy losses must be consistent with the used relativistic
equation of state. Some aspects of this problem was studied by Leinson and
Pérez [4], who have estimated relativistic effects of baryon recoil and parity
violation in the direct Urca processes including also effects of the baryon mass
difference. This approach is useful, for example, for the direct Urca reactions
changing the baryon strangeness because, in this case, the contribution of the
weak magnetism into the matrix element of the beta-decay is not well known
even for free particles. For nucleons, the relativistic regime should incorporate
the effects of weak magnetism and pseudoscalar interaction, which drastically
influence the neutrino emissivity of the corresponding reactions. To demon-
strate this, in the present paper we consider the totally relativistic direct Urca
process on nucleons. We utilize the Walecka-type relativistic model of baryon
matter [5], where the baryons interact via exchange of \( \sigma \), \( \omega \), and \( \rho \) mesons,
and perform the calculation of the neutrino energy losses in the mean field
approximation. This approximation is widely used in the theory of relativistic
nuclear matter, and allows to calculate in a self-consistent way the composi-
tion of the matter together with energies, and effective masses of the baryons.
In Section II we begin with considering the relativistic kinematics of the neu-
tron beta decay in the medium under beta equilibrium. We consider a free
gas model and some field theoretical models of nuclear matter to demonstrate
that the direct neutron decay in neutron stars is open only due to strong in-
teractions caused by isovector mesons. We shortly discuss the energies and
the wave functions of nucleons in the mean field approximation and demon-
strate the nonconservation of the charged vector current of nucleons in this
model. The matrix element of the neutron beta decay is derived in Section
III. In Section IV we calculate the neutrino energy losses caused by the di-
rect Urca on nucleons in the degenerate nuclear matter under chemical and
thermal equilibrium. In Section V we inspect the non-relativistic limit of the
neutrino energy losses in order to compare this with the expressions earlier
obtained in [3] and [4]. Efficiency of the relativistic approach is numerically
studied in Section VI. We evaluate the neutrino energy losses due to the direct Urca processes on nucleons in the multicomponent baryon matter under beta equilibrium and compare the relativistic result with that predicted by the known non-relativistic formula. We specially discuss the contributions of weak magnetism and pseudoscalar interaction. Summary and conclusion are in Section VII. In Appendix, we discuss some details of the model used for the nuclear matter.

In what follows we use the system of units \( \hbar = c = 1 \) and the Boltzmann constant \( k_B = 1 \). Summation over repeated Greek indexes is assumed.

2 Kinematics of the reaction and models of nuclear matter.

Before calculating the neutrino energy losses caused by the direct Urca processes, let us examine relativistic kinematics of the reaction, \( n \rightarrow p + l + \bar{\nu}_l \), in the degenerate matter under beta equilibrium. In what follows we consider massless neutrinos of energy and momentum \( k_1 = (\omega_1, k_1) \) with \( \omega_1 = |k_1| \). The energy-momentum of the final lepton \( l = e^-, \mu^- \) of mass \( m_l \) is denoted as \( k_2 = (\omega_2, k_2) \) with \( \omega_2 = \sqrt{k_2^2 + m_l^2} \). Thus, the energy and momentum conservation in the beta decay is given by the following equations

\[
E_n(p) - E_p(p') - \omega_1 - \omega_2 = 0 \\
p - p' - k_1 - k_2 = 0,
\]

where \( E_n(p) \) and \( E_p(p') \) are the in-medium energies of the neutron and the proton respectively.

The energy exchange in the matter goes naturally on the temperature scale \( \sim T \), which is small compared to typical kinetic energies of degenerate particles. Therefore the momenta of in-medium fermions can be fixed at their values at Fermi surfaces, which we denote as \( p_n, p_p \) for the nucleons and \( p_l \) for leptons respectively. Since the antineutrino energy is \( \omega_1 \sim T \), and the antineutrino momentum \( |k_1| \sim T \) is much smaller than the momenta of other particles, we can neglect the neutrino contributions in Eqs. (1). Then the momentum conservation, \( p_p + p_l = p_n \), implies the well known "triangle" condition,

\[
p_p + p_l > p_n,
\]

necessary for the Urca processes to operate. However, the momentum conservation is necessary but insufficient condition for the direct beta decay to occur. The energy conservation requires a one more condition

\[
E_n(p_n) - E_p(p_p) = \sqrt{(p_n - p_p)^2 + m_l^2},
\]

3
Squaring of both sides of this equation gives
\[(E_n (p_n) - E_p (p_p))^2 - (p_n - p_p)^2 = m_l^2. \quad (4)\]

If we denote the energy-momentum transfer from the nucleon as
\[K = (E_n - E_p, p_n - p_p), \quad (5)\]
the Eq. (4) can be readily recognized as \(K^2 \mu = m_l^2\). This condition naturally arises from the time-like momentum of the final lepton pair, \(K = k_1 + k_2 \approx k_2\), and can be satisfied only in the presence of the energy gap between spectrums of protons and neutrons.

Consider now the condition of beta equilibrium, \(\mu_n - \mu_p = \mu_l\), where \(\mu_n, \mu_p,\) and \(\mu_l\) are the chemical potentials of neutrons, protons, and leptons respectively. The chemical potentials of degenerate particles can be approximated by their individual Fermi energies, yielding
\[E_n (p_n) - E_p (p_p) = \sqrt{p_l^2 + m_l^2}. \quad (6)\]

Generally\(^1\) this equation is not the same as Eq. (3). However, if the "triangle" condition (2) is fulfilled then the direct neutron decay is open, and one has \(p_l = p_n - p_p\). In this case the Eq. (6) becomes identical to the energy conservation (3). Thus under beta equilibrium, the "triangle" condition is the necessary and sufficient for the direct neutron decay to occur. It should be emphasized however that the stated conditions are consistent only in the presence of the energy gap between protons and neutrons. Therefore the possibility of the direct neutron decay depends substantially on the model of nuclear matter.

