Electromagnetic Form Factors of the Nucleon in the Chiral Soliton Model

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Abstract

Several years ago it was pointed out that the chiral soliton model allows naturally for satisfactory agreement with the experimentally well-determined proton magnetic form factor $G_M^p$. The corresponding result for the proton electric form factor at that time was in serious disagreement with the data because the calculated $G_E^p$ showed as a rather stable feature a zero for $q^2$ near 10 (GeV/c)$^2$ which was hard to avoid for reasonable choices of parameters, while the data at that time showed no indication for such a behaviour. Meanwhile, new data have confirmed those $G_E^p$ predictions in a remarkable way, so it appears worthwhile to have another look at that model, especially concerning its flexibility with respect to the electric neutron formfactor $G_E^n$ while trying to maintain the satisfactory results for the proton form factors.

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Several years ago it was pointed out [1,2] that the chiral soliton model allows quite naturally for very satisfactory agreement with the experimentally well-determined proton magnetic form factor $G_p^M$ for momentum transfers $q^2$ up to 30 (GeV/c)$^2$. The corresponding result for the proton electric form factor at that time was in serious disagreement with the data because the calculated $G_p^E$ showed as a rather stable feature a zero for $q^2$ near 10 (GeV/c)$^2$ which was hard to avoid for reasonable choices of parameters, while the data at that time showed no indication for such a behaviour. The electric neutron square-radius $\langle r^2 \rangle_n^E$ for the parametrizations used at that time was too large (typically $\sim -0.25$ fm$^2$ as compared to the experimental value of $-0.114\pm 0.003$ fm$^2$) with a resulting electric neutron form factor rising to a maximum of about 0.09 as compared to the maximum of the Galster parametrization of about 0.05. Meanwhile, new data have confirmed the predictions for $G_p^E$ in a remarkable way, so it appears worthwhile to have another look at that model, especially concerning its flexibility with respect to the electric neutron formfactor $G_n^E$ while trying to maintain the satisfactory results for the proton form factors.

It is well known that the e.m. form factors obtained from the plain standard Skyrme model [3] are insufficient and that inclusion of vector meson contributions is necessary [4,5]. There are basically two simple versions to achieve this:


$$\mathcal{L}^{(\pi)} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)}$$

$$\mathcal{L}^{(2)} = \frac{f_\pi^2}{4} \int \left( -trL_\mu L^\mu + m_\pi^2 tr(U + U^\dagger - 2) \right) d^3x,$$

$$\mathcal{L}^{(4)} = \frac{1}{32\epsilon^2} \int tr[L_\mu, L_\nu]^2 d^3x$$

(where $L_\mu$ denotes the chiral gradients $L_\mu = U^\dagger \partial_\mu U$ ) is used with its standard constants: pion decay constant $f_\pi = 93$ MeV, pion mass $m_\pi = 138$ MeV, and the well-established Skyrme parameter $\epsilon = 4.25$. The coupling to the photon field is obtained through the local gauge transformation $U \to e^{i\epsilon Q} U e^{-i\epsilon Q}$ with the charge operator $Q = (\frac{1}{3} + \tau_3)/2$. The isoscalar part of the coupling arises from gauging the standard Wess-Zumino term.

To incorporate vector meson effects the resulting form factors then are multiplied by the factors

$$\Lambda_I(q^2) = \lambda_I \left( \frac{m_I^2}{m_I^2 + q^2} \right) + (1 - \lambda_I)$$

with $I=0,1$ for isoscalar and isovector form factors; $m_0, m_1$ are the $\omega$- and $\rho$-masses $m_\omega = 783$ MeV, $m_\rho = 770$ MeV, respectively; so the parameters $\lambda_0, \lambda_1$ allow for admixing of the vector meson poles to the purely pionic formfactors. The detailed expressions for the form factors are given explicitly in [1]. So, with $\epsilon$ kept fixed at its standard value, this most simple version contains two parameters: $\lambda_0$ and $\lambda_1$.

