We study a Randall-Sundrum model modified by a Gauss-Bonnet interaction term. We consider, in particular, a Friedmann-Robertson-Walker metric on the brane and analyse the resulting cosmological scenario. It is shown that the usual Friedmann equations are recovered on the brane. The equation of state relating the energy density and the pressure is uniquely determined by the matching conditions. A cosmological solution with negative pressure is found.

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I. INTRODUCTION

The possibility that our universe is a four dimensional brane embedded in a higher dimensional spacetime has been extensively discussed recently. The most popular model in this context is the one proposed by Randall and Sundrum [1]. This scenario is based on the metric

$$\text{d}s^2 = A^2(y) \eta_{ij} \text{d}x^i \text{d}x^j + \text{d}y^2,$$

(1.1)

where $\eta_{ij}$ is a flat Minkowski four dimensional metric on the brane and $A^2(y)$ is the warp factor depending only on $|y|$. Perturbations of this metric reproduce the expected $1/r$ Newtonian potential on the brane (the observed universe). This is due to the fact that the zero modes of the perturbation propagate on the brane only (they tend rapidly to zero in the fifth dimension). The other modes (the massive Kaluza-Klein modes) give merely a correction in $1/r^3$ to this potential.

One of the first development of this model was the generalisation of the Randall-Sundrum ansatz to include a wider class of metrics [2–15]. Different geometries were treated by considering solutions to the Randall-Sundrum model with metrics which, up to a non-constant conformal factor, can be written as

$$\text{d}s^2 = A^2(y) g_{ij}(x,y) \text{d}x^i \text{d}x^j + \text{d}y^2.$$

(1.2)

The requirement that the zero modes of the perturbations around these metrics are localised on the brane imposes further constraints on this class of geometries [3].

Since the Randall-Sundrum model is a string inspired picture [16], one would like to understand the implications of higher curvature terms in such brane world universe. These terms naturally arise in the string effective action beyond the first order in the string tension $\alpha'$. The inclusion of these terms are also of relevance to cosmology and inflation. It turns out that generic higher curvature terms lead in general to a delocalisation of gravity from the brane [17,18]. A combination of these terms in a Gauss-Bonnet form yields, however, the desired Randall-Sundrum behaviour of the zero modes of the perturbations. We should mention that the analyses of refs. [17,18] is carried out with a brane possessing a flat metric of the form given in (1.1). In this context, various other issues were also treated using higher curvature terms [19–27].

In this letter, we deal with a Randall-Sundrum model complemented by a Gauss-Bonnet density where the five dimensional metric is of the form in (1.2). We start by considering a metric on the brane with spherical symmetry. It is shown that the only possible solution in this case is a de Sitter or anti de Sitter spacetime for the brane. This is in contrast to the case without a Gauss-Bonnet term where black hole geometries are allowed [3]. Our bulk metric involves a warp factor that presents an oscillatory regime among other possibilities.

A second study consists in taking a Friedmann-Robertson-Walker metric on the brane. We recover the equations of ordinary cosmology on the brane. This is to be compared to previous brane world cosmology models [28–48] where, among other things, the square of the Hubble parameter is found to be proportional to the square of the energy density.
Here, the matching conditions are so restrictive that they determine the equation of state relating the energy density to the pressure. Various inflationary solutions with a cosmological constant on the brane are determined. Another solution with a time dependent energy density and pressure is also found. However, the pressure for this matter is negative and cannot describe an ordinary dust. On the other hand, a scalar field is found whose energy-momentum tensor could describe this behaviour.

II. THE MODEL

We consider a five dimensional spacetime with coordinates \((x^0 \equiv t, x^1, x^2, x^3, x^4 \equiv y)\) where \((t, x^1, x^2, x^3)\) denotes the usual four-dimensional spacetime and \(x^4 \equiv y\) is the coordinate of the fifth dimension, which is an orbifold \(S^1/Z_2\) where the \(Z_2\) action identifies \(y\) and \(-y\). The five dimensional indices are denoted by \(M, N, \ldots\) and the four dimensional brane world indices are \(i, j, \ldots = 0, \ldots, 3\). We will neglect the matter interaction and consider the five dimensional gravitational action

\[
S = \int d^5x \sqrt{-G} (\alpha R + \Lambda + \beta L_{GB})
\]  

where \(\alpha\) and \(\beta\) are two coupling constants and \(\Lambda\) is the five dimensional cosmological constant. The Gauss-Bonnet Lagrangian density is

\[
L_{GB} = R_{MNPQ}R^{MNPQ} - 4R_{MN}R^{MN} + R^2.
\]

