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Importance of reaction volume in hadronic collisions: Canonical enhancement

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Abstract. We study the canonical flavor enhancement. The theoretical motivation, and practical consequences, for both strangeness, and charm, are explored. We argue using qualitative and quantitative theoretical arguments, that this proposal to reevaluate strangeness signature of quark–gluon plasma is without merit.

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1. Introduction

We presents detailed study of the effect of volume dependence on strangeness and charm yields evaluated along the 'canonical statistical mechanics method' [1]. This '20 years after' work has been made necessary by claims that one can reinterpret the strange hadron signature of quark–gluon plasma in terms of the so called canonical enhancement/suppression [2, 3].

We will explain the need to amend the grand canonical method in subsection 2.1, and present the intuitive derivation of the canonical constraint in subsection 2.2, where we follow the approach of Ref. [1]. This can be generalized to more complex systems using the projection method [4, 5], which we demonstrate in subsection 2.3, and use in subsection 2.4 to obtain within the classical Boltzmann limit the suppression factors of multistrange hadrons [2, 3, 6]. This method can be extended and applied to solve more complex situation, for example conservation of several 'Abelian' quantum numbers [7, 8] (such as strangeness, baryon number, electrical charge) and the problem of particular relevance in this field, the exact conservation of color: all hadronic states, including QGP must be exactly color 'neutral' [9, 10].

After offering this thorough theoretical introduction in section 2, we study, in section 3, the magnitude of the different effects. We evaluate the magnitude of the canonical suppression in subsection 3.1. We demonstrate in subsection 3.2, after a rebasing which is converting the suppression into enhancement, that the canonical enhancement effect is in better agreement with the experimental results when the phase space considered is that of deconfined strange quarks. We then show, in subsection 3.3, that for charm flavor, the proposed mechanism is leading to a significant disagreement with experimental constraints on charm production, and thus charm must be different from strangeness in that we cannot use canonical enhancement argument.

In our closing section 4, we also evaluate the specific (per participant) A–A yield enhancement of multistrange baryons and antibaryons, comparing to the p–p system. We see that an enhancement is predicted well above the one observed experimentally, just like for the charm yield. It is generally accepted that the production pattern of charm must be explored within the realm of kinetic theory models. Logic demands that this observation extends also to the strange flavor sector. This then implies that the chemical equilibrium canonical reinterpretation of strangeness signature of quark–gluon plasma is not valid, and that the kinetic methods are applicable [11, 12].

2. Exact conservation of flavor quantum numbers

2.1. General considerations

At low reaction energy, or/and in small collision systems the yield of strangeness in each reaction is rather small, less than one pair of quarks produced per collision. This occasional pair can thermally (momentum distribution) equilibrate with the background of hadrons. In the discussion of the magnitude of this yield, we may be tempted to apply methods of grand canonical statistical ensemble equilibrium. However, these are wrong, as in their derivation a strong and important assumption is that the number of particles considered is large.

The statistical grand canonical flavor conservation condition is

$$\langle n_{\rm s}\rangle - \langle n_{\bar{\rm s}}\rangle = 0\,, \tag{1}$$

where the average is over the ensemble of physical systems, which in Gibbs sense are weakly connected, and can exchange particle number. Thus, each individual system does not conserve strangeness, the fluctuations of strange and antistrange quark number, in the subsystem, are independent of each other. In each subsystem, the magnitude of the average violation is the fluctuation in particle number:

$$n_{\rm s} - n_{\bar{\rm s}} \simeq \sqrt{n_{\rm s} + n_{\bar{\rm s}}}\,. \tag{2}$$

In heavy ion reactions where each collision system is completely disconnected from the other, use of grand canonical method is an idealization which allows the violation of the strangeness conservation law in the theoretical description of each individual collision reaction. This is a severe defect of the statistical method applied, which needs to be quantitatively understood and corrected. Only in a very large system, the average yield of strange quarks nearly equals the average yield of antistrange quarks, and the relative violation of strangeness conservation vanishes like $1/\sqrt{n_{\rm s}}$.

For many reaction systems of physical interest, the difference in strangeness and antistrangeness yield is not negligible. We thus must improve the statistical description enforcing exact strangeness conservation both for systems small and large. Strangeness is always produced in pairs and all experiments always will find (in absence of flavor changing weak interactions) that the micro canonical condition is satisfied,

$$n_{\rm s} - n_{\bar{\rm s}} = 0\,. \tag{3}$$

The yield of net strangeness will vanish exactly within our theoretical approach, as it does in nature. We will next show how it is possible to implement that the net strangeness conservation law is satisfied exactly in the statistical description of the physical properties, while using the power and convenience of statistical mechanics. This then has the minor defect that the number of pairs,

$$\langle n_{\rm s}\rangle + \langle n_{\bar{\rm s}}\rangle = 2\langle n_{\text{s-pair}}\rangle\,, \tag{4}$$

fluctuates in each collision. This may even not be a defect at all as the quantum mechanical laws which govern particle production also are leading to such fluctuations.

We refer to this situation, with exact conservation of some quantum number implemented, here specifically strangeness, as the canonical statistical ensemble. Each member of the ensemble conserves net strangeness exactly, while the number of pairs fluctuates, being exchanged between the members of the ensemble. The discussion above was for the case of vanishing strangeness quantum number, but could be easily repeated for the case of another arbitrary net value of the conserved quantum number.