2.1 Free gas model

Consider, for example, a degenerate free gas consisting of neutrons, protons, and electrons under beta equilibrium. In this case the energy gap exists only due to the mass difference of a neutron and a proton. If we denote the masses as \(M_n\) and \(M_p\) respectively, then the corresponding Fermi energies are \(E_n (p_n) = \sqrt{M_n^2 + p_n^2}\), and \(E_p (p_p) = \sqrt{M_p^2 + p_p^2}\). Due to charge neutrality the number density of electrons, \(n_e \propto p_l^3\), equals to the number density of protons, \(n_p \propto p_p^3\). This implies \(p_e = p_p\) and the equation of chemical equilibrium (6) becomes of the form
\[\sqrt{m_e^2 + p_e^2} + \sqrt{M_p^2 + p_p^2} = \sqrt{M_n^2 + p_n^2}. \quad (7)\]

\(^1\) We remind that, due to the modified Urca processes, the nuclear matter attains chemical equilibrium even if the direct beta decay is forbidden. In this case the Eq. (6) is valid but \(p_l \neq |p_n - p_p|\).
Solution of this equation

\[ p_p (p_n) = \frac{1}{2 \sqrt{M^2 + p_n^2}} \sqrt{(p_n^2 + M^2 - (M_p + m_e)^2)} \left( p_n^2 + M^2 - (M_p - m_e)^2 \right) \]

(8)

gives the proton Fermi momentum as a function of the neutron Fermi momentum in the beta-equilibrated gas of protons and neutrons. As required by the "triangle" condition, \( 2p_p \geq p_n \), the direct neutron decay in such a medium is open when \( p_p \) is larger than \( p_n / 2 \). So, solution of the equation

\[ p_p (p_n) = \frac{p_n}{2} \]

(9)

gives the critical value of the neutron Fermi momentum above which the direct neutron decay is forbidden. We find

\[ p_n^c = \frac{\sqrt{(M_p + M_n)^2 - m_i^2}}{\sqrt{2M_p^2 + 2m_i^2 - M_n^2}} = 2.381 \, \text{MeV.} \]

(10)

Thus the direct neutron decay is forbidden if the number density of neutrons is larger than

\[ n_n^c = \frac{(p_n^c)^3}{3\pi^2} = 5.932 \times 10^{31} \, \text{cm}^{-3} \]

(11)

This number density is much smaller than that typical for neutron star cores, therefore the direct neutron decay in the cooling neutron stars can occur only due to strong interactions.

Notice, some of models of strong interaction also forbid the direct neutron decay. Consider, for example, a simple model for the baryon matter [6], which contains fields for baryons and neutral scalar (\( \sigma \)) and vector (\( \omega \)) mesons. This model reproduces in a simple way the empirical NN scattering amplitude and the bulk nuclear properties. However, the neutral scalar and vector mesons equally interact with protons and neutrons, which therefore have identical energy spectrums. In this case, the energy and momentum in the direct neutron decay can not be conserved simultaneously.

To allow the direct neutron decay, the model of nuclear matter must be generalized to include some additional degrees of freedom and couplings, which are able to create a large energy gap between possible energies of the proton and neutron. For this purpose, besides isoscalar mesons \( \sigma \) and \( \omega \), the model should include also isovector mesons. The isovector meson couples differently to protons and neutrons, thus creating the energy gap necessary for the direct neutron decay.
2.2 Field theoretical model. Mean field approximation.

In the following we consider a self-consistent relativistic model of nuclear matter in which baryons interact via exchange of isoscalar mesons $\sigma$ and $\omega$ and an isovector meson $\rho$ (See Appendix). In the mean field approximation, when the contribution of mesons reduce to classical condensate fields $\langle \sigma \rangle = \sigma_0$, $\langle \omega^\mu \rangle = \omega_0 \delta^{\mu 0}$, $\langle b^\mu \rangle \equiv (0, 0, \rho_0) \delta^{\mu 0}$, only the baryon fields must be quantized. This procedure yields the following linear Dirac equation for the nucleon

$$
\left( i \partial_\mu \gamma^\mu - g_\omega \gamma^0 \omega_0 - \frac{1}{2} g_\rho \gamma^0 \rho_0 \tau_3 - (M - g_\sigma \sigma_0) \right) \Psi(x) = 0,
$$

(12)

Here and below we denote as $\tau_3$, and $\tau_\pm = (\tau_1 \pm i \tau_2)/2$ the components of isospin operator, which act on the isobaric doublet $\Psi(x)$ of nucleon field; $M = 939 \text{ MeV}$ is the bare nucleon mass$^2$.

The stationary and uniform condensate fields equally shift the effective masses

$$
M^* = M - g_\sigma \sigma_0
$$

(13)

but lead to different potential energies of the proton and neutron

$$
U_n = g_\omega \omega_0 - \frac{1}{2} g_\rho \rho_0, \quad U_p = g_\omega \omega_0 + \frac{1}{2} g_\rho \rho_0,
$$

(14)

thus creating the energy gap $U_n - U_p = -g_\rho \rho_0$ between possible energies of protons and neutrons.

The exact solutions of Eq. (12) can be found separately for protons and for neutrons. In our case of a stationary, uniform system, solution for the neutron is a spinor plane wave

$$
\psi_n(x) = N_n u_n \exp(-i E_n t + i p r),
$$

(15)

where the neutron energy is given by $E_n(p) = \sqrt{p^2 + M^*} + U_n$, and the spinor $u_n(P)$ should be found from the following equation

$$
\left( (E_n - U_n) \gamma^0 - \gamma P - M^* \right) u_n = 0.
$$

(16)

It is easily to understand that solution of this equation $u_n = u_n(P)$ is a free-like spinor constructed from the neutron kinetic momentum

$$
P^\mu = (E_n - U_n, p) = \left( \sqrt{p^2 + M^*}, p \right),
$$

(17)

$^2$ As explained in previous Section, the mass difference of the proton and the neutron can be neglected considering the nucleon as an isobaric doublet.
For the proton, one has
\[
\psi_p(x) = N_p u_p (P') \exp \left( -iE_p t + i\mathbf{p}' \cdot \mathbf{r} \right),
\]  
where the spinor \( u_p (P') \) obeys the equation
\[
\left( (E_p - U_p) \gamma^0 - \gamma \mathbf{p}' - M^* \right) u_p = 0
\]  
and depends respectively on the proton kinetic momentum
\[
P'^\mu = (E_p - U_p, \mathbf{p}') = \left( \sqrt{p'^2 + M^*^2}, \mathbf{p}' \right).
\]  
In what follows we denote by \( \varepsilon = \sqrt{p^2 + M^*^2}, \varepsilon' = \sqrt{p'^2 + M^*^2} \) the kinetic energy of the neutron and the proton respectively. So the normalization factors are of the form
\[
N_n = \frac{1}{\sqrt{2\varepsilon}}, \quad N_p = \frac{1}{\sqrt{2\varepsilon'}}.
\]  
and the single-particle energies are \( E_n (\mathbf{p}) = \varepsilon + U_n \), and \( E_p (\mathbf{p}') = \varepsilon' + U_p \).