The equations of motion corresponding to our action are\(^1\)

\[
\mathcal{E}_{MN} \equiv \alpha \left(R_{MN} - \frac{1}{2}g_{MN}R\right) - \frac{1}{2}\Lambda g_{MN} + 2\beta \left(R_{MP}^{\quad PQ} R_{PQNS} + 2R_{PQ}^{\quad MMPS} R_{NPS} - 2R_{MP} R_{NP}^{\quad M} + RR_{MN} - \frac{1}{4}g_{MN} L_{GB} \right) = 0.
\]

As it is well-known, these equations possess the following solutions [49,50]

\[
R_{MNPQ} = -\sigma \left(G_{MP}G_{NQ} - G_{MQ}G_{NP} \right),
\]

where \(\sigma\) takes two possible values as given by

\[
\sigma = \frac{1}{4\beta} \left(\alpha \pm \sqrt{\alpha^2 - \frac{2}{3} \beta \Lambda}\right).
\]

This last equation can be inverted to get an expression for \(\Lambda\)

\[
\Lambda = \frac{1}{2} \sigma \left(\alpha - 2\beta \sigma\right).
\]

This value of \(\Lambda\) will be needed in the rest of the paper.

As a start, we would like to analyse the above equations of motion in the context of the brane world scenario. We consider, for this purpose, a spherically symmetric line element as given by

\[
ds^2 = A^2(y) \left(-N(r)dt^2 + M(r)dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)\right) + dy^2.
\]

and examine the equations of motion

\[
\mathcal{H}^M_N \equiv \mathcal{E}^M_N + T^M_N = 0,
\]

where the non-zero components of the energy-momentum tensor \(T^M_N\) are

\(^1\)Our conventions are such that \(R^M_{NPQ} = \partial_P\Gamma^M_{NQ} + \Gamma^M_{PR}\Gamma^R_{NQ} - (P \leftrightarrow Q)\) and \(R_{MN} = R^Q_{MQN} \).  

with \( \lambda \) denoting the cosmological constant on the brane. We obtain four different equations corresponding to the components \( \mathcal{H}_0^0, \mathcal{H}_1^1, \mathcal{H}_2^2 = \mathcal{H}_3^3 \) and \( \mathcal{H}_4^4 \). Subtracting \( \mathcal{H}_1^1 \) from \( \mathcal{H}_0^0 \) leads to

\[
M(r) = \frac{1}{N(r)}.
\]

(2.10)

Substituting for \( M(r) \) and subtracting \( \mathcal{H}_2^2 \) from \( \mathcal{H}_0^0 \) yields

\[
N(r) = 1 + \mu r^2 + \frac{\nu}{r}.
\]

(2.11)

where \( \mu \) and \( \nu \) are two constants of integration.

After substituting for \( N(r) \), the equation corresponding to \( \mathcal{H}_4^4 = 0 \) can be cast in the form

\[
12 \left( \mu + A'^2 \right) \left[ \alpha A^2 - 4\beta \left( \mu + A'^2 \right) \right] - \Lambda A^4 = \frac{12\nu^2\beta}{r^6}.
\]

(2.12)

Our notation is explained in the footnote below\(^2\). It is clear here that one must have \( \nu = 0 \). This condition, however, is not needed if \( \beta = 0 \) (see for example \([3]\)). With \( \nu = 0 \), the last equation takes then the simple form

\[
\mu + A'^2 = \sigma A^2,
\]

(2.13)

where \( \sigma \) is as previously defined. The solution to our last differential equation is

\[
A(y) = \frac{\mu}{4\gamma\sigma} \exp \left( \pm \sqrt{\sigma} |y| \right) + \gamma \exp \left( \mp \sqrt{\sigma} |y| \right).
\]

(2.14)

Here \( \gamma \) is an integration constant.

