Covariant Effective Field Theory
for Bulk Properties of Nuclei

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Abstract

Recent progress in Lorentz-covariant quantum field theories of the nuclear many-body problem (quantum hadrodynamics, or QHD) is discussed. The importance of modern perspectives in effective field theory and density functional theory for understanding the successes of QHD is emphasized. The inclusion of hadronic electromagnetic structure and of nonanalytic terms in the energy functional is also considered.

I. INTRODUCTION

A central goal of nuclear theory is to describe atomic nuclei in terms of their constituents. This goal is especially relevant now, in view of the new accelerators that will probe nuclei with high energies and high precision using electrons, hadrons, and heavy ions. These accelerators are expected to reveal new physics involving the properties of hadrons inside nuclei, the nuclear matter phase diagram, the role of relativity, and the dynamics of the quantum vacuum. Moreover, we hope to learn not only about hadron dynamics but also about the role of explicit quark and gluon degrees of freedom.

Although the Schrödinger equation has been basically successful in describing nuclei at low energies, this framework must be extended if we are to compare calculations with the data of the future. A more complete treatment of hadronic systems should include relativistic nucleon-nucleon interactions, dynamical mesons and baryon resonances, modifications of the hadron structure in the nucleus, and the dynamics of the quantum vacuum, while respecting the general principles of quantum mechanics, Lorentz covariance, gauge invariance, cluster decomposition, and microscopic causality. These physical effects will be relevant regardless of the degrees of freedom used to describe the system, and they must be studied simultaneously and consistently to draw definite conclusions about nuclear dynamics at high temperatures, high densities, and short distances.

In principle, quantum chromodynamics (QCD) should provide such a description, since QCD is the fundamental theory of the strong interaction. Nevertheless, QCD predictions at nuclear length scales with the precision of existing (and anticipated) data are not now available, and this state of affairs will probably persist for some time. Even if it becomes
possible to use QCD to describe nuclei directly, this description is likely to be awkward, since quarks cluster into hadrons at low energies, and hadrons (not quarks or gluons) are the degrees of freedom actually observed in experiments.

Given the present inadequacy of both the Schrödinger equation and QCD for formulating an improved description of nuclear dynamics, we must consider alternatives. Since hadrons are the relevant experimental degrees of freedom, it is important to see if practical, reliable, and accurate hadronic descriptions can be developed for the energy, density, and temperature regimes obtainable with the new experimental facilities. In any lagrangian approach, one must first decide on the generalized coordinates, and hadronic degrees of freedom—baryons and mesons—are the most appropriate for the vast majority of nuclear phenomena.

Lorentz-covariant, meson–baryon effective field theories (“quantum hadrodynamics” or QHD) have proven to be useful for quantitative descriptions of the nuclear many-body problem [1–8]. When applied within the framework of modern effective field theory (EFT) and density functional theory (DFT), they provide a realistic description of bulk nuclear properties and the spin-orbit force throughout the Periodic Table [4,9,6]. This success arises from the presence of large scalar and vector mean fields, which imply that there are large relativistic interaction effects in nuclei under normal conditions [7]. There is evidence from QCD sum rules that these large fields are dynamical consequences of the underlying chromodynamics [10,11]. Moreover, similar relativistic effects are responsible for the efficient description of spin observables in medium-energy proton–nucleus scattering using the Relativistic Impulse Approximation, and they are consistent with the major role played by scalar and vector meson exchange in modern boson-exchange models of the nucleon–nucleon (NN) interaction. All of these features motivate further investigations into the application of QHD to the nuclear many-body problem.

Since QHD contains strong couplings, one needs a consistent underlying framework and a systematic expansion procedure to perform reliable calculations. One possible framework involves renormalizable QHD models, which are characterized by a finite number of coupling constants and masses. Since the number of parameters is finite, these models provide a self-contained, Lorentz covariant, causal framework for extrapolating known nuclear information to nuclear matter under extreme conditions of density, temperature, and flow velocity. Nevertheless, renormalizable models make specific assumptions about the form of the interactions and the role of hadronic degrees of freedom in the dynamics of the quantum vacuum. These assumptions must be tested by comparing detailed calculations with experiment. Several such calculations have been performed in recent years, which have indicated that the constraint of renormalizability is too restrictive [4,12]. This has prevented the development of a systematic expansion procedure at finite density and has also stimulated the search for alternatives to renormalizable models.

