Generating Vector Boson Masses

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Abstract
If the Higgs particle is never found, one will need an alternative theory for vector boson masses. I propose such a theory involving an antisymmetric tensor potential coupled to a gauge field.
The Standard Model of electroweak interactions is, as its name implies, a well-established theory both from theoretical and experimental viewpoints. However, there are still a few unanswered questions about it. Perhaps the biggest of them is about the Higgs sector of the theory. The basis of the misgivings is that no elementary scalar has ever been found, and there is no positive evidence at currently available energies of a Higgs particle. On the other hand various analyses set the upper bound of the Higgs mass only a little out of our current reach. This suggests that perhaps it is time we prepared ourselves for the situation that no Higgs particle is ever found within the predicted range of energies.

What is the purpose of the Higgs sector? It is intimately related to spontaneous symmetry breaking. It breaks the global electroweak $SU(2) \times U(1)$ symmetry, thus generating masses for vector bosons. It also contributes to the mass generation mechanism for fermions via chiral symmetry breaking. There have been various attempts to treat these effects without the use of an elementary scalar particle. For example, in technicolor theories [1], one postulates a new strong interaction which binds techniquark-antiquark pairs into a chiral condensate that breaks the electroweak symmetry. In another scenario [2], one uses the top quark rather than techniquarks to form condensates. Both these routes try to replace the Higgs sector by something else. In this letter I make a similar attempt and propose an alternative mechanism for generating vector boson masses. This mechanism is somewhat similar in its aspects to the theory of a heavy Higgs particle [3], since the abelian theory is dual to an abelian Higgs model with infinitely massive Higgs. However, any connection between the theory proposed here and that of heavy Higgs bosons (or that of nonlinear $\sigma$-models) is yet to be discovered. This letter is intended to be a digest of results; details will be published elsewhere.

The proposed alternative involves the non-abelian generalization of the theory of a two-form potential [4] interacting with a gauge field. The abelian theory has been known to render the photon massive [5,6] and has the added virtue of not having a residual degree of freedom (à la the Higgs particle). Attempts at producing massive non-abelian gauge bosons this way have come up short of expectations because of the complexity and difficulty involved in non-abelianizing the vector gauge symmetry associated with the two-form (see, for example, [7] and [8]). In what follows I offer solutions to some of the well-known problems with this system.

The Unadorned Theory: The starting point of this theory is a naive non-abelianization of the mass generation mechanism [5] for electrodynamics via a $B \wedge F$ coupling between a two-form $B_{\mu\nu}$ and the field-strength $F_{\mu\nu}$ of the gauge field. One adds a kinetic term for
$B_{\mu\nu}$ and the usual $F^2$ to get the desired result. (For simplicity I do not include fermionic matter in this model.) To move on to an $SU(N)$ gauge group the derivative operator $\partial_\mu$ is replaced by the covariant derivative $D_\mu$ and $F_{\mu\nu} = [D_\mu, D_\nu]$, resulting in the action

$$S = \int d^4x \text{Tr} \left( -\frac{1}{6} H_{\mu\nu\lambda} H^{\mu\nu\lambda} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m}{2} \epsilon^{\mu\nu\rho\lambda} B_{\mu\nu} F_{\rho\lambda} \right),$$

(1)

where $H_{\mu\nu\lambda} = D_\lambda B_{\mu\nu} + \text{cyclic}$, with $B_{\mu\nu}$ in the adjoint representation of $SU(N)$. The classical equations of motion derived from this action are invariant under an additional global field redefinition $B_{\mu\nu} \rightarrow B_{\mu\nu} + \alpha F_{\mu\nu}$, where $\alpha$ is a constant. For the moment I exclude other terms from the action by requiring this invariance. (It is possible that this invariance will be broken in the quantized theory. That however does not affect the applicability of this theory to mass generation, as I will discuss below.)