Model B [2] : The vector mesons are included explicitely as dynamical degrees of freedom in the lagrangian. In the minimal version the axial vector mesons are eliminated in chiral invariant way [6,7]. This leaves two gauge coupling constants $g_\rho, g_\omega$ for $\rho$- and $\omega$-mesons.

$$\mathcal{L} = \mathcal{L}^{(\pi)} + \mathcal{L}^{(\rho)} + \mathcal{L}^{(\omega)}$$
\[ \mathcal{L}^{(\rho)} = \int \left( -\frac{1}{8} tr \rho_{\mu\nu} \rho^{\mu\nu} + \frac{m_{\rho}^2}{4} tr(\rho_{\mu} - i \frac{\tau}{2g_{\rho}} (l_{\mu} - r_{\mu}))^2 \right) d^3x, \quad (5) \]

\[ \mathcal{L}^{(\omega)} = \int \left( -\frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{m_{\omega}^2}{2} \omega_{\mu} \omega^{\mu} + 3g_{\omega} \omega_{\mu} B^{\mu} \right) d^3x, \quad (6) \]

with the topological baryon current \( B_{\mu} = \frac{1}{24\pi^2} \epsilon_{\mu\nu\rho\sigma} tr L^{\nu} L^{\rho} L^{\sigma} \), and \( l_{\mu} = \xi^\dagger \partial_{\mu} \xi, \quad r_{\mu} = \partial_{\mu} \xi \xi^\dagger \), where \( \xi^2 = U \).

The contributions of the vector mesons to the electromagnetic currents arise from the local gauge transformations

\[ \rho^{\mu} \to e^{iQ_V} \rho^{\mu} e^{-iQ_V}, \quad \omega^{\mu} \to \omega^{\mu} + \frac{Q_{0}}{g_{0}} \partial^{\mu} \epsilon \quad \text{(7)} \]

(with \( Q_{0} = 1/6 \), \( Q_V = \tau_3/2 \)). The resulting form factors are expressed in terms of three static and three induced profile functions which characterize the rotating hedgehog soliton with baryon number \( B = 1 \).

Because the Skyrme term \( \mathcal{L}^{(4)} \) at least partly accounts for static \( \rho \)-meson effects its strength in Model B must be strongly reduced, or could even be omitted. So, in this model we consider \( e \) as an additional parameter. The coupling constant \( g_{\rho} \) can be fixed by the KSRF relation \( g_{\rho} = m_{\rho}/(2\sqrt{2} f_{\rho}) = 2.925 \), but small deviations from this value are tolerable. The \( \omega \)-mesons introduce two gauge coupling constants, \( g_{\omega} \) to the baryon current in \( \mathcal{L}^{(\rho)} \), and \( g_{0} \) for the isoscalar part of the charge operator. Within the \( SU(2) \) scheme we can in principle allow \( g_{0} \) to differ from \( g_{\omega} \) and thus exploit the freedom in the e.m. coupling of the isoscalar \( \omega \)-mesons. However, as the isoscalar part of the electromagnetic current is given by the baryonic current, it is natural to expect \( g_{\omega} \approx g_{0} \).

A difficulty of all nucleon models is to relate the form factors evaluated in the nucleon rest frame to their momentum-transfer dependence in the Breit frame moving relative to the rest frame with velocity \( v \), with

\[ \gamma^2 = (1 - v^2)^{-1} = 1 + \frac{q^2}{(2M)^2}, \quad (8) \]

where \( M \) is the nucleon mass. Unfortunately, the simple boost prescription [8,9]

\[ G_{M}^{\text{Breit}}(q^2) = \gamma^{-2} G_{M}^{\text{rest}}(\gamma^{-2} q^2), \quad G_{E}^{\text{Breit}}(q^2) = G_{E}^{\text{rest}}(\gamma^{-2} q^2) \quad (9) \]

has a serious flaw: it generally violates the superconverge law expected for nucleon formfactors [10]

\[ q^2 G^{\text{Breit}}(q^2) \to 0 \quad \text{for} \quad q^2 \to \infty. \quad (10) \]

This is due to the fact that the boost in Eq.(9) maps \( G^{\text{rest}}(4M^2) \to G^{\text{Breit}}(q^2 \to \infty) \), and \( G^{\text{rest}}(4M^2) \), although being very small, generally does not vanish exactly. This shows up, of course, in a very drastic way, if the resulting formfactors are divided by the standard dipole