One guiding principle in the search for alternatives is that hadronic theories should respect the underlying symmetries of QCD, such as chiral symmetry. Much work in recent years has focused on reconciling the successful picture of nuclear structure mentioned earlier with chiral symmetry. What has been learned is that the large scalar field present in nuclei, which has its origin in the strong mid-range NN attraction, is primarily a consequence of correlated two-pion exchange between the nucleons [13–15]. This dynamics is implemented most efficiently by using a nonlinear realization of the chiral symmetry to describe the pion–nucleon interactions [16] and by adding an additional, effective, scalar-isoscalar, chiral
scalar field to simulate the correlated two-pion exchange [17,3]. One conclusion from this work is that even within the context of hadronic field theories, one is led to the introduction of effective fields to describe the relevant dynamics at large distances. This observation, together with the difficulties encountered in the consistent application of renormalizable models, suggests that one should formulate hadronic many-body theories as effective field theories from the outset.

The scalar and vector meson fields will therefore be considered as effective fields, and their dynamics can be implemented in two different ways: first, by including these fields in an effective lagrangian, one has both an obvious mean-field (factorized) limit, leading to a Dirac–Hartree description similar to that discussed above, as well as the possibility of developing systematic improvements to this approximation; alternatively, by including these fields in an energy functional, one can apply many of the well-known techniques of (nonrelativistic) density functional theory to a covariant energy functional.

The application of effective relativistic field theory to the nuclear many-body problem is in its adolescence, with new developments occurring rapidly. Progress in this area will require the synthesis of ideas from both many-body theory and relativistic quantum field theory. The following sections describe in detail some of the recent progress and likely future directions, such as the application of these effective theories at the one-loop level to electroweak interactions with nuclei; the development of systematic techniques for going beyond one-loop to include exchange, correlation, and vacuum effects; and studies of how to extend the formalism to higher densities (and temperatures). The role of chiral symmetry and the resulting pion dynamics is also important, but we emphasize that the nuclear many-body problem involves aspects that go beyond chiral symmetry, such as vector-meson dominance and the broken scale invariance of QCD, which manifests itself in the dynamics of the scalar-isoscalar sector.

II. AN EFFECTIVE CHIRAL LAGRANGIAN FOR NUCLEI AND NUCLEAR MATTER

As discussed in Ref. [3], we have recently constructed an effective hadronic lagrangian consistent with the symmetries of quantum chromodynamics and intended for applications to finite-density nuclear systems [18,19]. The degrees of freedom are (valence) nucleons, pions, and the low-lying non-Goldstone bosons, which account for the intermediate-range nucleon–nucleon interactions and conveniently describe the nonvanishing expectation values of bilinear products of nucleon fields. Chiral symmetry is realized nonlinearly [16], with a light scalar meson included as a chiral singlet to describe the mid-range nucleon–nucleon attraction. The low-energy electromagnetic structure of the nucleon is described using vector-meson dominance, so that ad hoc form factors are not needed.

The effective lagrangian is expanded in powers of the fields and their derivatives, with the terms organized using Georgi’s “naive dimensional analysis” [20]. Results for finite nuclei and nuclear matter have been calculated at one-baryon-loop order, using the single-nucleon structure determined within the theory. This leads essentially to a factorized parametrization of the ground-state energy functional. Although the form of the scalar effective potential is motivated from broken scale invariance [18], we find that only the first few terms in an
expansion of powers of the scalar field are important at normal nuclear densities, and thus
the primary source of constraints on the scalar dynamics is the nuclear structure physics,
which determines the free parameters in the scalar potential. These parameters implicitly
contain the effects of many-body forces, hadron substructure, and vacuum dynamics. The
parameters are obtained from fits to nuclear properties and show that naive dimensional
analysis is a useful principle, and that a truncation of the effective lagrangian at the first
few powers of the fields and their derivatives is justified \cite{3,5}. This is because the relevant
expansion parameters are the ratios of the meson mean fields in nuclei to the nucleon mass,
and these ratios are roughly 1/4 to 1/3 at the nuclear densities of interest. Recently, we
have verified that bulk and single-particle nuclear data can determine roughly five, isoscalar,
non-gradient parameters, one gradient parameter, and one isovector parameter \cite{5}.

### III. ELECTROWEAK INTERACTIONS WITH NUCLEI

One of the advantages of the effective field theory is that the structure of the hadrons
can be included with increasing detail by adding more and more nonrenormalizable inter-
actions in a derivative expansion. For example, the long-range parts of the single-nucleon
electromagnetic form factors (that is, magnetic moments and mean-square radii) can be
described using an effective lagrangian that includes vector-meson dominance and direct
couplings of photons to nucleons \cite{19}. The parameters describing the nucleon structure are
adjusted to reproduce free nucleon data (so that external form factors need not be intro-
duced), and the lagrangian dynamics then determines how this structure is modified inside
the nucleus. Moreover, with the DFT approach, accurate nuclear ground-state densities
and single-particle wave functions (for states near the Fermi surface) can be obtained, and the
electromagnetic current is automatically conserved by virtue of the U(1) gauge invariance
of the effective lagrangian (and the resulting field equations).