Firstly I shall discuss the attractive features of this theory. The quadratic part of this action is exactly what is needed to give the gauge boson a tree level mass. This mass is equal to $m$ and occurs as a pole in the gauge boson propagator upon summing all tree level two point diagrams with insertions of $B_{\mu\nu}^a$ propagators (i.e., by diagonalizing the fields at zero gauge coupling). The classical counting of modes is as follows — if I denote the conjugate momenta to $A_\mu$ and $B_{\mu\nu}$ respectively by $\Pi^\mu$ and $\Pi^{\mu\nu}$, the constraints are

$$\Pi^0 \approx 0, \quad \Pi^{0i} \approx 0,$$

$$D_j \Pi^j + [B_{ij}, \Pi^{ij}] \approx 0,$$

$$D_j \Pi^{ij} + \frac{m}{2} \epsilon^{ijk} F_{jk} \approx 0.$$

(2)

This forms a closed, reducible system of constraints. The closure is easily verified using Poisson brackets $\{A_i, \Pi^j\} = \delta^j_i$, $\{B_{ij}, \Pi^{kl}\} = \frac{1}{2} (\delta^k_i \delta^l_j - \delta^k_j \delta^l_i)$. (The factor of $\frac{1}{2}$ arises from a conflict in conventions, while $D_{[\mu} D_{\nu]} = D_\mu D_\nu - D_\nu D_\mu$, $B_{[\mu\nu]} = B_{\mu\nu} \equiv \frac{1}{2} (B_{\mu\nu} - B_{\nu\mu})$.)

If I define $b_i := \frac{1}{2} \epsilon_{ijk} B_{jk}$ and the conjugate momenta $\Pi_i^{(b)}$, the last set of constraints are equivalent to the abelian $\vec{\nabla} \times \vec{\Pi}^{(b)} + \frac{m}{2} \epsilon^{ijk} F_{jk} \approx 0$. This shows that the constraints are not independent and one needs to fix a gauge, say $\nabla^j B_{ij} = 0 = \vec{\nabla} \times \vec{b}$. This shows that there is a surviving longitudinal mode coming from the $B$ field. This is of course the mode that couples with the gauge field to produce a massive boson.

Obviously, the masses of the gauge bosons are all equal in this model. If this is to be applicable to electroweak phenomena, $m$ has to be replaced by a mass matrix. One takes the viewpoint that the symmetry breaking is induced by some mechanism not yet included in this model (possibly by couplings to fermions), that selects out a $U(1)$
group from $SU(2) \times U(1)$. Since the end result should be one massless photon and three massive gauge bosons, this is equivalent to diagonalizing a $4 \times 4$ mass matrix $m_{ab}$ (over the $SU(2) \times U(1)$ Lie algebra), which has exactly one vanishing eigenvalue. Diagonalization leads to the difference between $W^\pm$ and $Z$ masses and gives the Weinberg angle. Finally, this model is power-counting renormalizable and also seems to be renormalizable at the one-loop level, as I point out in the next section.

Symmetries: The principal obstacle to writing a dynamical theory of a non-abelian two-form is the lack of symmetries. While the abelian theory has a vector gauge symmetry under $B_{\mu \nu} \rightarrow B_{\mu \nu} + \partial_{[\mu} \Lambda_{\nu]}$, this symmetry does not seem to survive non-abelianization. In the absence of this symmetry, the constraints (2) (specifically the last one of the set) are very difficult to implement on the fields, and questions arise concerning the renormalizability and unitarity of the model. This situation is similar to quantizing a spontaneously broken gauge theory in the unitary gauge. The main point of this letter is to propose that this symmetry is actually hidden in the theory given by (1). I introduce an auxiliary field $C_\mu$, also in the adjoint representation of the gauge group, and rewrite the action as

$$S = \int d^4 x \text{Tr} \left( -\frac{1}{6}(H_{\mu \nu \lambda} - [F_{[\mu \nu}, C_{\lambda]}])(H^{\mu \nu \lambda} - [F^{[\mu \nu}, C^{\lambda]}]) - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{m}{2} \epsilon^{\mu \nu \rho \lambda} B_{\mu \nu} F_{\rho \lambda} \right).$$