We are left with one equation to solve, namely \( \mathcal{H}_0^0 \)

\[
6\alpha A \left( \mu + A'^2 \right) + 6 \left[ \alpha A^2 - 4\beta \left( \mu + A'^2 \right) \right] \frac{d^2A}{dy^2} - \Lambda A^3 + 2\lambda A^3 \delta(y) = 0.
\]

(2.15)

This equation involves second derivatives of \( A(y) \) which generate delta functions as explained in the footnote. Using equation (2.13) in \( \mathcal{H}_0^0 \) and matching the delta functions yields the fine tuning conditions

\[
\lambda = -6 \left( \alpha - 4\beta \sigma \right) \frac{A'(0)}{A(0)}.
\]

(2.16)

Substituting then for \( A(y) \) in (2.15), fixes the five dimensional cosmological constant \( \Lambda \) to its original value \( \Lambda = 12\sigma \left( \alpha - 2\beta \sigma \right) \). Notice that the warp factor \( A(y) \) can present various behaviours depending on the value of \( \sigma \). In particular if \( \sigma \) is negative then an oscillatory regime is obtained.

### III. COSMOLOGICAL SOLUTIONS

The metric for this study is taken to have the form

\[
ds^2 = A^2(y) \left\{ -dt^2 + a(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right] \right\} + dy^2.
\]

(3.1)

The non-vanishing components of the energy-momentum tensor \( T_{ij}^{M} \) are now given by

\(^2\)Since \( A \) is a function of \( |y| \) we have \( \frac{dA}{dy} = A' \frac{d|y|}{dy} \) where \( A' \) denotes the derivative of \( A \) with respect to its argument \( |y| \) and \( \frac{d|y|}{dy} = 2\Theta(y) - 1 \) where \( \Theta(y) \) is the Heaviside function. Notice that \( \left( \frac{dA}{dy} \right)^2 = 1 \) and we have \( \left( \frac{dA}{dy} \right)^2 = A'^2 \). On the other hand \( \frac{d^2A}{dy^2} = A'' + 2A' \delta(y) \) where \( A'' \) denotes the second derivative of \( A \) with respect to its argument \( |y| \).
\[ \begin{align*}
T_0 &= -\rho (t) \delta (y) \\
T_1 &= T_2 = T_3 = p (t) \delta (y).
\end{align*} \] (3.2)

The equations of motion we would like to solve are still \( \mathcal{H}_N^M = \mathcal{E}_N^M + T_N^M = 0 \). There are three different types of equations \( \mathcal{H}_0^0, \mathcal{H}_1^1 = \mathcal{H}_2^2 = \mathcal{H}_3^3 \) and \( \mathcal{H}_4^4 \). The first of these, \( \mathcal{H}_0^0 \), is

\[ 6 \left[ a^2 \left( \alpha A^2 - 4 \beta A'^2 \right) + 4 \beta \left( k + \dot{a}^2 \right) \right] \frac{d^2 A}{dy^2} - 6 \alpha A \left( k + \dot{a}^2 \right) + 6 \alpha a^2 AA'^2 - \Lambda A^3 a^2 - 2 p a^2 A'^3 \delta (y) = 0, \] \] (3.3)

where \( \dot{a} \) is the derivative of \( a \) with respect to \( t \). Matching the delta functions in this last equation yields

\[ \rho = \frac{6 A' (0)}{a^2 [A (0)]^3} \left[ a^2 \left( \alpha [A (0)]^2 - 4 \beta [A' (0)]^2 \right) + 4 \beta \left( k + \dot{a}^2 \right) \right]. \] \] (3.4)

The second equation, \( \mathcal{H}_1^1 \), is given by the expression

\[ 2 \left[ 3 a^2 \left( \alpha A^2 - 4 \beta A'^2 \right) + 8 \beta a \ddot{a} + 4 \beta \left( k + \dot{a}^2 \right) \right] \frac{d^2 A}{dy^2} - 2 \alpha A \left( k + \dot{a}^2 \right) + 6 \alpha a^2 AA'^2 - \Lambda a^2 A^3 - 4 \alpha a \ddot{a}A + 2 p a^2 A'^3 \delta (y) = 0. \] \] (3.5)