\[ G_{D}(q^2) = 1/(1 + q^2/0.71)^2, \quad (11) \]
which is the common way to present them. So it is vital for a comparison with experimentally
determined form factors for \( q^2 \gg M^2 \) to modify the boost prescription in such a way that
agreement with the data for \( G_M^p/\mu_p G_D \) at the highest available values of \( q^2 \) is improved.
A simple way to achieve this is to allow the mass \( M \) in the boost Eq.(9) to be larger than the
experimental value of the nucleon mass, which further decreases the absolute value of
\( G_M^{\text{rest}}(4M^2) \). One might even argue this to be consistent with the soliton model because the
classical soliton masses typically are around 1.5 GeV. Further improvement of the high-\( q^2 \)
behaviour could be achieved by enforcing superconvergence through subtraction of the small
constants \( G^{\text{rest}}(4M^2) \) from the form factors as described in [1]. In any case, however, the
high-\( q^2 \) behaviour is not a profound consequence of the model but rather reflects the boost
prescription. So, in the following, we do not enforce superconvergence.

Of course, it is unfortunate that in this way the physical information provided by the high-
\( q^2 \) limit of \( G_M^p/\mu_p G_D \) is lost, but there is no hope anyway, why such low-energy effective
models should give a profound answer for that high-\( q^2 \) limit. However, the functional form
of the different form factors relative to each other remains largely intact. Therefore in both
versions, Model A and Model B, we introduce the boost mass \( M \) in Eq.(9) as one additional
parameter to adjust to the high-\( q^2 \) data for \( G_M^p/\mu_p G_D \).

The parameters \( \lambda_0 \) and \( \lambda_1 \) in Model A determine the amount of vector dominance in the
isoscalar and isovector channels, respectively. Their difference, \( \lambda_0 - \lambda_1 \), therefore is crucial
for the magnitude of the electric neutron formfactor \( G_n^E \). In [1] they were taken equal for
simplicity \( (\lambda_0 = \lambda_1 = 0.75) \), which resulted in the quadratic neutron radius being much too
large. Improving on this point requires stronger vector dominance for the \( \omega \)-meson \( (\lambda_0 \to 1) \)
with \( \lambda_1 \) still around 0.75. We present in Fig. 1 results for Model A with \( \lambda_0 = 0.92, \lambda_1 = 0.78, \)
and \( M=1.5 \) GeV. The form of \( G_E^n \) is very similar to the Galster parametrization [11], but
still exceeds it by about 20\% near the maximum. Attempts to further lower it (by increasing
the difference \( \lambda_0 - \lambda_1 \) ) result in simultaneous decrease of \( G_M^p G_D/\mu_p G_M^p \), moving it further to the
left of the recent data set of [12,13]. So, in this very restricted model we find it difficult to
bring \( G_E^n \) down to the Galster result, while maintaining overall agreement with both proton
form factors. (The sharp rise of \( G_M^p G_D/\mu_p G_M^p \) beyond \( 10 \) (GeV/c)^2 is due to the small finite
value of \( G^{\text{rest}}(4M^2) \). It could be removed by enforcing superconvergence.)

In Model B the amount of vector dominance for the \( \rho \)-meson is fixed by the ratio
\( g_\rho/(m_\rho f_\pi) \), while for the \( \omega \)-mesons it is determined by \( g_\omega/g_0 = \lambda_0 \). So, the results from
Model A imply that the constraint \( g_\omega = g_0 \) in Model B should lead to better results for \( G_n^E \),
which indeed proves to be the case. For satisfactory overall agreement of the proton form
factors we find it helpful to keep a small Skyrme term in the lagrangian (note that it is \( \sim e^{-2} \),
so with \( e=12 \) it has about 10\% of its standard strength). We present in Fig. 2 two fits: (B1)
with \( e=12, g_\rho=2.6, g_\omega=g_0=1.4, \) and \( M=1.89 \) GeV; and (B2): with \( e=12, g_\rho=2.64, g_\omega=0.9, \)
\( g_0=1.1 g_\omega \), and \( M=2.1 \) GeV. Again it proves difficult to further lower the electric neutron
form factor, with the ratio \( g_\omega/g_0=1 \) fixed (in fit B1), and trying to keep \( G_M^p G_D/\mu_p G_M^p \) within
the data. The form of \( G_E^n \) in this case differs slightly from the Galster form, the maximum is
shifted to lower \( q^2 \), and the following decrease is steeper, so \( G_E^n \) for \( q^2 > 1 \) (GeV/c)^2 is smaller
than the Galster result. However, \( G_E^n \) is very sensitive to the ratio \( g_\omega/g_0 \) and allowing for
a 10\% increase (fit B2) brings its maximum down to the Galster value. Readjustment of
\( g_\rho \) and \( g_\omega \) allows to maintain the agreement with both proton form factors. The resulting
\( G_E^n \) develops a small dip beyond its main decrease so it actually passes through zero near
For small $q^2$, $G^n_E$ still exceeds the Galster parametrization (because the maximum is shifted to the left), so the absolute values of the resulting neutron square radii are still too large (cf. Table 1).