2.2. Grand canonical and canonical partition functions

The grand partition function in the classical Boltzmann limit for strange particles has the form,

$$\begin{aligned}\ln \mathcal{Z}_{\rm s}^{\rm HG} \equiv Z_{\rm HGs}^{(1)} = \frac{VT^3}{2\pi^2}\Big[&(\lambda_{\rm s}\lambda_{\rm q}^{-1}+\lambda_{\rm s}^{-1}\lambda_{\rm q})\gamma_{\rm s}\gamma_{\rm q}F_{\rm K} + (\lambda_{\rm s}\lambda_{\rm q}^{2}+\lambda_{\rm s}^{-1}\lambda_{\rm q}^{-2})\gamma_{\rm s}\gamma_{\rm q}^{2}F_{\rm Y}\\ &+(\lambda_{\rm s}^{2}\lambda_{\rm q}+\lambda_{\rm s}^{-2}\lambda_{\rm q}^{-1})\gamma_{\rm s}^{2}\gamma_{\rm q}F_{\Xi} + (\lambda_{\rm s}^{3}+\lambda_{\rm s}^{-3})\gamma_{\rm s}^{3}F_{\Omega}\Big]\,.\end{aligned} \tag{5}$$

In the phase space function F_i, all kaon (K), hyperon (Y), cascade (Ξ) and omega (Ω) resonances plus their antiparticles are taken into account:

$$\begin{aligned}F_{\rm K} &= \sum_j g_{{\rm K}_j} W(m_{{\rm K}_j}/T);\ \ {\rm K}_j = {\rm K}, {\rm K}^*, {\rm K}_2^*, \ldots,\quad m \le 1780\ {\rm MeV}\,,\\ F_{\rm Y} &= \sum_j g_{{\rm Y}_j} W(m_{{\rm Y}_j}/T);\ \ {\rm Y}_j = \Lambda, \Sigma, \Sigma(1385), \ldots,\ m \le 1940\ {\rm MeV}\,,\\ F_{\Xi} &= \sum_j g_{\Xi_j} W(m_{\Xi_j}/T);\ \ \Xi_j = \Xi, \Xi(1530), \ldots,\quad m \le 1950\ {\rm MeV}\,,\\ F_{\Omega} &= \sum_j g_{\Omega_j} W(m_{\Omega_j}/T);\ \ \Omega_j = \Omega, \Omega(2250)\,.\end{aligned} \tag{6}$$

The g_i are the spin–isospin degeneracy factors, $W(x) = x^2 K_2(x)$, where K_2 is the modified Bessel function.

The chemical fugacities, as introduced in Eq. (5), allow to count separately the quark content $(\lambda_{\rm q}, \lambda_{\rm s})$ and the yield of quark–antiquark pairs $(\gamma_{\rm s}, \gamma_{\rm q})$. Specifically,

$$\langle n_{\rm s}\rangle - \langle n_{\bar{\rm s}}\rangle = \lambda_{\rm s}\frac{\partial}{\partial\lambda_{\rm s}}\ln\mathcal{Z}_{\rm s}^{\rm HG}\,, \tag{7}$$

$$\langle n_{\rm s}\rangle + \langle n_{\bar{\rm s}}\rangle = 2\langle n_{\text{s-pair}}\rangle = \gamma_{\rm s}\frac{\partial}{\partial\gamma_{\rm s}}\ln\mathcal{Z}_{\rm s}^{\rm HG}\,. \tag{8}$$

To emphasize that any flavor (in particular s, c) or even baryon number is under consideration here, we generalize slightly the notation s → f. We also expand the exponential of the one particle partition function $Z^{(1)}$ in Eq. (5):

$$\mathcal{Z}_{\rm f} = e^{Z_{\rm f}^{(1)}} = \sum_{n=0}^{\infty}\frac{1}{n!}\left(Z_{\rm f}^{(1)}\right)^n\,. \tag{9}$$

The flavor and antiflavor terms within $Z_{\rm f}^{(1)}$ are additive in Eq. (5), and we consider at first only singly-flavored particles, in a self explanatory simplified notation:

$$Z_{\rm f}^{(1)} = \gamma[\lambda_{\rm f}\tilde{F}_{\rm f} + \lambda_{\rm f}^{-1}\tilde{F}_{\bar{\rm f}}]\,, \qquad \tilde{F}_i = \frac{VT^3}{2\pi^2}F_i. \tag{10}$$

Combining Eq. (10) with Eq. (9), we obtain:

$$\mathcal{Z}_{\rm f} = \sum_{n,k=0}^{\infty} \frac{\gamma^{n+k}}{n!k!}\lambda_{\rm f}^{n-k}\tilde{F}_{\rm f}^n\tilde{F}_{\bar{\rm f}}^k\,. \tag{11}$$

When $n \neq k$, the sum in Eq. (11) contains contributions with unequal number of f and $\bar{\rm f}$ terms. Only when $n = k$, we have contributions with exactly equal number of f and $\bar{\rm f}$ terms. We recognize that only $n = k$ terms contribute to the canonical partition function with exactly conserved flavor quantum number,

$$Z_{\rm f=0} = \sum_{n=0}^{\infty} \frac{\gamma^{2n}}{n!n!}(\tilde{F}_{\rm f}\tilde{F}_{\bar{\rm f}})^n = I_0(2\gamma\sqrt{\tilde{F}_{\rm f}\tilde{F}_{\bar{\rm f}}})\,, \tag{12}$$

where we have introduced the modified Bessel function I_0.