Again, matching the delta functions coming from \( \frac{d^2 A}{dy^2} \) and those coming from the energy-momentum tensor gives

\[ p = -\frac{2 A' (0)}{a^2 [A (0)]^3} \left[ 3 a^2 \left( \alpha [A (0)]^2 - 4 \beta [A' (0)]^2 \right) + 8 \beta a \ddot{a} + 4 \beta \left( k + \dot{a}^2 \right) \right]. \] \] (3.6)

Using the expression of \( \rho \) we deduce the following expression for the Hubble parameter

\[ H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{[A (0)]^3}{24 \beta A' (0)} \rho - \frac{k}{a^2} + \left( -\frac{\alpha [A (0)]^2}{4 \beta} + [A' (0)]^2 \right). \] \] (3.7)

This is the first Friedmann relation of ordinary cosmology. Similarly, combining the expression of \( \rho \) with that corresponding to \( p \), yields the second Friedmann equation

\[ \ddot{a} = -\frac{[A (0)]^3}{48 \beta A' (0)} (3 \rho + p) a + \left( -\frac{\alpha [A (0)]^2}{4 \beta} + [A' (0)]^2 \right) a. \] \] (3.8)

Differentiating \( H^2 \) and using the expression of \( \dot{a} \) results in the usual conservation equations

\[ \dot{\rho} a + 3 (p + \rho) \dot{a} = 0. \] \] (3.9)

Therefore, our gravitational theory with the Gauss-Bonnet term leads to ordinary Friedmann equations.

Once the matching is carried out, we can deal with the two equations \( \mathcal{H}_1^1 \) and \( \mathcal{H}_0^0 \) away from \( y = 0 \). Substracting \( \mathcal{H}_1^1 \) from \( \mathcal{H}_0^0 \) gives then

\[ (k + \dot{a}^2 - a \ddot{a}) (\alpha A - 4 \beta A'') = 0. \] \] (3.10)

If the first factor \( (k + \dot{a}^2 - a \ddot{a}) \) vanishes then an interesting solution is given by

\[ a (t) = \frac{k}{4 \kappa \tau^2} \exp (\pm \tau t) + \kappa \exp (\mp \tau t), \] \] (3.11)

where \( \tau \) and \( \kappa \) are two integration constants.

Upon substituting for \( a (t) \) in \( \mathcal{H}_4^4 \), we obtain

\[ 12 \left( A'^2 - \tau^2 \right) [\alpha A^2 - 2 \beta \left( A'^2 - \tau^2 \right)] - \Lambda A^4 = 0. \] \] (3.12)

This equation is exactly the one found in (2.12) with \( \nu = 0 \) and where \( \mu \) is replaced by \(-\tau^2\). The solution, \( A (y) \), to this equation is therefore as given in (2.14) upon replacing \( \mu \) by \(-\tau^2\). Putting the expression of \( a (t) \) in \( \mathcal{H}_0^0 \) leads to the differential equation
Again, this equation is that found in (2.15) away from the position of the brane and where \( \mu \) is replaced by \(-\tau^2\). Therefore, replacing for \( A(y) \) in this last differential equation fixes the bulk cosmological constant to be \( \Lambda = 12\sigma (\alpha - 2/3\beta) \).

We should mention that this solution leads to the following relation between the energy density \( \rho \) and the pressure \( p \)

\[
p = -\rho = \lambda
\]  

where \( \lambda \) is as given in (2.16). The Hubble parameter for this solution is given by

\[
H^2 = -\frac{k}{a^2} + \tau^2
\]

with \( a(t) \) as given in (3.11). Similarly, we find that

\[
\frac{\ddot{a}}{a} = \tau^2
\]  

We distinguish, therefore, two cases. The first corresponds to \( \tau^2 > 0 \) and leads to an inflationary regime whenever one of the two exponentials in \( a(t) \) dominates. The second situation arises when \( \tau^2 < 0 \). In this case the scale factor is given by

\[
a(t) = \epsilon \cos \left( \frac{\sqrt{-k}}{\varepsilon} t + \varphi \right),
\]

where \( \epsilon \) and \( \varphi \) are two real integration constants. The corresponding \( A(y) \) is given by equation (2.14) where \( \mu \) is replaced by \(-k/\varepsilon^2\). Of course, this solution is valid only when \( k \) is negative and describes a repeatedly collapsing universe. The Hubble parameter in this case is \( H^2 = -k/a^2 + k/\varepsilon^2 \).