At the Fermi surfaces the single-particle energies are \( E_n (\mathbf{p}_n) = \sqrt{M^*^2 + p_n^2} + U_n \), and \( E_p (\mathbf{p}_p) = \sqrt{M^*^2 + p_p^2} + U_p \). In this case the Eq. (4) takes the form
\[
\left( \sqrt{M_n^*^2 + p_n^2} - \sqrt{M_p^*^2 + p_p^2} + U_n - U_p \right)^2 - (\mathbf{p}_n - \mathbf{p}_p)^2 = m^2_f. \]  
Typically \( U_n - U_p = -g_\rho \rho_0 \sim 100 \text{ MeV} \), so this equation can be readily satisfied simultaneously with the momentum conservation \( \mathbf{p}_n - \mathbf{p}_p = \mathbf{p}_l \).

2.3 Nonconservation of the charged vector current of nucleons.

The Lagrangian density (A.1) ensures a conserved isovector current [7]
\[
T^\mu = \frac{1}{2} \bar{\psi} \gamma^\mu \tau \psi + \mathbf{b}^\mu \times \mathbf{B}^{\nu \mu}, \quad \partial_\mu T^\mu = 0,
\]  
where \( \mathbf{B}^{\mu \nu} = \partial^\mu \mathbf{b}^\nu - \partial^\nu \mathbf{b}^\mu \) is the \( \rho \) meson field strength tensor. Besides the directly nucleon contribution the conserved current (23) includes also the contribution of the isovector field \( \mathbf{b}^\mu \), which obeys the field equations
\[
\partial_\nu \mathbf{B}^{\nu \mu} + m_\rho^2 \mathbf{b}^\mu = \frac{1}{2} g_\rho \bar{\psi} \gamma^\mu \tau \psi; \quad \partial^\nu \mathbf{b}_\nu = 0.
\]  
By the use of Eq. (24) the condition \( \partial_\mu T^\mu = 0 \) may be transformed as
\[
i \partial_\mu \left( \bar{\psi} \gamma^\mu \tau \psi \right) = g_\rho \mathbf{b}_\mu \times \bar{\psi} \gamma^\mu \tau \psi.
\]
In the mean field approximation this gives
\[ i\partial_{\mu} (\bar{\psi} \gamma^\mu \tau^+ \psi) = -g_{\rho} \rho_0 \bar{\psi} \gamma^0 \tau^+ \psi. \] (26)

By introducing the covariant derivative
\[ D_\mu = \left( \frac{\partial}{\partial t} - ig_{\rho} \rho_0, \nabla \right), \] (27)
we can recast the Eq. (26) to the following form
\[ D_\mu (\bar{\psi} \gamma^\mu \tau^+ \psi) = 0. \] (28)

At the level of matrix elements this can be written as
\[ \bar{u}_p (P') q_\mu \gamma^\mu u_n (P) = 0, \] (29)
where \( q_\mu \) is the kinetic momentum transfer
\[ q^\mu = (E_n - E_p + g_{\rho} \rho_0, \mathbf{p} - \mathbf{p}') = (\varepsilon - \varepsilon', \mathbf{p} - \mathbf{p}') \] (30)
Thus the matrix element of the transition current is orthogonal to the kinetic momentum transfer but not to the total momentum transfer from the nucleon\(^3\). Note that this effect originates not from a special form (23) of the conserved isovector current in the medium but is caused by the energy gap between the proton and neutron spectrums. This follows directly from the Dirac equation (12), which ensures the Eq. (29) with
\[ q^\mu = P^\mu - P'^\mu = (E_n - E_p - U_n + U_p, \mathbf{p} - \mathbf{p}'), \] (31)
which coincides with Eq. (30) because \( U_n - U_p = -g_{\rho} \rho_0 \).

3 Matrix element of the neutron beta decay.

In the lowest order in the Fermi weak coupling constant \( G_F \), the matrix element of the neutron beta decay is of the form
\[ \langle f \mid (S - 1) \mid i \rangle = -i \frac{G_F C}{\sqrt{2}} N_n N_p \bar{u}_l (k_2) \gamma_\mu (1 + \gamma_5) \nu (-k_1) p \langle P' \mid J^{\mu} (0) \mid P \rangle_n \times \times (2\pi)^4 \delta (E_n - E_p - \omega_1 - \omega_2) \delta (\mathbf{p} - \mathbf{p}' - \mathbf{k}_1 - \mathbf{k}_2), \] (32)

\(^3\) Some consequences of this fact for the weak response functions of the medium are discussed in [4]
were $C = \cos \theta_C = 0.973$ is the Cabibbo factor. The effective charged weak current in the medium consists of the polar vector and the axial vector, $J^\mu (x) = V^\mu (x) + A^\mu (x)$.

Our goal now is to derive the nucleon matrix element of the charged weak current in the medium. Consider first the polar-vector contribution. By the use of the isovector current (23) one can construct the conserved electromagnetic current in the medium

$$J^\mu_{em} = \frac{1}{2} \bar{\psi} \gamma^\mu \psi + T_3^\mu + \frac{1}{2M} \partial_\nu \left( \bar{\psi} \lambda \sigma^{\mu \nu} \psi \right), \quad \partial_\mu J^\mu_{em} = 0. \quad (33)$$

The last term in Eq. (33) is the Pauli contribution, where $2\sigma^{\mu \nu} = \gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu$ and

$$\lambda = \lambda_p \frac{1}{2} (1 + \tau_3) + \lambda_n \frac{1}{2} (1 - \tau_3). \quad (34)$$

In the mean field approximation, we replace the magnetic formfactors of the nucleon with anomalous magnetic moments of the proton and the neutron, $\lambda_p = 1.7928$ and $\lambda_n = -1.9132$.