The argument of I_0 has a physical meaning, it is the yield of flavor pairs $N_{\rm pair}^{\rm GC}$ in grand canonical ensemble, evaluated with grand canonical flavor conservation, Eq. (1). To see this, we evaluate:

$$0 = \frac{\partial}{\partial\lambda_{\rm f}}\ln\mathcal{Z}_{\rm f} = \frac{\partial}{\partial\lambda_{\rm f}}\left(\gamma[\lambda_{\rm f}\tilde{F}_{\rm f} + \lambda_{\rm f}^{-1}\tilde{F}_{\bar{\rm f}}]\right)\,. \tag{13}$$

We obtain:

$$\lambda_{\rm f}|_0 = \sqrt{\tilde{F}_{\bar{\rm f}}/\tilde{F}_{\rm f}}\,, \qquad \ln\mathcal{Z}_{\rm f}|_{\lambda_{\rm f}=\lambda_{\rm f}|_0} = 2\gamma\sqrt{\tilde{F}_{\rm f}\tilde{F}_{\bar{\rm f}}} \equiv 2\,N_{\rm pair}^{\rm GC}\,. \tag{14}$$

In order to evaluate, using Eq. (12), the number of flavor pairs in the canonical ensemble, we need to average the number n over all the contributions to the sum in Eq. (12). To obtain the extra factor n, we perform the differentiation with respect to γ^2 and obtain the canonical ensemble f-pair yield,

$$\langle N_{\rm f}^{\rm CE}\rangle \equiv \gamma^2\frac{d}{d\gamma^2}\ln Z_{\rm f=0} = \gamma\sqrt{\tilde{F}_{\rm f}\tilde{F}_{\bar{\rm f}}}\,\frac{I_1(2\gamma\sqrt{\tilde{F}_{\rm f}\tilde{F}_{\bar{\rm f}}})}{I_0(2\gamma\sqrt{\tilde{F}_{\rm f}\tilde{F}_{\bar{\rm f}}})} = N_{\rm pair}^{\rm GC}\frac{I_1(2N_{\rm pair}^{\rm GC})}{I_0(2N_{\rm pair}^{\rm GC})}\,, \tag{15}$$

where we have used $I_1(x) = dI_0(x)/dx$. The first term is identical with the result we obtained in the grand canonical formulation, Eq. (14). The second factor I_1/I_0 is the effect of exact conservation of the number of flavor pairs.

2.3. Projection method

For the case of 'Abelian' quantum numbers, e.g., flavor or baryon number, the projection method arises from the general relation between the grand canonical and canonical partition function:

$$\mathcal{Z}(\beta,\lambda,V) = \sum_{f=-\infty}^{\infty}\lambda^{\rm f}Z_{\rm f}(\beta,V)\,. \tag{16}$$

In the canonical partition function $Z_{\rm f}$, some discrete (flavor, baryon) quantum number has the value f. Substituting $\lambda = e^{i\varphi}$, we obtain:

$$Z_{\rm f}(\beta,V;n_{\rm f}) = \int_0^{2\pi}\frac{d\varphi}{2\pi}e^{-in_{\rm f}\varphi}\mathcal{Z}(\beta,\lambda=e^{i\varphi},V)\,. \tag{17}$$

In case of Boltzmann limit, and including singly charged particles only, we obtain for net flavor n_f, from Eq. (11):

$$Z_\mathrm{f}(\beta,V;n_\mathrm{f}) = \sum_{n,k=0}^{\infty} \frac{\gamma^{n+k}}{n!k!} \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{i(n-k-n_\mathrm{f})\varphi} \tilde{F}_\mathrm{f}^n \tilde{F}_{\bar{\mathrm{f}}}^k \,. \tag{18}$$

The integration over φ yields the $\delta(n-k-n_\mathrm{f})$-function. Replacing $n = k + n_\mathrm{f}$, we obtain:

$$Z_\mathrm{f}(\beta,V;n_\mathrm{f}) = \sum_{k=0}^{\infty} \frac{\gamma^{2k+n_\mathrm{f}}}{k!(k+n_\mathrm{f})!} \tilde{F}_\mathrm{f}^{k+n_\mathrm{f}} \tilde{F}_{\bar{\mathrm{f}}}^k \,. \tag{19}$$

The power series definition of the modified Bessel function I_f is:

$$I_{n_\mathrm{f}}(z) = \sum_{k=0}^{\infty} \frac{(z/2)^{2k+n_\mathrm{f}}}{k!(k+n_\mathrm{f})!} \,. \tag{20}$$

Thus, we obtain:

$$Z_\mathrm{f}(\beta,V;n_\mathrm{f}) = \left(\frac{\tilde{F}_\mathrm{f}}{\tilde{F}_{\bar{\mathrm{f}}}}\right)^{n_\mathrm{f}/2} I_{n_\mathrm{f}}(2\gamma\sqrt{\tilde{F}_\mathrm{f}\tilde{F}_{\bar{\mathrm{f}}}}) \,. \tag{21}$$

The case $n_\mathrm{f} = 0$, we considered earlier Eq. (12), is reproduced. We note that for integer n_f, we have $I_{n_\mathrm{f}} = I_{-n_\mathrm{f}}$. We used n_f as we would count baryon number, thus in flavor counting, n_f counts the flavored quark content, with quarks counted positively and antiquarks negatively. This remark is relevant in numerical studies when the factors $\tilde{F}_\mathrm{f}$, $\tilde{F}_{\bar{\mathrm{f}}}$ contain baryochemical potential.

2.4. Suppression of multistrange particle yield

Multistrange particles can be introduced as additive terms in the exponent of Eq. (17). This allows us to evaluate their yields [2]. However, the canonical partition function is dominated by singly strange particles and we will assume, in the following, that it is sufficient to only consider these, in order to obtain the effect of canonical flavor conservation. This assumption is consistent with use of classical Boltzmann statistics. In fact, expanding the Bose distribution for kaons, one finds that the next to leading order contribution, which behaves as strangeness $n_\mathrm{s} = \pm 2$ hadron, is dominating in the projection the influence of all multistrange hadrons.

In order to find yields of rarely produced particles such as is, e.g., Ω(sss), we show the omega term explicitly:

$$Z_\mathrm{f}(\beta,V;n_\mathrm{f}=0) = \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{\tilde{F}_\mathrm{f} e^{i\varphi} + \tilde{F}_{\bar{\mathrm{f}}} e^{-i\varphi} + \lambda_\Omega e^{3i\varphi} \tilde{F}_\Omega + \cdots} \,. \tag{22}$$

The unstated terms in the exponent are the other small abundance multi-flavored particles. The fugacities not associated with strangeness, as well as the yield fugacity γ_s, are incorporated in Eq. (22) into the phase space factors $\tilde{F}_i$ for simplicity of notation.