This action now has two gauge symmetries

$$A_\mu \rightarrow U A_\mu U^{-1} - \partial_\mu U U^{-1},$$

$$B_{\mu \nu} \rightarrow U B_{\mu \nu} U^{-1},$$

$$C_\mu \rightarrow U C_\mu U^{-1};$$

and

$$A_\mu \rightarrow A_\mu,$$

$$B_{\mu \nu} \rightarrow B_{\mu \nu} + D_{[\mu} \Lambda_{\nu]},$$

$$C_\mu \rightarrow C_\mu + \Lambda_\mu,$$

where $\Lambda_\mu$ is any vector field transforming in the adjoint representation. The action (1) is then seen as the gauge-fixed ($C_\mu = 0$) version of this action. Since the action (3) contains no quadratic term for $C_\mu$, one needs to introduce one in order to compute Feynman diagrams of the theory. This is most simply done with a fake mass term $\text{Tr}(\frac{1}{2\eta} C_\mu C^\mu)$. One then takes $\eta \rightarrow 0$ after calculating with a given cutoff. This decouples $C_\mu$ from the theory and diagrams containing internal $C_\mu$ propagators vanish. This is good from the point of view
of renormalizability of the theory. The gauge field sector is renormalizable, of course, and one needs to consider only the sector involving the $B_{\mu\nu}$ field. The couplings appearing in the action (1) are all dimension four, propagators for both $A_\mu$ and $B_{\mu\nu}$ fall off as $1/k^2$ at large momenta, and it follows (and can be explicitly checked) that possible counterterms appearing at the one loop level are at most dimension four. The renormalization of this theory is quite involved and I shall discuss it in more detail elsewhere. The dimensionality of the counterterms leads one to hope that the theory is renormalizable even when the global symmetry is broken. It also turns out that the relevant Ward identity for the two-point function of $B_{\mu\nu}$ implies that only the longitudinal mode of the $B_{\mu\nu}$ field is affected by higher order corrections. As a result one should expect $B_{\mu\nu}$ to remain longitudinal at higher orders in perturbation theory. Therefore, even if higher order diagrams break the global symmetry, they will not produce a Higgs particle, nor invalidate this theory as a mechanism for generating vector boson masses. More specifically, no counterterm of the form $B^2$ or $B \wedge B$ arises as higher loop corrections. Finally, one hopes that the symmetries (4) and (5) in the action (3) are enough to decouple non-propagating modes from the theory. While that seems obvious at the classical level, it needs to be verified after quantization.

**Comments and Summary:** There are various open problems associated with this theory. One of the more important ones is the question of couplings of $B_{\mu\nu}$ to fermions. Since the theory with $C_\mu = 0$ does not have the vector gauge symmetry (5), fermions may couple to $B_{\mu\nu}$ via couplings either of the form $\bar{\psi}\sigma^{\mu\nu}B_{\mu\nu}\psi$ or $\bar{\psi}\gamma^5\sigma^{\mu\nu}B_{\mu\nu}\psi$. (One replaces $B_{\mu\nu}$ by $(B_{\mu\nu} - D_{[\mu} C_{\nu]})$ in order to see the gauge symmetry (5) explicitly.) These couplings then obey the gauge symmetries but break chiral symmetry. One expects contributions to magnetic (or electric) dipole moments of fermions from such couplings.

In the abelian version of this theory there is no residual degree of freedom, i.e., there is no Higgs particle. Analysis of the non-abelian theory points to the same conclusion. However, the possibility of a condensate corresponding to the Higgs particle cannot be ruled out.

It is tempting to apply this theory to QCD, as the aforementioned global transformation allows one to remove a $\Theta F \wedge F$ term from the Lagrangian, even for arbitrarily small values of the bare mass parameter $m$. However, it is not clear if this actually solves the strong CP problem, or just sweeps it under the rug.

I have not ascertained the full implications of this theory on the phenomenology of electroweak or strong interactions. It seems that the most immediately obvious impact will
be on processes that involve the production or exchange of Higgs bosons in the standard model. Tree level processes should otherwise be indistinguishable form those in the standard model. One should also expect differences arising from the possible fermion couplings mentioned above.

In conclusion, I have proposed in this letter a mechanism for generating masses for non-abelian vector bosons that apparently does not have an excitation corresponding to the Higgs particle. SU(2) × U(1) symmetry breaking can also be incorporated into this model without invoking a Higgs particle. This model also seems to be renormalizable, and should be of interest to field theorists and phenomenologists alike — to theorists because of the rich structures in both the classical and quantum theories, and to phenomenologists because processes corresponding to this model differ from the standard model predictions and can be tested in high-energy experiments. In the event that the Higgs particle is never found, the model proposed here should be a viable alternative for the Higgs sector of the standard model.

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References