By the conserved-vector-current theory (CVC), the nucleon matrix element of the charged vector weak current is given by

$$p \langle P' | V^\mu | P \rangle_n = p \langle P' | J^\mu_{em} | P \rangle_p - n \langle P' | J^\mu_{em} | P \rangle_n. \quad (35)$$

For the electromagnetic transitions one has

$$p \langle P' | J^\mu_{em} (0) | P \rangle_p = \bar{u}_p (P') \left( \gamma^\mu + \frac{1}{2M} \lambda_p \sigma^{\mu \nu} q_\nu \right) u_p (P), \quad (36)$$

$$n \langle P' | J^\mu_{em} (0) | P \rangle_n = \bar{u}_n (P') \left( \frac{1}{2M} \lambda_n \sigma^{\mu \nu} q_\nu \right) u_n (P) \quad (37)$$

with $q_\mu = P - P'$, as given by Eq. (30). Thus, in the mean field approximation, we obtain

$$p \langle P' | V^\mu (0) | P \rangle_n = \bar{u}_p (P') \left[ \gamma^\mu + \frac{\lambda_p - \lambda_n}{2M} \sigma^{\mu \nu} q_\nu \right] u_n (P). \quad (38)$$

The second term in Eq. (38), describes the weak magnetism effects. By the use of the Dirac Eqs. (16) and (19) for the nucleon spinors we find

$$p \langle P' | q_\mu V^\mu (0) | P \rangle_n = 0 \quad (39)$$

in accord with Eq. (29).

Consider now the axial-vector charged current. This current is responsible for both the $np$ transitions and the pion decay. In the limit of chiral symmetry,
\( m_\pi \to 0 \), the axial-vector current must be conserved. In the medium with \( \rho \) meson condensate, this implies

\[
\lim_{m_\pi \to 0} D_\mu A^\mu (x) = 0,
\]

where the covariant derivative \( D_\mu \) is defined by Eq. (27). At the finite mass of a pion, \( m_\pi \), the axial-vector charged current is connected to the field \( \pi^- (x) = (\pi_1 + i\pi_2) / \sqrt{2} \) of \( \pi^- \) meson. For a free space, this relation is known as the hypothesis of partial conservation of the axial current (PCAC). In the medium the PCAC takes the form

\[
D_\mu A^\mu (x) = m^2_\pi f_\pi \pi^- (x),
\]

where \( m_\pi = 139 \text{ MeV} \) is the mass of \( \pi^- \)-meson, and \( f_\pi \) is the pion decay constant.

With allowing for interactions of the pions with nucleons and \( \rho \) mesons the Lagrangian density for the pion field is of the form \[7\]

\[
\mathcal{L}_\pi = \frac{1}{2} \left[ (\partial_\mu \pi - g_\rho b_\mu \times \pi) \cdot (\partial^\mu \pi - g_\rho b^\mu \times \pi) - m^2_\pi \pi \cdot \pi \right] + ig_\pi \bar{\psi}_p \gamma_5 \tau \cdot \pi \psi.
\]

(42)

In the mean field approximation this results in the following equation for the field of \( \pi^- \) meson

\[
\left( (i\partial^0 + g_\rho \rho_0)^2 - (i\nabla)^2 - m^2_\pi \right) \pi^- (x) = -\sqrt{2}g_\pi \bar{\psi}_p \gamma_5 \psi_p,
\]

(43)

where \( g_\pi \) is the pion-nucleon coupling constant.

For the nucleon transition of our interest, the Eq. (41) gives

\[
p\langle P' | q_\mu A^\mu (0) | P \rangle_n = im^2_\pi f_\pi p \langle P' | \pi^- (0) | P \rangle_n
\]

(44)

Here the right-hand side can be calculated by the use of Eq. (43). We obtain

\[
p\langle P' | q_\mu A^\mu (0) | P \rangle_n = -\sqrt{2}m^2_\pi f_\pi g_\pi \bar{u}_p (P') \gamma_5 u_n (P).
\]

(45)

This equation allows to derive the nucleon matrix element of the axial-vector charged current. Really, to construct the axial-vector matrix element of the charged current, caused by the nucleon transition, we have only two independent pseudovectors, consistent with invariance of strong interactions under \( T_2 \) isospin transformation, namely: \( \bar{u}_p (P') \gamma^\mu \gamma_5 u_n (P) \), and \( \bar{u}_p (P') q^\mu \gamma_5 u_n (P) \). This means that the matrix element of the axial-vector charged current is of following general form

\[
p\langle P' | A^\mu (0) | P \rangle_n = C_A \bar{u}_p (P') (\gamma^\mu \gamma_5 + F_q q^\mu \gamma_5) u_n (P).
\]

(46)
Here, in the mean field approximation, we set $C_A \approx 1.26$, while $F_q$ is the form-factor to be chosen to satisfy the Eq. (45), which now reads

$$C_A \left( -2M^* + F_q q^2 \right) \bar{u}_p (P') \gamma_5 u_n (P) = -\frac{\sqrt{2}m^2 f_{\pi} g_{\pi}}{m_{\pi}^2 - q^2} \bar{u}_p (P') \gamma_5 u_n (P).$$ (47)

To obtain this we used the Dirac Eqs. (16) and (19) for the nucleon spinors. Thus

$$C_A \left( 2M^* - F_q q^2 \right) = \frac{\sqrt{2}m^2 f_{\pi} g_{\pi}}{m_{\pi}^2 - q^2}.$$ (48)

In the mean field approximation, we assume that the coupling constants are independent of the momentum transfer. By setting $q^2 = 0$ in Eq. (48) we obtain the Goldberger - Treiman relation

$$f_{\pi} g_{\pi} = \sqrt{2} M^* C_A.$$ (49)

By inserting this in (48) we find

$$F_q = -\frac{2M^*}{(m_{\pi}^2 - q^2)}.$$ (50)

Thus, with taking into account Eqs. (38), (46), and (50), the total matrix element of the neutron beta decay is found to be

$$\mathcal{M}_{fi} = -i \frac{G_F C}{\sqrt{2}} \bar{u}_l (k_2) \gamma_\mu \left( 1 + \gamma_5 \right) \nu \left( -k_1 \right) \times$$

$$\times \bar{u}_p (P') \left[ C_V \gamma_\mu + \frac{1}{2M} C_M \sigma^{\mu\nu} q_\nu + C_A \left( \gamma^\mu \gamma_5 + F_q q^\mu \gamma_5 \right) \right] u_n (P),$$ (51)

where, in the mean field approximation, we assume

$$C_V = 1, \quad C_M = \lambda_p - \lambda_n \simeq 3.7, \quad C_A = 1.26.$$ (52)

Note that the matrix element obtained is of the same form as that for the neutron decay in a free space, but with the total momentum transfer replaced with the kinetic momentum transfer. Due to the difference in the neutron and proton potential energy, the kinetic momentum transfer

$$q = P - P' = (\varepsilon - \varepsilon', p - p')$$ (53)

to be used in the matrix element (51) differs from the total momentum of the final lepton pair

$$K = (\varepsilon - \varepsilon' + U_n - U_p, p - p')$$ (54)

This ensures $K^2 > 0$, while $q^2 = (\varepsilon - \varepsilon')^2 - (p - p')^2 < 0$. 