The number of Ω is obtained differentiating $\ln Z_\mathrm{f}(\beta,V)$, with respect to λ_Ω, and subsequently neglecting the sub dominant terms in the exponent. We obtain:

$$\langle n_\Omega \rangle = \frac{\tilde{F}_\Omega}{I_0} \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{3i\varphi} e^{\tilde{F}_\mathrm{f} e^{i\varphi} + \tilde{F}_{\bar{\mathrm{f}}} e^{-i\varphi}} \,. \tag{23}$$

The integral is just $Z_{\rm f}(\beta, V; n_{\rm f} = -3)$, Eq. (21), since we need to balance the three strange quarks in the particle observed by the balance in the background of singly strange particles (kaons and hyperons):

$$\langle n_\Omega \rangle = \tilde F_\Omega \left(\frac{\tilde F_{\rm f}}{\tilde F_{\bar{\rm f}}} \right)^{-3/2} \frac{I_3(2N^{\rm GC}_{\rm pair})}{I_0(2N^{\rm GC}_{\rm pair})} \,. \tag{24}$$

We recall that, according to Eq. (14), the middle term is just the fugacity factor $\lambda_{\rm s}^3$. The first two factors, in Eq. (24), constitute the grand canonical yield, while the canonical Ω-suppression factor is the last term. A full treatment of the canonical suppression of multistrange particles in small volumes has been used to obtain particle yields in elementary interactions [13].

Similarly, one finds that the Ξ suppression has the factor I_2/I_0, while as discussed for the general example of flavor pair yield, the single strange particle yield is suppressed by the factor I_1/I_0. The yield of all flavored hadrons in the canonical approach (superscript 'C') can be written as function of the yield expected in the grand canonical approach in the general form,

$$\langle s^\kappa \rangle^{\rm C} = \tilde F_\kappa \left(\frac{\tilde F_{\rm f}}{\tilde F_{\bar{\rm f}}} \right)^{\kappa/2} \frac{I_{|\kappa|}(2N^{\rm GC}_{\rm pair})}{I_0(2N^{\rm GC}_{\rm pair})} = \langle s^\kappa \rangle^{\rm GC} \frac{I_{|\kappa|}(2N^{\rm GC}_{\rm pair})}{I_0(2N^{\rm GC}_{\rm pair})} \,, \tag{25}$$

with $\kappa = \pm 3$, ± 2, and ± 1 for Ω, Ξ, and Y, K, respectively. On the left hand side, in Eq. (25), the power indicates the flavor content in the particle considered with negative numbers counting antiquarks. We note, inspecting the final form of Eq. (25), that the canonical suppression of particles and antiparticles is the same. However, a particle/antiparticle asymmetry can occur if baryon/antibaryon asymmetry is present.

The simplicity of this result originates in the assumption that the single strange particle contribution to strangeness conservation are dominant. A more complex evaluation taking all multistrange hadrons into account, but considering kaons as Boltzmann particles is theoretically inconsistent.

3. Canonical strangeness and charm suppression

3.1. The suppression function

The canonical flavor yield suppression factor,

$$\eta \equiv \frac{I_1(2\gamma\sqrt{\tilde F_{\rm f} \tilde F_{\bar{\rm f}}})}{I_0(2\gamma\sqrt{\tilde F_{\rm f} \tilde F_{\bar{\rm f}}})} = \frac{I_1(2N^{\rm GC}_{\rm pair})}{I_0(2N^{\rm GC}_{\rm pair})} < 1 \,, \tag{26}$$

depends in a complex way on the volume of the system, or alternatively said, on the grand canonical number of pairs, $N^{\rm GC}_{\rm pair}$. The suppression function $\eta(N) \equiv I_1(2N)/I_0(2N)$ is shown in Fig. 1, as function of N. For $N > 1$, we see (dotted lines) that the approach to the grand canonical limit is relatively slow, it follows the asymptotic form,

$$\eta \simeq 1 - \frac{1}{4N} - \frac{1}{128N^2} + \dots \,, \tag{27}$$

while for $N \ll 1$, we see a nearly linear rise:

$$\eta = N - \frac{N^3}{2} + \dots \tag{28}$$

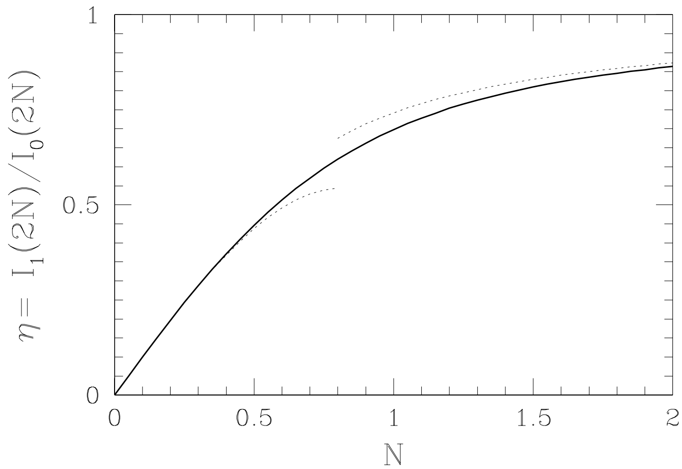

Figure 1. Solid line: canonical yield suppression factor as function of the grand canonical particle yield N. Dotted lines: asymptotic expansion presented in text.