The other solution to \( (k + \ddot{a}^2 - a\ddot{a}) = 0 \) is given by

\[
a(t) = \pm \sqrt{-kt} + \delta
\]  

with \( \delta \) an integration constants. This solution is physical only for \( k \) negative. Substituting this expression of \( a(t) \) in \( \mathcal{H}_4^1 \) leads to the differential equation

\[
A'^2 = \sigma A^2,
\]

where \( \sigma \) is as defined in (2.5). Therefore \( A(y) \) is given by

\[
A(y) = \psi \exp \left( \sqrt{\sigma} |y| \right) \quad \text{or} \quad A(y) = \omega \exp \left( -\sqrt{\sigma} |y| \right)
\]

for two integration constants \( \psi \) and \( \omega \). The solution in (3.18) and (3.20) automatically satisfies \( \mathcal{H}_4^0 \). Furthermore, we have \( p = -\rho = \lambda \) and the Hubble parameter is \( H^2 = -k/a^2 \).

Let us now return to the second possibility as allowed by equation (3.10), namely when \( (\alpha A - 4\beta A') = 0 \). This case is of course present only when \( \beta \) is different from zero and we have the solution

\[
A(y) = \xi \exp \left( \frac{1}{2} \sqrt{\frac{\alpha}{\beta}} |y| \right) + \theta \exp \left( -\frac{1}{2} \sqrt{\frac{\alpha}{\beta}} |y| \right),
\]

with \( \xi \) and \( \theta \) are two integration constants. Substituting this solution in \( \mathcal{H}_4^1 \) leads to

\[
a^3 \left( 3\alpha^2 - 2\beta \Lambda \right) \left\{ \left[ \xi \exp \left( \frac{1}{2} \sqrt{\frac{\alpha}{\beta}} |y| \right) \right] + \theta \exp \left( -\frac{1}{2} \sqrt{\frac{\alpha}{\beta}} |y| \right) \right\}^4 - 6\xi^2 \theta^2
\]

\[-48\ddot{u} \left[ \beta^2 (k + \ddot{a}^2) + \alpha \beta \xi \theta a^2 \right] - 6\xi \theta a \left[ 8\alpha \beta (k + \ddot{a}^2) + \xi \theta a^2 (5\alpha^2 + 2\beta \Lambda) \right] = 0.
\]

It is clear, from the separation of variables, that one must have

\[
\Lambda = \frac{3\alpha^2}{2\beta}
\]  

(3.23)
and the above differential equation reduces then to
\[ [\beta (k + \dot{a}^2) + \alpha \xi \theta a^2] (\beta \ddot{a} + \alpha \xi \theta a) = 0 \]  
(3.24)

Regardless of which factor vanishes first, the solution to this last differential equation takes the form
\[ a(t) = \zeta \exp \left( -\sqrt{-\frac{\alpha \xi \theta}{\beta}} t \right) + \chi \exp \left( -\sqrt{\frac{\alpha \xi \theta}{\beta}} t \right) \]  
(3.25)

where \( \zeta \) and \( \chi \) are two integration constants and if the first factor of the differential equation vanishes then
\[ \beta k + 4 \alpha \xi \theta \zeta \chi = 0 \]  
(3.26)

With \( \Lambda = \frac{(3\alpha^2)}{(2\beta)} \), the equation corresponding to \( H_0^2 \) yields
\[ \left( \alpha A - 4\beta A'' \right) \left[ a^2 \left( \alpha A^2 - 4\beta A'^2 \right) + 4\beta (k + \dot{a}^2) \right] = 0 \]  
(3.27)

and is therefore automatically satisfied.