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The square of the matrix element of the reaction summed over spins of initial and final particles is found to be:

\[
|M_{fi}|^2 = 32G_F^2c^2 \left[ (C_A^2 - C_V^2) M^* (k_1 k_2) + (C_A - C_V)^2 (k_1 P_2) (k_2 P_1) \right. \\
+ (C_A + C_V)^2 (k_1 P_1) (k_2 P_2) \\
+ 2C_M^* M^* \left[ 2C_A ((k_1 P_1) (k_2 P_2) - (k_1 P_2) (k_2 P_1)) \\
+C_V \left( (k_1 k_2) (P_1 P_2 - M^*^2) - (k_1 P_1 - k_1 P_2) (k_2 P_1 - k_2 P_2) \right) \right] \\
- \frac{C_M^2}{M^2} \left[ M^*^2 (k_2 P_2) (3 (k_2 P_2) - (k_2 P_1)) \right. \\
+ M^*^2 (k_1 P_1) (3 (k_2 P_1) - (k_2 P_2)) + (k_1 k_2) \left( P_1 P_2 - M^*^2 \right)^2 \\
- (k_1 P_1 + k_1 P_2) (k_2 P_1 + k_2 P_2) (P_1 P_2) \left] \right. \\
+ C_A^2 F_q \left( 2M^* + F_q (M^*^2 - (P_1 P_2)) \right) \left[ (k_1 k_2) \left( M^*^2 - (P_1 P_2) \right) \\
- (k_1 P_1 - k_1 P_2) (k_2 P_1 - k_2 P_2) \right] \right]
\] (55)

with \( P_1 = (\varepsilon, p) \) and \( P_2 = (\varepsilon', p') \).

### 4 Neutrino energy losses

We consider the total energy which is emitted into neutrino and antineutrino per unit volume and time. Within beta equilibrium, the inverse reaction \( p + l \rightarrow n + \nu_l \) corresponding to a capture of the lepton \( l \), gives the same emissivity as the beta decay, but in neutrinos. Thus, the total energy loss \( Q \) for the Urca processes is twice more than that caused by the beta decay. Taking this into account by Fermi’s ”golden” rule we have

\[
Q = 2 \int \frac{d^3k_2 d^3k_1 d^3p d^3p'}{(2\pi)^{12}2\omega_2 2\omega_1 2\varepsilon 2\varepsilon'} |M_{fi}|^2 \omega_1 f_n (1 - f_p) (1 - f_\mu) \\
\times (2\pi)^4 \delta (E_n (p) - E_p (p') - \omega_1 - \omega_2) \delta (p - p' - k_1 - k_2). 
\] (56)

Antineutrinos are assumed to be freely escaping. The distribution function of initial neutrons as well as blocking of final states of the proton and the lepton \( l \) are taken into account by the Pauli blocking-factor \( f_n (1 - f_p) (1 - f_\mu) \). The Fermi-Dirac distribution function of leptons is given by

\[
f_l (\omega_2) = \frac{1}{\exp (\omega_2 - \mu_l) / T + 1}, \quad (57)
\]
while the individual Fermi distributions of nucleons are of the form

\[ f_n(\varepsilon) = \frac{1}{\exp\left(\frac{\varepsilon + U_n - \mu_n}{T}\right) + 1}, \]

\[ f_p(\varepsilon') = \frac{1}{\exp\left(\frac{\varepsilon' + U_p - \mu_p}{T}\right) + 1}. \]

By neglecting the chemical potential of escaping neutrinos, we can write the condition of chemical equilibrium as \( \mu_l = \mu_n - \mu_p \). Then by the use of the energy conservation equation, \( \varepsilon + U_n = \varepsilon' + U_p + \omega_2 + \omega_1 \), and taking the total energy of the final lepton and antineutrino as \( \omega_2 + \omega_1 = \mu_l + \omega' \) we can recast the blocking-factor as

\[ f_n(\varepsilon) (1 - f_p(\varepsilon')) (1 - f_l(\omega_2)) \equiv f_n(\varepsilon) (1 - f_n(\varepsilon - \omega')) (1 - f_l(\mu_l + \omega' - \omega_1)), \]

where \( \omega' \sim T \).

Furthermore, since the antineutrino energy is \( \omega_1 \sim T \), and the antineutrino momentum \( |k_2| \sim T \) is much smaller than the momenta of other particles, we can neglect the neutrino contributions in the energy-momentum conserving delta-functions

\[ \delta (\varepsilon + U_n - \varepsilon' - U_p - \omega_1 - \omega_2) \delta (p - p' - k_1 - k_2) \]

\[ \simeq \delta (\varepsilon + U_n - \varepsilon' - U_p - \omega_2) \delta (p - p' - k_2) \]

and perform integral over \( d^3p' \) to obtain \( p' = p - k_2 \) in the next integrals.

Nucleons and leptons in the neutron star core are strongly degenerate, therefore the main contribution to the integral (56) comes from narrow regions near the corresponding Fermi momenta. Thus we can set \( |p| = p_n, |k_2| = p_l \) in all smooth functions under the integral.