Overall, when the the yield of particles is small, we have using Eq. (28):

$$N_{\rm f}^{\rm CE} \simeq (N_{\rm f}^{\rm GC})^2\,. \tag{29}$$

The chemical equilibrium yield, at small abundances, is quadratic in grand canonical particle yield, which for $m > T$ is, expanding the K_2-Bessel function,

$$N_{\rm f}^{\rm GC} = \frac{g_{\rm f}}{2\pi^2} T^3 V \sqrt{\frac{\pi m_{\rm f}^3}{2T^3}} e^{-m_{\rm f}/T}\,. \tag{30}$$

Thus, when the yield of particles is small, e.g., when $m_{\rm f} \gg T$, the canonical result applies:

$$N_{\rm f}^{\rm CE} = \frac{g_{\rm f}^2}{4\pi^3} T^3 m_{\rm f}^3 V^2 e^{-2m_{\rm f}/T}\,. \tag{31}$$

This result resolves an old puzzle first made explicit by Hagedorn, who queried the quadratic behavior of the pair particle yield, compared to Boltzmann yield, $Y \propto e^{-2m/T} \simeq (e^{-m/T})^2$ being concerned about rarely occurring astrophysical pair production processes [14].

The benchmark result, seen in Fig. 1, is that when one particle pair would be expected to be present in grand canonical chemical equilibrium the actual canonical yield is suppressed, the true phase space yield is 0.6 pairs. This suppression occurs when the exact strangeness conservation is enforced due to reduction of the accessible phase space by particle–antiparticle correlation.

We now look at the suppression of multistrange particles by the suppression factors $\eta_3(N) = I_3(2N)/I_0(2N)$, for Ω, and $\eta_2(N) = I_2(2N)/I_0(2N)$, for Ξ. For small values of N, we obtain:

$$\eta_\kappa \equiv \frac{I_\kappa(2N)}{I_0(2N)} \to N^\kappa \frac{1}{\kappa!}\left(1 - \frac{\kappa}{\kappa+1} N^2\right)\,. \tag{32}$$

This result is easily understood on physical grounds: for example when the expected grand canonical yield is three strangeness pairs, it is quite rare that all three strange quarks go into an Ω. This is seen in Fig. 2 (short dashed curve), and in fact this will occur 1/5 as often as we would expect computing the yield of Ω, ignoring the

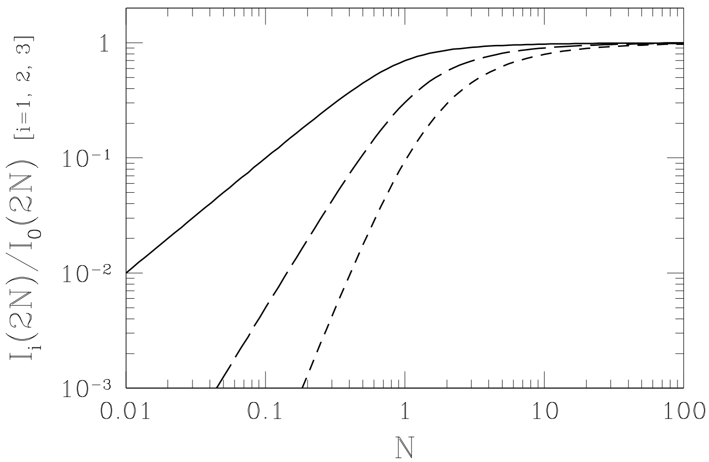

Figure 2. Canonical yield suppression factor I_κ/I_0 as function of the grand canonical particle yield N. Short-dashed line: suppression of triply strange hadrons; long dashed: suppression of doubly flavored hadrons; and solid line, the suppression of singly flavored hadrons.

canonical conservation of strangeness. The other lines, in Fig. 2, correspond to the other suppression factors, long dashed is $\eta_2(N) = I_2(2N)/I_0(2N)$ and the solid line is $\eta(N) = I_1(2N)/I_0(2N)$. They are shown dependent on the number N of strange pairs expected in the grand canonical equilibrium. We see that the suppression effect increases with strangeness content, and that for $N > 5$, it practically vanishes.

3.2. Hadronic gas compared to quark–gluon plasma

We first consider how big a volume we need, in order to find (using grand canonical ensemble counting) one pair of strange particles. As unit volume, we choose $V_{\rm h} = (4\pi/3)\,1\,{\rm fm}^3$. The flavor and antiflavor phase space is symmetric in the deconfined state. In the Boltzmann limit,

$$\tilde{F}_{\rm f} = \tilde{F}_{\bar{\rm f}} = \frac{3VTm_{\rm f}^2}{\pi^2} K_2(m_{\rm f}/T)\,. \tag{33}$$

In Fig. 3, the dashed line shows the volume required for one pair using the strange quark phase space, which does not depend on $\lambda_{\rm q}$, and has been obtained choosing $m_{\rm s} = 160$ MeV and $T = 160$ MeV. Just a little less than one hadronic volume suffices, one finds one pair in $V_{\rm h}$ for $m_{\rm s} = 200$ MeV.

For the hadronic phase space, counting as before strange quark content as positively 'flavor charged', we obtain using Eqs. (5, 6):

$$\tilde{F}_{\rm f} = \lambda_{\rm q}^{-1}\tilde{F}_{\rm K} + \lambda_{\rm q}^{2}\tilde{F}_{\rm Y}\,, \tag{34}$$

$$\tilde{F}_{\bar{\rm f}} = \lambda_{\rm q}\tilde{F}_{\rm K} + \lambda_{\rm q}^{-2}\tilde{F}_{\rm Y}\,. \tag{35}$$

All these quantities $\tilde{F}_i$ are proportional to the reaction volume. With $\lambda_{\rm s}$ chosen to conserve strangeness, Eq. (14),

$$\frac{V}{V_{\rm h}} = \frac{2\pi^2}{V_{\rm h}T^3\gamma_{\rm q}\gamma_{\rm s}\sqrt{(F_{\rm K}+\lambda_{\rm q}^3F_{\rm Y})(F_{\rm K}+\lambda_{\rm q}^{-3}F_{\rm Y})}}\,. \tag{36}$$