Thus it is possible within this framework to study (e,e'N) reactions with the same single-
particle potentials in the initial and final state (thereby ensuring current conservation), with
realistic nuclear wave functions, and with the medium-modified nucleon structure specified
unambiguously. While this approach may be too simple, it has the virtue of including all of
these effects simultaneously within the framework of a single lagrangian. Moreover, meson-
exchange currents consistent with gauge invariance can be derived systematically, and one
can extend the analysis to include coupled channels as a way of introducing absorptive effects.
One of the challenges in this program is to extend the description of the nucleon structure
so that it is useful at the higher momentum transfers that are relevant to experiments at
Jefferson Laboratory.

Initial studies of the weak currents within this framework focused on axial-vector cur-
rents, including meson-exchange currents \cite{21}. Although the leading exchange currents (in
inverse powers of the nucleon mass) can be deduced from SU(2)\textsubscript{L} × SU(2)\textsubscript{R} current algebra, it
is necessary to have a lagrangian with a consistent power counting to extend these results to
higher orders. It is also known that in models based on a linear representation of the chiral
symmetry (such as the σ model), it is impossible to satisfy the three constraints of PCAC,
the Goldberger–Treiman relation, and the correct charge algebra simultaneously (at least
at any tractable level of approximation) \cite{22}. For the nonlinear chiral lagrangian discussed
above, however, PCAC is built into the lagrangian, and the axial coupling between pions and nucleons contains a free parameter, which allows all three constraints to be satisfied. Moreover, as previously mentioned, the other parameters in the lagrangian can be chosen to yield accurate nuclear wave functions that can be used to compute matrix elements of these exchange currents within a single framework. Finally, the pion–pion and pion–nucleon part of the lagrangian is exactly the same as that used in chiral perturbation theory [23].

The weak vector and axial-vector currents have already been determined through all orders of the pion field and up to two derivatives of pion fields, together with contributions involving both rho mesons and pions. (For computation of axial exchange currents, one requires only purely mesonic terms and terms that are bilinear in the baryon fields.) The corresponding weak charges reproduce the standard charge algebra, and the Goldberger–Treiman relation is satisfied at the tree level. The two-body, axial-vector, meson-exchange amplitudes satisfy PCAC [21]. The remaining task is to deduce the exchange currents in forms that will be useful in calculations using either relativistic (four-component) or nonrelativistic (two-component) nuclear wave functions. One can then analyze the exchange-current contributions in some selected weak reactions and compare the results to earlier calculations.

IV. CALCULATIONS BEYOND ONE-LOOP ORDER

One can also improve the description of the many-body dynamics within the effective lagrangian approach. At the one-loop level, the nucleons move in classical meson fields, and the vacuum contributions are easy to handle because they can be separated from the contributions of valence nucleons and then parametrized by polynomials in the meson fields in the lagrangian (or energy functional). Although the one-loop calculations provide a realistic description of bulk and single-particle nuclear properties, which can be understood within the framework of density functional theory (see below), it is still imperative to study exchange and correlation corrections in a systematic way in order to make reliable predictions for other nuclear observables. Moreover, it is important to verify that the “natural” values of the parameters obtained from fits at the one-loop level do not become unnatural when higher-order, many-body effects are included. Whereas the valence-nucleon contributions to these corrections are analogous in structure to those in nonrelativistic many-body theory, the vacuum dynamics is more complicated. This is because one must now consider counterterms involving the nucleon mass and wave function, as well as the nucleon–meson vertices, and one must deal with “mixed” vacuum and valence terms, such as the strong nuclear Lamb shift [24].