It is of interest to also look at the magnetic neutron form factor $G^n_M$. In order to get rid of the problems with superconvergence we consider the ratio of the normalized proton and neutron form factors $G^n_M \mu_p / (G^n_M \mu_n)$. In Fig.3 we present these ratios for both models, together with data from [14,15]. Both models consistently predict this ratio to increase above 1 by 20-40% for $q^2 > 1 (\text{GeV/c})^2$. This increase is the more pronounced the lower the value of $G^n_E$ near 1 (\text{GeV/c})^2 is. The present data do not show such an increase, in fact they indicate the opposite tendency. This conflict was already noticed in [1].

In Table 1 we list the quadratic radii and magnetic moments as they arise from the three fits given above. Notoriously low are the magnetic moments, as is well known in chiral soliton models. Quantum corrections may partly be helpful in this respect (see [16]), as they certainly are for the absolute values of the masses. Of course both models can be extended; the addition of 6th order terms in Model A, the explicite inclusion of axial vector mesons in Model B provide more flexibility through additional parameters. It is, however, remarkable that in their minimal versions as described above they are able to provide quite satisfactory results for both proton and the electric neutron form factors. In fact, the sharp drop in $G^n_E$ was predicted by these models, and it would be very interesting to have also new data for $G^n_M$ concerning the conflict indicated in Fig.3. Evidently, the weakest point of these considerations is the transition from the rest- to the Breit frame. Although it looks quite natural, the Ji-prescription Eq.(9) is very unsatisfactory, and it would be highly desirable to have superconvergence incorporated in a cogent way. As long as this problem has not been settled there is little hope to gain profound insight from high-$q^2$ ($>10 (\text{GeV/c})^2$) data for e.m.form factors.

<table>
<thead>
<tr>
<th></th>
<th>Model A</th>
<th>Model B1</th>
<th>Model B2</th>
<th>Exp.</th>
</tr>
</thead>
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<td>$\langle r^2 \rangle^p_E$</td>
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<td>0.807</td>
<td>0.782</td>
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<td>$\langle r^2 \rangle^n_E$</td>
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<td>-0.203</td>
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<tr>
<td>$\langle r^2 \rangle^n_M$</td>
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<td>0.776</td>
<td>0.739</td>
<td>0.77</td>
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<tr>
<td>$\mu_p$</td>
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<td>1.71</td>
<td>1.49</td>
<td>2.79</td>
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<tr>
<td>$\mu_n$</td>
<td>-1.42</td>
<td>-1.27</td>
<td>-1.05</td>
<td>-1.91</td>
</tr>
</tbody>
</table>

TABLE I. Nucleon quadratic radii and magnetic moments as obtained from Models A and B, for the fits given in the text, compared to their experimental values [22].
REFERENCES


FIG. 1. Magnetic and electric formfactors of the proton $G_M^p/(\mu_p G_D)$ and $G_E^p/\mu_p G_M^p$ (left) and electric formfactor $G_E^n$ of the neutron (right), for Model A with $\lambda_0=0.92$, $\lambda_1=0.78$, and $M=1.5$ GeV. The data for $G_M^p$ are from [12,13,17–20]. The data for $G_E^n$ are from [21] for Paris potential and from [11] for Lomon wavefunction, the dotted line is the corresponding Galster parametrization.

FIG. 2. The same as Fig.1 for Model B: Fit B1 (dashed lines): $e=12$, $g_\rho=2.6$, $g_\omega=g_0=1.4$, and $M=1.89$ GeV; Fit B2 (full lines): $e=12$, $g_\rho=2.64$, $g_\omega=0.9$, $g_0=1.1g_\omega$, and $M=2.1$ GeV. The dotted line for $G_E^n$ again is the Galster parametrization.
FIG. 3. The ratio of normalized magnetic neutron and proton form factors $\frac{G_M^p \mu_p}{(G_M^p \mu_n)}$ for model A (dotted line) and for model B (fit B1: dashed line, fit B2: full line). The data are from [14,15].