The result is shown as solid line in Fig. 3, as function of $\lambda_{\rm q}$, for $\gamma_{\rm q} = 1, \gamma_{\rm s} = 1$. We recall that at SPS and RHIC energies, we have $\lambda_{\rm q} < 1.6$. We see that for small $\lambda_{\rm q}$, we

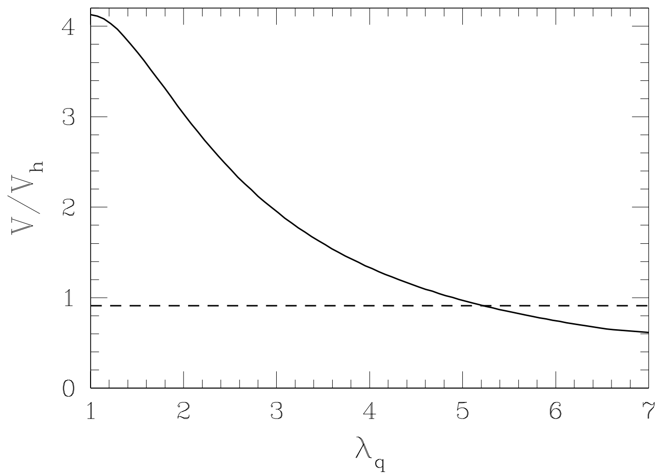

Figure 3. Volume needed for one strange quark pair using grand canonical counting as function of $\lambda_{\rm q}$ for $T = 160$ MeV, $\gamma_{\rm q} = 1, \gamma_{\rm s} = 1$, $V_{\rm h} = (4\pi/3)\,1\,{\rm fm}^3$. Solid line: hadron gas phase space, dashed line: quark phase space with $m_{\rm s} = 160$ MeV.

need much greater volumes to find one strange quark pair, and thus we recognize that the hadron gas phase space is significantly smaller in absence of dense baryon number. In a more colloquial language, strangeness 'production' is easier in the channel KΛ than in K$\overline{\rm K}$.

This strong difference in the magnitude of the phase space between the confined and deconfined phase, seen in Fig. 3, makes the effect of canonical suppression different when we compare quark–gluon plasma with hadronic gas. Thus in what follows the yield of strange hadrons is dependent on the nature of the phase from which emission occurs.

It has been proposed to exploit the canonical suppression, which grows with strangeness content, in order to explain the increase of strange hadron production, which also grows with strangeness content of the particle [2, 3]. To do this, we must turn things 'upside down' by rebasing all yields to unity at a unit volume. We first consider more closely how big an effect we get for singly strange hadrons for quark–gluon plasma and hadronic gas. In Fig. 4, the quark phase (solid line) and hadron phase (dashed line), the suppression results are renormalized multiplicatively to cross for $V = V_{\rm h}$ unity. Since quark phase space is bigger, it has 'less space left' to grow to reach saturation, and hence the production enhancement is by a factor two, while for the hadron case there is 'more catch up left' to do and thus the enhancement is larger, we see that it is by a factor three.

Experimentally, the enhancement of strangeness production comparing p–p and A–A interactions is nearly be the factor two, which result is obtained from the yields of produced particles in terms of the Wróblewski ratio [15],

$$W_{\rm s} = \frac{2\langle {\rm s}\bar{\rm s}\rangle}{\langle {\rm u}\bar{\rm u}\rangle + \langle {\rm d}\bar{\rm d}\rangle}\,. \tag{37}$$

Only newly made s$\bar{\rm s}$, u$\bar{\rm u}$ and d$\bar{\rm d}$ quark pairs are counted. If strangeness were to be as easily produced as light u, d quarks, we would find $W_{\rm s} \to 1$. To obtain the experimental value for $W_{\rm s}$, a careful study of produced hadron yields is required. We refer to results obtained using a semi-theoretical method [8], in which numerous

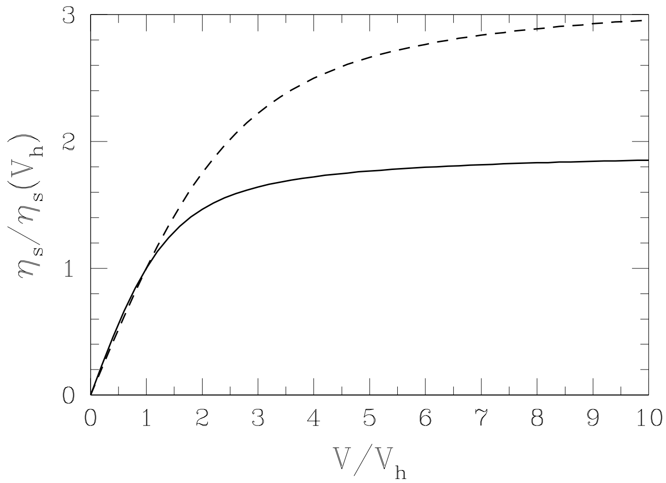

Figure 4. Canonical yield enhancement at large volumes compared to unit hadron volume $V_{\rm h} = (4\pi/3)\,1\,{\rm fm}^3$. Solid line QGP phase, dashed line HG.

particle yields are described within the framework of a statistical model. In elementary collisions pp, p$\bar{\rm p}$, e^+e^-, a value $W_{\rm s} \simeq 0.22$ is obtained, strangeness is thus relatively strongly suppressed. On the other hand, in nuclear A–A′ collisions $W_{\rm s}$ nearly doubles compared to p–p interactions at the same collision energy.

Assuming that the enhancement is due to canonical effects, the result shown in Fig. 4 is implying that strange quarks originate from deconfined state. This conclusion is very strong since there is no way that the experimental ratio $W_{\rm s}$ is enhanced by factor three, or that reaction volume in p–p interactions is much smaller than $V_{\rm h}$. On cannot stop but to smile at this. Namely, turning things 'upside down' one succeeds to argue that the smaller strangeness enhancement (not a factor three, but two) is indication of deconfinement! Of course such arguments, based on comparison of equilibrium oranges with equilibrium apples, are very tentative: using as input the actual yield of strangeness in p–p interactions, the canonical enhancement is by a factor three in any scenario, as the observed experimental yield absorbs the uncertainty about the phase space, see section 4. However, the presentation we have made in Fig. 4 follows the line of argument of [2, 3], and illuminates the arguments made in this work.