Recent work [25] has focused on applying the loop expansion to the chiral lagrangian discussed above. While we do not expect the loop expansion to be practical for many-body calculations, it nevertheless provides a systematic framework for studying the interplay of short-range (vacuum) and long-range (many-body) effects. Moreover, the insight gained from the loop expansion has allowed us to extend the analysis to more relevant approximations, such as those involving the summation of rings and ladders. What one finds (perhaps not surprisingly) is that in a large class of approximations, it is possible to separate the short-range contributions from the long-range contributions, and to show that the former
can be written in terms of products of fields that are already present in the lagrangian. In short, this implies that once one decides on a “canonical” form for the lagrangian (which means that one chooses a set of field variables that eliminates redundant terms in a well-defined—but nonunique—fashion), the resulting parameters implicitly include short-range effects to all orders in the interaction, and these effects need not be calculated explicitly. The long-range, many-body effects from valence nucleons, which must be calculated explicitly, resemble well-known contributions from nonrelativistic many-body theory, like summations of ladders and rings. The only differences are that the baryons now have Dirac wave functions, the meson propagation is retarded, and there are modifications to the meson propagation coming from nonlinear meson interactions that incorporate short-range physics. Thus the nuclear dynamics can be organized in a fashion very similar to the nonrelativistic nuclear many-body problem, with the only differences being the relativistic modifications mentioned above and a small number of unknown parameters that describe the short-range dynamics (which is manifested through many-body forces). This new approach to the separation of “valence” and “vacuum” effects solves a problem that has existed in QHD for more than 20 years.

Obviously, there is still a tremendous amount of work that must be done within this program. Our preliminary calculations at the two-loop level and in a simplified Dirac–Brueckner–Hartree–Fock calculation show that it remains possible to reproduce the empirical properties of nuclear matter with parameters that are natural; this is further evidence of the dominance of the Hartree contributions to these parameters. Nevertheless, these Brueckner calculations must be improved, as there are still several ambiguities that must be removed: a consistent cutoff procedure, which truncates integrals systematically at the scale of the “heavy” non-Goldstone-boson masses must be implemented (and checked for sensitivity), and the retardation and short-range modifications to the meson propagators must be included. A systematic method for constructing a canonical form of the lagrangian (with no redundant terms) must also be found. One must also develop the formulation of “conserving approximations” [26,27] that go beyond the simple one-loop level and that allow for the inclusion of retarded interactions.

V. ENERGY FUNCTIONAL ANALYSIS OF MEAN-FIELD THEORIES OF NUCLEI

As an alternative approach to the relativistic nuclear many-body problem, we consider an energy functional that depends on valence-nucleon wave functions and classical scalar and vector fields. Extremization of this functional leads to coupled equations for finite nuclei and nuclear matter, and the success of relativistic mean-field models discussed earlier shows that these variables allow an efficient description of bulk and single-particle nuclear properties.

Although the energy functional contains classical meson fields, this framework can accommodate physics beyond the simple Hartree (or one-baryon-loop) approximation. This is achieved by combining aspects of both density functional theory [28–31] (DFT) and effective field theory [32–34] (EFT). In a DFT formulation of the relativistic nuclear many-body problem, the central object is an energy functional of scalar and vector densities (or more generally, vector four-currents). Extremization of the functional gives rise to Dirac equations for occupied orbitals with local scalar and vector potentials, not only in the Hartree
approximation, but in the general case as well. (Note that the Dirac eigenvalues do not correspond precisely to physical energies in the general case [28]). Rather than work solely with the densities, we can introduce auxiliary variables corresponding to the local potentials, so that the energy functional depends also on meson fields. The resulting DFT formulation takes the form of a Hartree calculation, but correlation effects can be included, if the proper density functional can be found. Our procedure is analogous to the well-known Kohn–Sham [35] approach in DFT, with the local meson fields playing the role of Kohn–Sham potentials; by introducing nonlinear couplings between these fields, we can implicitly include density dependence in the single-particle potentials.

Moreover, by introducing the meson fields, we can incorporate the ideas of EFT. The exact energy functional has kinetic energy and Hartree parts (which are combined in the relativistic formulation) plus an “exchange-correlation” functional, which is a nonlocal, nonanalytic functional of the densities that contains all the other many-body and relativistic effects. At the present stage of our investigations, we do not try to construct the latter functional explicitly from a lagrangian (which would be equivalent to solving the full many-body problem), but instead approximate the functional using an expansion in classical meson fields and their derivatives. The parameters introduced in the expansion can be fit to experiment, and if we have a systematic way to truncate the expansion, the framework is predictive. Thus a conventional mean-field energy functional fit directly to nuclear properties, if allowed to be sufficiently general, will automatically incorporate effects beyond the Hartree approximation, such as those due to short-range correlations. These observations serve to justify existing relativistic mean-field models containing nonlinear meson self-interactions, which are successful, but which involve lagrangians or energy functionals that have customarily been truncated at some low order without any justification.