The energy of the final lepton is close to its Fermi energy \( \mu_l = \mu_n - \mu_p \). Here the chemical potentials of nucleons can be approximated by their individual Fermi energies \( \mu_n = \varepsilon_n + U_n, \mu_p = \varepsilon_p + U_p \). This allows us to transform the energy-conserving \( \delta \)-function as

\[ \delta (\varepsilon_n - \sqrt{p_n^2 + p_l^2} - 2p_n p_l \cos \theta_l + M^* + U_n - U_p - \mu_l) \]

\[ = \frac{\varepsilon_n}{p_n p_l} \delta \left( \cos \theta_l - \frac{1}{2p_n p_l} (p_n^2 - p_p^2 + p_l^2) \right), \]
where $\theta_l$ is the angle between the momentum $\mathbf{p}_n$ of the initial neutron and the momentum $\mathbf{p}_l$ of the final lepton. Notice, when the baryon and lepton momenta are at their individual Fermi surfaces, the $\delta$-function (62) does not vanish only if $p_p + p_t > p_n$.

Further we use the particular frame with $Z$-axis directed along the neutron momentum $\mathbf{p}_n$. Then

\[
P_1 = (0, 0, p_n, \varepsilon_n) \\
k_1 = \omega_1 (\sin \theta_\nu, 0, \cos \theta_\nu, 1) \\
k_2 = (p_l \sin \theta_l \cos \varphi_l, p_l \sin \theta_l \sin \varphi_l, p_l \cos \theta_\nu, \mu_l)
\]

(63)

The energy-momentum of the final proton is defined by conservation laws:

\[
P_2 = (-p_l \sin \theta_l \cos \varphi_l, -p_l \sin \theta_l \sin \varphi_l, p_n - p_l \cos \theta_\nu, \varepsilon_p)
\]

(64)

Insertion of (63) and (64) in the square of the matrix element (55) yields a rather cumbersome expression, which, however, is readily integrable over solid angles of the particles.

Since we focus on the actually important case of degenerate nucleons and leptons, we may consider the neutrino energy losses to the lowest accuracy in $T/\mu_l$. Then the remaining integration reduces to the factor

\[
\int d\omega_1 \omega_1^3 d\omega' d\varepsilon f_n (\varepsilon) (1 - f_n (\varepsilon - \omega')) (1 - f_l (\mu_l + \omega' - \omega_1))
\]

\[
\simeq \int_{-\infty}^{\infty} d\omega' \frac{\omega'}{\exp \omega'/T - 1} \int_0^{\infty} d\omega_1 \frac{\omega_1^3}{1 + \exp (\omega_1 - \omega')/T} = \frac{457}{5040} \pi^6 T^6.
\]

(65)

Finally the neutrino emissivity is found to be of the form:
\[ Q = \frac{457\pi}{10080} G_F^2 C^2 T^6 \Theta (p_l + p_p - p_n) \left\{ \left( C_A^2 - C_V^2 \right) M^* \mu_l \right. \\
+ \frac{1}{2} \left( C_V^2 + C_A^2 \right) \left[ 4 \varepsilon_n \varepsilon_p \mu_l - (\varepsilon_n - \varepsilon_p) \left( (\varepsilon_n + \varepsilon_p)^2 - p_l^2 \right) \right] \\
+ C_V C_M \frac{M^*}{M} \left[ 2 (\varepsilon_n - \varepsilon_p) p_l^2 - (3 (\varepsilon_n - \varepsilon_p)^2 - p_l^2) \mu_l \right] \\
+ C_A \left( C_V + 2 \frac{M^*}{M} C_M \right) (\varepsilon_n + \varepsilon_p) \left( p_l^2 - (\varepsilon_n - \varepsilon_p)^2 \right) \\
+ C_M^2 \frac{1}{4 M^2} \left[ 8 M^* \left( \varepsilon_n - \varepsilon_p \right) \left( p_l^2 - (\varepsilon_n - \varepsilon_p) \mu_l \right) \right. \\
+ \left( p_l^2 - (\varepsilon_n - \varepsilon_p)^2 \right) \left( 2 \varepsilon_n^2 + 2 \varepsilon_p^2 - p_l^2 \right) \mu_l \\
- \left( p_l^2 - (\varepsilon_n - \varepsilon_p)^2 \right) (\varepsilon_n + \varepsilon_p)^2 \left( 2 \varepsilon_n - 2 \varepsilon_p - \mu_l \right) \\
\left. \right. \\
\left. \left. - C_A^2 M^* \Phi \left( 1 + m_n^2 \Phi \right) \left[ \mu_l \left( (\varepsilon_n - \varepsilon_p)^2 + p_l^2 \right) - 2 (\varepsilon_n - \varepsilon_p) p_l^2 \right] \right\} \right] \tag{66} \]

with \( \Theta (x) = 1 \) if \( x \geq 0 \) and zero otherwise. In the above, the last term, with

\[ \Phi = \frac{1}{m_n^2 + p_l^2 - (\varepsilon_n - \varepsilon_p)^2}, \tag{67} \]

represents the contribution of the pseudoscalar interaction. The "triangle" condition \( p_p + p_l > p_n \), required by the step-function, is necessary for conservation of the total momentum in the reaction and exhibits the threshold dependence on the proton concentration.

5 Non-relativistic limit

Under beta-equilibrium, the superthreshold proton fraction in the core of neutron stars appears at large densities, when Fermi momenta of nucleons are of the order of their effective mass. Therefore the "triangle" condition is inconsistent with the non-relativistic limit. Nevertheless, it is necessary to show how the expression given by Lattimer et al. [3] can be formally obtained from Eq. (66). For this purpose we consider the non-relativistic limit of Eq. (66) by neglecting the "triangle" condition.

The non-relativistic approximation is valid when \( p_n, p_p, p_l \ll M^* \). However, the smallness of the particle momenta is not enough to point out unambiguously the leading terms in Eq. (66). The relative contributions of various terms depend also on electron abundance in the medium. If the electron fraction is as large that \( M^* \mu_l \gg p_n^2 \), then we obtain the result of Lattimer et al. [3]:

\[ Q_L = \frac{457\pi}{10080} G_F^2 C^2 \left( C_V^2 + 3 C_A^2 \right) T^6 M^* \mu_l \Theta (p_l + p_p - p_n), \tag{68} \]
widely used in simulations of neutron stars. However, when $p_n^2 \sim M^* \mu_l$, what actually takes place in the non-relativistic nucleon matter under beta-equilibrium, we obtain

$$Q_{nr} = \frac{457 \pi}{10080} G_F^2 C^2 T^6 \left[ (C_V^2 + 3C_A^2) M^* \mu_l - (C_V^2 + C_A^2) M^* p_n^2 \right] \Theta (p_l + p_p - p_n).$$

Here the additional term is due to the nucleon recoil. When the electron (proton) fraction is small, this contribution is comparable to the terms of Eq. (68).