3.3. Canonical charm yields

Not everybody is tempted to use statistical equilibrium when considering the yield of charm. The charmed quark mass is sufficiently high to stop even the greatest of optimists from claiming that thermal collisions could equilibrate the yield. On the other hand, since the mass is so large, the thermal grand canonical abundance is relatively small. Thus, the few hard collisions occurring between colliding partons also suffice to reproduce so much charm that it can easily be well above the chemical equilibrium yield.

The yield of charm, in Pb–Pb interactions at 158A GeV, is estimated from lepton background at 0.5 pairs per central collision [16]. We can use the small N expansion, Eq. (32). The corresponding A–A canonical enhancement factor, compared to p–A, is $N_{\rm AA}/N_{\rm pA} \simeq 100\ A$. (Here, N is now grand canonical yield of 'open' charm, and not strangeness). Experimental results are scaling with A^α , $\alpha < 1.3$, thus there is no

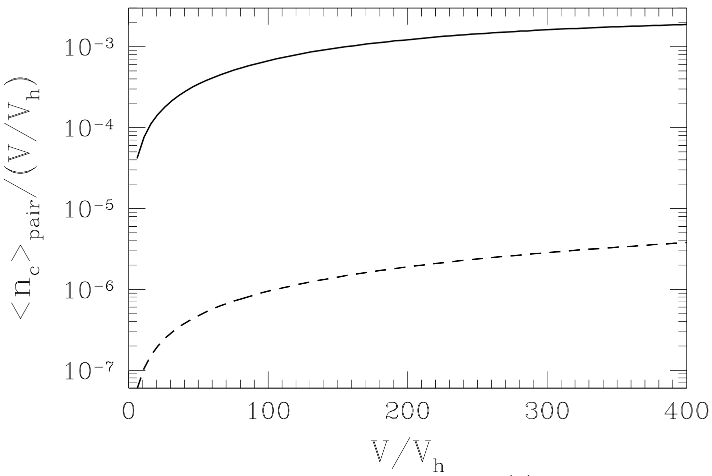

Figure 5. Canonical yield of open charm quark pairs $\langle n\rangle_{\rm pair}$ per unit volume as function of volume, in units of $V_{\rm h} = 4\pi/3\,1\,{\rm fm}^3$. Solid line: QGP with $m_{\rm c} = 1.3$ GeV, dashed line HG at $\mu_{\rm b} = 210$ MeV, both phases at $T = 145$ MeV.

space for canonical enhancement/suppression for charm production of this magnitude.

To be more specific, we show, in Fig. 5, the specific yield per unit volume as function of volume of charm $\langle n\rangle_{\rm pair}$. The canonical effect is the deviation from a constant value and it is significant, $\mathcal{O}(100)$. Even at $V = 400V_{\rm h}$ the infinite volume grand canonical limit is not yet attained, for the case of the larger phase space of QGP (solid line), the total charm yield is 0.8 charm pairs. The absolute yield in both phases is strongly dependent on temperature used, here $T = 145$ MeV, corresponding to SPS hadronization condition. In quark–gluon plasma, we took $m_{\rm c} = 1.3$ GeV. The hadronic gas phase space includes all known charmed mesons and baryons, with light quark abundance controlled by $\mu_{\rm b} = 210$ MeV, $\mu_{\rm s} = 0$.

While choosing a slightly higher value of T, we could increase the equilibrium yield of charm in hadronic gas to the quark–gluon plasma level [17], this does not eliminate the effect of canonical suppression of charm production if chemical equilibrium is assumed for charm in the elementary interactions. We are simply so deep in the 'quadratic' domain of the yield, see Eq. (32), that playing with parameters changes nothing, since we are constrained in Pb–Pb interactions by experiment to have a charm yield below one pair. Then, the expected yield in p–p and p–A interactions is well below measurement, the canonical suppression is overwhelming. Charm yield is surely not in chemical equilibrium either at small or large volumes, most probably in both limits.

4. Final remarks

We have discussed the subtle differences in particle yields that arise in equilibrium statistical mechanics when, within a finite system, the conservation of flavor is enforced exactly. We addressed the recent proposal [2, 3], that the enhancement of strange particles may be also described in chemical equilibrium model using the nonlinear canonical volume dependence discovered 20 years ago [1].

We have shown that, for single strange hadrons, this effect is actually much better agreeing with data when the phase space of strangeness is that of deconfined quarks,

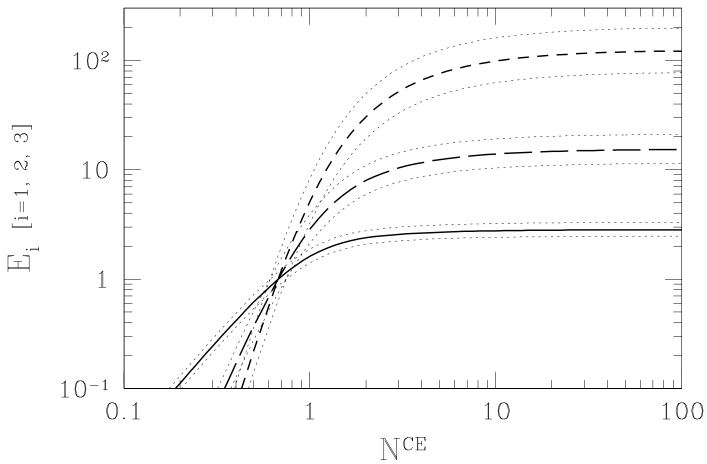

Figure 6. Canonical yield enhancement factor $E_i,\ i=1,2,3$ as function of the canonical pair particle yield N^{CE}. Solid line, E_1 the enhancement of singly flavored hadrons, relative to the yield 0.66 ± 0.07, expected in p–p reactions. Similarly, long dashed: E_2 enhancement of doubly flavored hadrons; and short-dashed line: E_3 enhancement of triply strange hadrons. Dotted lines correspond to the errors arising from the error in the strangeness yield, to which the results are normalized.