We rely on the special characteristics of nuclear ground states in a relativistic formulation, namely, that the mean scalar and vector potentials $\Phi$ and $W$ are large on nuclear energy scales but are small compared to the nucleon mass $M$ [18,36]. This implies that the ratios $\Phi/M$ and $W/M$ provide useful expansion parameters. Moreover, as is illustrated in Dirac–Brueckner–Hartree–Fock (DBHF) calculations [37–39], the scalar and vector potentials (or self-energies) are nearly state independent and are dominated by the Hartree contributions. Thus the Hartree contributions to the energy functional should dominate, and an expansion of the exchange-correlation functional in terms of mean fields should be a reasonable approximation. This “Hartree dominance” also implies that it should be a good approximation to associate the single-particle Dirac eigenvalues with the observed nuclear energy levels, at least for states near the Fermi surface [28]. Of course, the mean-field expansion cannot accommodate all of the nonlocal and nonanalytic aspects of the exchange-correlation functional, and important future work must be done to see how these additional effects can be best incorporated.

Given a suitable truncation scheme, the first step is to analyze a general energy functional (through some level of truncation) to determine the characteristics that generate successful nuclear phenomenology. One wants to accurately reproduce nuclear charge densities, binding-energy systematics, and single-particle energy levels. If one recalls that the Kohn–Sham approach is formulated to reproduce precisely the ground-state density, and that the relativistic Hartree contributions are expected to dominate the Dirac single-particle potentials, these observables are precisely the ones for which meaningful comparisons with
experiment should be possible. Moreover, experience has shown that these observables can be replaced by a set of nuclear matter properties plus constraints on the meson masses. Finally, it is possible to analytically invert the field equations to solve directly for the model parameters in terms of the nuclear matter input properties [40]. This allows for a systematic and complete study of the parameter space, so that parameter sets that accurately reproduce nuclear observables can be found, and models that fail to reproduce nuclear properties can be excluded. For example, one learns that favored parameter sets typically involve small but significant nonlinear meson self-interactions. What remains to be done is to identify optimal linear combinations of parameters that are the most tightly constrained by the data and use these to produce some “favored” parameter sets.

One of the primary directions for future study will involve the assumption of “naturalness,” namely, that once the appropriate mass scales have been identified, the coefficients of various terms in the energy functional, when expressed in dimensionless form, should all be of order unity. Naturalness allows us to estimate the size of the terms omitted from the energy functional and also implies that one should include all possible terms allowed by the symmetries through a given level of truncation; thus nonlinear vector–vector and vector–scalar interactions should be as important as scalar–scalar interactions in producing accurate nuclear observables, and we indeed find that this is the case. Nevertheless, because the general energy functional contains all powers in the fields and densities, one has great freedom to make field redefinitions, and the relevant question is which representation of the interactions leads to the most natural and efficient truncation scheme. We have found that simple nucleon–meson Yukawa couplings supplemented by meson self-interactions is one way to provide a description with natural parameters, but it may also be possible to use more complicated meson–nucleon couplings (like \( \psi \psi \phi \)) or higher powers of the nuclear scalar and vector densities (“contact terms”) [41].

We also plan to investigate the relationship between the energy functional and underlying effective lagrangians (such as the one discussed in Section II) by performing microscopic calculations beyond the Hartree level. Although there are some outstanding formal issues in the derivation of relativistic density functional theory (such as the existence of a relativistic Hohenberg–Kohn theorem, or variational principle [29]), the primary technical issue is how best to approximate the exchange, correlation, and vacuum corrections in terms of the Kohn–Sham potentials. For example, exchange terms introduce momentum dependence into the baryon self-energies in infinite matter, but the Kohn–Sham potentials used to approximate these self-energies should be state independent. Is there an optimal momentum to use to approximate the state dependence? Is it better to introduce small nonlocalities? Preliminary studies on these questions have been done, but these issues are still a long way from being resolved.

**VI. THE NUCLEAR SYMMETRY ENERGY IN COVARIANT DENSITY FUNCTIONALS**

A power-counting analysis shows that only one isovector parameter in covariant mean-field density functionals is determined by conventional fits to nuclear properties [5]. This implies that the density dependence of the symmetry energy is not constrained by a good fit,
but only some average value is. The usual parametrization of isovector interactions in terms of simple rho-meson exchange leads to a much larger density dependence than is obtained in most nonrelativistic Skyrme models. An observable consequence is a significantly larger prediction for the neutron skin in heavy nuclei. Relativistic and nonrelativistic Brueckner calculations of asymmetric nuclear matter using realistic interactions all predict a density dependence consistent with the lower values, which suggests that the covariant parametrization may be deficient. It is important to examine the effect on the symmetry energy of more general isovector terms and of explicit pion-exchange contributions to the energy functional. These results could have impact on the structure of neutron stars.

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