6 Efficiency of the relativistic approach

Neutrino energy losses caused by the direct Urca on nucleons depend essentially on the composition of beta-stable nuclear matter. Therefore, in order to estimate the relativistic effects, we consider the model of nuclear matter, which includes nucleon and hyperon degrees of freedom (See Appendix). The parameters of the model are chosen as suggested in Ref. [8] to reproduce the nuclear matter equilibrium density, the binding energy per nucleon, the symmetry energy, the compression modulus, and the nucleon effective mass at saturation density $n_0 = 0.16 \text{ fm}^{-3}$. The composition of neutrino-free matter in beta equilibrium among nucleons, hyperons, electrons and muons is shown in Fig. 1 versus the baryon number density $n_b$, in units of $n_0$ (left panel). The right panel represents the results of self-consistent calculation of the effective baryon masses and individual Fermi momenta of the baryons.

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4 In order to apply the non-relativistic Eq. (68) to relativistic nucleons the authors [1] suggest replacing the effective Dirac mass with the relativistic Fermi energy, $M_{n,p}^* \rightarrow \sqrt{p_{n,p}^2 + M^{*2}}$. The latter is termed as "the effective Landau mass". This trick can not be justified from the theoretical point of view. Firstly because the mass term and the energy term in the Dirac equation are of different matrix structure. Secondly, by the correct definition, the effective Landau mass has no relation to relativism but is intended to take into account interactions between particles near the Fermi surface. In this meaning the effective mass (13) is actually the effective Landau mass.
Fig. 1. The left panel shows individual concentrations for matter in beta equilibrium among nucleons, hyperons, electrons and muons as a function of the density ratio $n_b/n_0$. The right panel represents the self-consistent effective baryon masses (solid lines) and individual Fermi momenta of the baryons (dashed lines).

At number densities smaller than the saturation density the matter consists mostly of neutrons with a small admixture of equal amounts of protons and electrons, however, the proton fraction grows along with the matter density. Near saturation density the electron chemical potential reaches the muon mass, and the muon fraction appears also growing along with the matter density together with the electron fraction up to $n_b \approx 0.3 \, fm^{-3}$. Above this density of nuclear matter, constituents other than neutrons, protons, electrons, and muons appear. The $\Sigma^-$ hyperons appear when their lowest energy state first lies below $\mu_n + \mu_l$. Due to the charge neutrality, increasing of the $\Sigma^-$ fraction suppresses the lepton abundance. Above the density $n_b \approx 0.48 \, fm^{-3}$, beta equilibrium requires appearance of $\Lambda$ hyperons because the lowest energy state for a $\Lambda$ lies lower than $\mu_n$. At densities larger than $n_b \approx 0.91 \, fm^{-3}$ fraction of $\Sigma^0$ hyperons also exists.
Fig. 2. The left panel shows the neutrino emissivity of the direct Urca processes among nucleons and electrons. The right panel is the same, but for the muon reactions. The emissivity is shown versus the density ratio \( n_b/n_0 \) for the matter composition represented in Fig. 1. The curves begin at the threshold density. Solid curves represent the total relativistic emissivity, as given by Eq. (66). The short-dashed curves are the non-relativistic emissivity given by Eq. (68). The dot-dashed curves show the relativistic emissivity without the contributions of weak magnetism and pseudoscalar interaction, and the long-dashed curves are the emissivity without the pseudoscalar contribution. All the emissivities are given in units \( 10^{27} T_9^6 \text{ erg cm}^{-3} \text{s}^{-1} \), where the temperature \( T_9 = T/10^9 \text{ K} \).

In Fig. 2 we compare the relativistic neutrino emissivity (66) with the non-relativistic energy losses, as given by formula (68). Due to a steady decrease of the effective mass of the nucleon the non-relativistic formula (short-dashed curve) predicts a decreasing of the emissivity along with the density increase. Appearance of hyperons in the system suppresses the nucleon fractions and lepton abundance. Therefore at densities, where the number of hyperons is comparable with the number of protons, the relativistic emissivity (solid curve) reaches the maximum and then also has a tendency to decrease. The relativistic emissivity, however, is found to be substantially larger than that predicted in the non-relativistic approach.

To inspect the contribution of weak magnetism and the pseudoscalar interaction we demonstrate two additional graphs. The long-dashed curve demonstrates the energy losses obtained from Eq. (66) by formal setting \( \Phi = 0 \). This
eliminates the pseudoscalar contribution. The dot-dashed curve is obtained by formal replacing $\Phi = 0$ and $C_M = 0$, which eliminates both the weak magnetism and pseudoscalar contributions. A comparison of these curves demonstrates the weak magnetism effects. The contribution of the pseudoscalar interaction can be observed by comparing the total neutrino energy losses (solid curve) with the long-dashed curve, which is calculated without this contribution. We see that the weak magnetism effects approximately doubles the relativistic emissivity, while the pseudoscalar interaction slightly suppresses the energy losses.

7 Summary and Conclusion

In the mean field approximation, we have studied the neutrino energy losses caused by the direct Urca processes on nucleons in the degenerate baryon matter under beta equilibrium. We have shown that the direct Urca processes in a superdense matter of neutron star cores are kinematically allowed only due to isovector mesons, which differently interact with protons and neutrons. By creating the energy gap between proton and neutron spectrums the isovector mesons support a time-like total momentum transfer from the nucleon, as required by kinematics of the reaction. In the mean field approximation, we derived the matrix element of the nucleon transition current, which is found to be a function of the space-like kinetic momentum transfer. We have calculated the neutrino energy losses caused by the direct Urca processes on nucleons. Our Eq. (66) for neutrino energy losses exactly incorporates the effects of nucleon recoil, parity violation, weak magnetism, and pseudoscalar interaction. To quantify the relativistic effects we consider a self-consistent relativistic model, widely used in the theory of relativistic nuclear matter. The relativistic energy losses are up to four times larger than those given by the non-relativistic approach. In our analysis, we pay special attention to the effects of weak magnetism and pseudoscalar interaction in the neutrino energy losses. We found that weak magnetism approximately doubles the neutrino emissivity, while the pseudoscalar interaction slightly suppresses the energy losses, approximately by 10%. The mean field Eq. (66) may be considered as a starting point for studying of the correlation effects.