and thus such an interpretation strongly supports the quark–gluon plasma hypothesis as the source of hadrons, rather the conventional confined hadron gas source. However, we have pointed out that the chemical equilibrium hypothesis applied in this context is a very uncertain one. Specifically, we recall that in all systems studied the full chemical strangeness equilibrium has not been attained [8, 12, 13].

How is it then possible that the multistrange hadron yield has been explained? Compared to the grand canonical ensemble, we see, in Fig. 2, 'upside down' suppression/enhancement factor which depends sensitively on the choice of the (grand-canonical) yield of strange pairs $N\propto V$. Thus, with an appropriate choice of a reference point $V_{\rm h}$ and T these factors can be fine tuned as is in fact done in Ref. [2, 3], within a eyeball fit.

We try here a more refined approach. For p–p reactions at the top SPS energy the strange pair yield is known, $\langle n_{\text{s-pair}}\rangle = 0.66\pm0.07$ pairs [15]. We take the results shown in Fig. 2 and convert the ordinate to be the canonical yield, $N^{\rm CE}=NI_1(2N)/I_0(2N)$, and normalize the yields at the observed 0.66 ± 0.07 pairs, thus showing the 'canonical enhancement', $E_i,\ i=1,2,3$, with reference to the p–p collision system in Fig. 6. The errors (dotted lines) correspond to the errors in the strangeness yield, to which the results are normalized.

The single strange hadron enhancement, solid line in Fig. 6, is by factor three. By coincidence the canonical strangeness enhancement shown is of the same magnitude as expected in kinetic theory models of strangeness production. However, the canonical (equilibrium) enhancement is very rapid, as seen in Fig. 4, and expressed equivalently in Fig. 6. It is reached in a few elementary collision volumes, or equivalent when a few strange quark pairs are present. On the other hand, this yield rise will be significantly delayed if the chemical equilibrium can be attained only at 30–50 elementary volumes, where new physics comes into play. The shape of the enhancement curve as function of the volume then also shows where the gluon fusion mechanism of strangeness production sets in [11], where one would expect that the deconfinement begins, as function of reaction volume at given collision energy. The experimental results from NA52 experiment [18] shows a rather sudden strangeness enhancement threshold at $\simeq$ 50 participants, just where WA57 recently reports a sudden onset of $\overline{\Xi}$ yield enhancement [19].

Moreover, we see in Fig. 6 the long-dashed line describing the double strange (cascade) canonical enhancement which is by a factor E_2 =12–21, and the short-dashed line describing the enhancement of triply strange $\Omega, \overline{\Omega}$ by a factor E_3 =80–200, well above kinetic model expectations. The experimental enhancement results are reported by the WA97 experiment, with base obtained in p–Pb and p–Be collision system [20]. The p–Be interaction is dominated by the p–n interaction, where the valance neutron 'binding' the two alpha-nuclei is scattered from, thus these results can be used here. The reported enhancement is considerably smaller, for the $\Omega + \overline{\Omega}$, $E_3 < 20$ and for $\overline{\Xi}$ $E_2 < 8$, with a yet smaller enhancement for Ξ. We believe that the WA97 results are in disagreement with the expected large canonical multi strange hadron effects. This disagreement is already clearly visible looking at p–Pb yields which are not enhanced compared to p–Be yields. The canonical enhancement is highly sensitive to the relatively large increase in the number of participants, when these two systems are compared, the quark–gluon plasma kinetic mechanism with larger participant threshold is not sensitive, as seen in the experiment.

In passing, we address the more complex case of $\phi(\mathrm{s\bar{s}})$ enhancement. This particle has only 'hidden' strangeness, it does not follow the E_2 enhancement curve. If the ϕ production mechanism in p–p and A–A reactions are the same and involve s–$\bar{\mathrm{s}}$ pairs performed in the fireball, the enhancement of ϕ production follows the general strangeness pair enhancement. Experimentally expressed per participant the $\phi(\mathrm{s\bar{s}})$ enhancement is by factor 3.6 [21], comparing p–p with Pb–Pb.

For charm, we have also obtained a canonical enhancement well above experimental expectations. We have seen, in Fig. 5, that a large change is expected in the canonical charm yield per unit of volume (which is equivalent to yield per participant) when chemical equilibrium is subsumed. Experimental results do not show that the yield of charm is rising that fast. This implies that heavy charm quarks are not in chemical equilibrium, and their production has to be studied in kinetic theory of parton collision processes.

If attainment of chemical equilibration is seen as a fundamental process driven by an unknown 'demon' which operates within statistical hadronization, charm should not be different from strangeness. Thus, if charm is excluded from equilibrium, this means that there is indeed no 21st century Maxwell 'equilibration demon' control of charm, and by extension, also not of strangeness.

In conclusion, we argued and/or have shown that the canonical strangeness enhancement:

1) lacks internal theoretical consistency, considering both strangeness and charm;

2) it is more consistent with the observed strangeness enhancement when QGP phase space is used;

3) that its behavior as function of volume disagrees with the available experimental results;

4) that the effect for multistrange hadrons is much greater than the experimental results suggest;

5) is absent, though expected, comparing p–Be with p–Pb results.

This work has demonstrated that the chemical equilibrium canonical suppression/enhancement reinterpretation of quark–gluon plasma strange hadron signature is without scientific merit.

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