If the first factor in (3.24) vanishes then we have
\[ p = -\rho = 0 \]  
(3.28)

The Hubble parameter for this case is
\[ H^2 = -\frac{k}{a^2} - \frac{\alpha \xi \theta}{\beta} \]  
(3.29)

On the other hand if the second factor in (3.24) vanishes then we have the equation of state
\[ p = -\frac{1}{3} \rho = -8 \frac{A'(0)}{[A(0)]^3} \frac{1}{a^2} (\beta k + 4 \alpha \xi \theta \zeta \chi) \]  
(3.30)

where \( A(y) \) is given by (3.21). The corresponding expression for the Hubble parameter is
\[ H^2 = -\frac{\alpha \xi \theta}{\beta} \left( 1 - \frac{4 \zeta \chi}{a^2} \right) \]  
(3.31)

with \( a(t) \) as given in (3.25). In both cases we have
\[ \frac{\ddot{a}}{a} = -\frac{\alpha \xi \theta}{\beta} \]  
(3.32)

We notice that if \( -\alpha \xi \theta / \beta \) is positive then the universe is accelerating otherwise \( a(t) \) oscillates in time.

**IV. DISCUSSION**

Among the solutions found in this analyses, special attention should be paid to that described by a time dependent energy density and pressure as in (3.30). However, the equations of states \( p = -\rho/3 \) cannot correspond to an ordinary dust as the pressure \( p \) is negative. Let us suppose that the only matter present on the brane is a self interacting scalar field with energy-momentum tensor
\[ T_{ij} = \left[ \partial_i \phi \partial_j \phi - \frac{1}{2} g_{ij} \left( \partial^k \phi \partial_k \phi + V(\phi) \right) \right] \delta(y) \]  
(4.1)

where \( g_{ij} \) is a Friedmann-Robertson-Walker metric on the brane. If the field \( \phi \) depends only on time then one has
\[ \rho = -T_0^0 = \frac{1}{2} \left[ \dot{\phi}^2 + V \right] \delta(y) \]
\[ p = T_1^1 = T_2^2 = T_3^3 = \frac{1}{2} \left[ \dot{\phi}^2 - V \right] \delta(y) \]  
(4.2)
The field $\phi$ is subject to the equations of motion

$$\frac{1}{\sqrt{-g}} \partial_i (\sqrt{-g} \partial^i \phi) - \frac{1}{2} V' = 0 \quad (4.3)$$

with $V'$ being the derivative of the potential $V$ with respect to $\phi$.

The equation of state $p = -\rho/3$ leads to

$$\dot{\phi}^2 = \frac{1}{2} V \quad \Rightarrow \quad V' = 4 \ddot{\phi} \quad (4.4)$$

while the equation of motion of the scalar field yields

$$\ddot{\phi} + 3 \frac{\dot{a}}{a} \dot{\phi} = -\frac{1}{2} V' \quad (4.5)$$

Combining these last two equations results in

$$\ddot{\phi} + \frac{\dot{a}}{a} \dot{\phi} = 0 \quad . \quad (4.6)$$

Therefore $\dot{\phi} = \upsilon/a$, for some integration constant $\upsilon$. Using the expression of $a(t)$ as given in (3.25), we find that

$$\phi(t) = \frac{\upsilon}{\sqrt{\alpha \xi \theta \beta}} \arctan \left[ \sqrt{\frac{\xi}{\chi}} \exp \left( \sqrt{-\frac{\alpha \xi \theta}{\beta}} t \right) \right] + \varpi \quad , \quad (4.7)$$

where $\varpi$ is an integration constant. In order to be able to express the potential $V$ as a function of $\phi$, it is convenient to rewrite this last equation in the form

$$\exp \left( \sqrt{-\frac{\alpha \xi \theta}{\beta}} t \right) = \sqrt{\frac{\chi}{\xi}} \tan \left[ \frac{1}{\upsilon} \sqrt{-\frac{\alpha \xi \theta \xi \chi}{\beta}} (\phi - \varpi) \right] \quad . \quad (4.8)$$

Since the potential is given by $V = 2\dot{\phi}^2 = 2\upsilon^2/a^2$, we find that

$$V(\phi) = \frac{\upsilon^2}{2\xi \chi} \sin^2 \left[ \frac{2}{\upsilon} \sqrt{-\frac{\alpha \xi \theta \xi \chi}{\beta}} (\phi - \varpi) \right] \quad . \quad (4.9)$$

This potential has an infinite number of minima.

As it is known, cosmological scenarios with negative pressure are used in explaining the current acceleration of our universe (see for example [51]). The scalar field providing this pressure is known as quintessence [52]. Similar models are also constructed in the context of brane world cosmology [53–56]. It seems that we have found here another model for quintessence where the scalar field is governed by a very simple potential.

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