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A Field theoretical models of nuclear matter

We employ a self-consistent relativistic model of nuclear matter in which baryons, \( B = n, p, \Sigma^-, \Sigma^0, \Sigma^+, \Lambda \), interact via exchange of \( \sigma, \omega, \rho \) mesons \cite{5}. In principle, the pion fields should be also included in the model. However, the expectation value of the pion field equals zero, giving no contribution to the mean fields. Therefore, only non-redundant terms are exhibited in the Lagrangian density

\[
L = \sum_B \bar{B} \left( i \gamma^\mu \left( \partial_\mu - g_{\omega B} \gamma^\mu \omega^\mu \right) - \left( M_B - g_{\sigma B} \sigma \right) \right) B \\
- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega^\mu - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{1}{2} m_\rho^2 b_\mu b^\mu \\
+ \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 \right) - U (\sigma) + \bar{l} (i \gamma_\mu \partial^\mu - m_l) l,
\]

(A.1)

which includes the interaction of baryon fields \( B \) with a scalar field \( \sigma \), a vector field \( \omega_\mu \) and an isovector field \( b_\mu \) of \( \rho \)-meson. In the above, \( B \) are the Dirac spinor fields for baryons, \( b_\mu \) is the isovector field of \( \rho \)-meson. We denote as \( \tau \) the isospin operator, which acts on the baryons of the bare mass \( M_B \). The leptons are represented only by electrons and muons, \( l = e^-, \mu^- \), which are included in the model as noninteracting particles. The field strength tensors for the \( \omega \) and \( \rho \) mesons are \( F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \) and \( B_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu \), respectively. The potential \( U (\sigma) \) represents the self-interactions of the scalar field and is taken to be of the form

\[
U (\sigma) = \frac{1}{3} b M (g_{\sigma N} \sigma)^3 + \frac{1}{4} c (g_{\sigma N} \sigma)^4.
\]

(A.2)

In what follows we consider the mean field approximation widely used in the theory of relativistic nuclear matter. In this approximation, the meson fields are replaced with their expectation values

\[
\sigma \rightarrow \langle \sigma \rangle \equiv \sigma_0, \quad \omega^\mu \rightarrow \langle \omega^\mu \rangle \equiv \omega_0 \delta_\mu 0, \quad b^\mu \rightarrow \langle b^\mu \rangle \equiv (0, 0, \rho_0) \delta_\mu 0.
\]

(A.3)

In this case only the baryon fields must be quantized. This procedure yields the following linear Dirac equation for the baryon field

\[
\left( i \partial_\mu \gamma^\mu - g_{\omega B} \gamma^0 \omega_0 - \frac{1}{2} g_{\rho B} \gamma^0 \rho_0 \tau_3 - (M - g_{\sigma B} \sigma_0) \right) B (x) = 0,
\]

(A.4)

The effective baryon mass is \( M_B^* = M_B - g_{\sigma B} \sigma_0 \), and the single-particle energies and self-consistent potentials are given by

\[
U_B = g_{\omega B} \omega_0 + g_{\rho B} t_{3B} \rho_0.
\]

(A.5)
Here $\omega_0$, $b_0$, and $\sigma_0$ are, respectively, nonzero average values of the $\omega$-, $\rho$-, and $\sigma$-meson fields, and $g_{\omega B}$, $g_{\rho B}$, and $g_{\sigma B}$ are the strong interaction couplings of the baryons to different meson fields, and $t_{3B}$ is the third component of isospin for the baryons. The number density for baryon species $B$ is related to the corresponding Fermi momentum as

$$n_B = \frac{p^3_B}{3\pi^2}.$$  \hspace{1cm} (A.6)

Thus, the average meson fields are solutions of the following equations

$$m^2_{\omega} \omega_0 = \sum_B g_{\omega B} n_B,$$
$$m^2_{\rho} \rho_0 = \sum_B g_{\rho B} t_{3B} n_B,$$
$$m^2_{\sigma} \sigma_0 = -\frac{dU(\sigma)}{d\sigma} + \sum_B \frac{g_{\sigma B}}{2\pi^2} \left( M^*_B \rho_B \sqrt{p^2_B} + M^2_B - M^3_B \arcsin \frac{p_B}{M_B} \right).$$  \hspace{1cm} (A.7)

For a fixed total baryon number density

$$n_b = \sum_B n_B,$$  \hspace{1cm} (A.8)

the additional conditions needed to obtain a solution are given by the charge neutrality requirement

$$n_e + n_\mu + n_{\Sigma^-} = n_p + n_{\Sigma^+}$$  \hspace{1cm} (A.9)

and the beta equilibrium relations

$$\mu_n = \mu_p + \mu_e, \quad \mu_\mu = \mu_e, \quad \mu_{\Sigma^-} = \mu_n + \mu_e,$$
$$\mu_{\Sigma^0} = \mu_\Lambda = \mu_n, \quad \mu_{\Sigma^+} = \mu_p.$$  \hspace{1cm} (A.10)

For nucleons, the strong interaction constants [8]

$$\frac{g_{\sigma N}}{m_\sigma} = 3.15 \text{ fm}, \quad \frac{g_{\omega N}}{m_\omega} = 2.20 \text{ fm}, \quad \frac{g_{\rho N}}{m_\rho} = 2.19 \text{ fm},$$

$$b = 0.008659, \quad c = -0.002421,$$  \hspace{1cm} (A.11)

with $m_\omega = 783$ MeV, $m_\rho = 770$ MeV, $m_\sigma = 520$ MeV, are determined by reproducing the nuclear matter equilibrium density $n_0 = 0.16 \text{ fm}^{-3}$, and the binding energy per nucleon ($\sim 16$ MeV), the symmetry energy ($\sim 30 - 35$ MeV), the compression modulus ($200 \text{ MeV} \leq K_0 \leq 300$ MeV), and the nucleon effective mass $M^* = (0.6 - 0.7) \times 939$ MeV at $n_0$. For hyperons we take $g_{\sigma H} = 0.6 g_{\sigma N}$, $g_{\rho H} = 0.6 g_{\rho N}$, $g_{\omega H} = 0.658 g_{\omega N}$. 

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References


