ON EXCEPTIONAL NON-RENORMALIZATION PROPERTIES OF $\mathcal{N} = 4$ SYM$_4$

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Abstract

We discuss non-renormalization properties of some composite operators in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory.

Recently considerable attention was attracted to the $\mathcal{N} = 4$ super Yang-Mills theory basically due to the prominent role it plays among the models realizing the holographic AdS/CFT duality [1]. In the superconformal phase the dynamics of the gauge theory is encoded in the correlation functions of the composite gauge invariant operators, which might exhibit in general a non-trivial behavior under the RG flow. In particular it is of great interest to determine the 4-point correlation functions (both holographic and weak coupling) of the $\mathcal{N} = 4$ supercurrent (stress-tensor) multiplet $L$ and its OPE; the latter contains an information about many other composite operators present in the theory.

A superconformal primary operator generating $L$ is a scalar $O^I$ of dimension 2 transforming in the irrep 20 of the $R$-symmetry group $SU(4)$, $I = 1, \ldots, 20$. Presently both the holographic [2] and the weak coupling [3] 4-point correlators of $O^I$ and their OPE studies [4, 5, 6] are available.\footnote{For studies of other correlation functions from stress-tensor multiplet see e.g. [7].} Surprisingly composite operators\footnote{They saturate the bound of the so-called series A) of unitary irreps of $SU(2, 2|4)$ and transform non-trivially under $R$-symmetry [8].} with vanishing anomalous dimensions were found [4] though naively unitarity allows the latter to appear in quantum interacting theory.

This note is based on the paper [6] and reviews a statement that the OPE of two primary operators from the multiplet $L$ can contain superconformal
primary operators with a non-vanishing anomalous dimension only in the singlet of SU(4).

It was found non-perturbatively [9] that the “quantum” part of the four-point function of \( O^I \) comprising all possible quantum corrections to the free-field result is given by a single function \( F(v, u) \) of conformal cross-ratios, which we choose to be \( v = \frac{x_1^2 x_2^2}{x_1^2 x_3^2 x_2^2 x_3^2} \) and \( u = 1 - \frac{x_1^2 x_2^2}{x_1^2 x_2^2 x_3^2 x_4^2} \). Under SU(4) the product of two \( O^I \) decomposes as \( 20 \times 20 = 1 + 20 + 105 + 84 + 15 + 175 \). The “quantum” part of the four-point function of the operators \( O^I \) projected on different irreps is

\[
\langle O(x_1)O(x_2)O(x_3)O(x_4) \rangle_i = \frac{1}{x_{12}^4 x_{34}^4} P_i(v, u) \frac{v F(v, u)}{(1 - u)^2},
\]

where \( P_i(v, u) \) are certain polynomials [4, 5]. Every irrep \( i \) of SU(4) in the OPE of two \( O^I \) represents a contribution from an infinite tower of operators \( O^i_{\Delta, l} \), where \( \Delta \) is the conformal dimension of the operator, \( l \) is its Lorentz spin. The corresponding contribution to the four-point function can then be represented as an expansion of the type

\[
\langle O(x_1)O(x_2)O(x_3)O(x_4) \rangle_i = \sum_{\Delta, l} a_{\Delta, l}^i \mathcal{H}_{\Delta, l}(x_{1,2,3,4}).
\]

Here \( \mathcal{H}_{\Delta, l}(x_{1,2,3,4}) \) denotes the (canonically normalized) Conformal Partial Wave Amplitude (CPWA) for the exchange of an operator \( O^i_{\Delta, l} \) and \( a_{\Delta, l}^i \) is a normalization constant. We treat the CPWA as a double series of the type

\[
\mathcal{H}_{\Delta, l} = \frac{1}{x_{12}^4 x_{34}^4} v^\frac{\Delta}{2} \sum_{n, m=0}^{\infty} c_{nm}^{\Delta, l} u^n v^m,
\]

where the dimension \( \Delta \) was split into a canonical part \( \Delta_0 \) and an anomalous part \( h: \Delta = \Delta_0 + h \). Assigning the grading parameter \( T = 2n + m \) to the monomial \( v^n u^m \) one can show that the monomials in (3) with the lowest value of \( T \) have \( T = \Delta_0 \), where \( \Delta_0 \) is the canonical (free-field) dimension of the corresponding operator.

Comparing (1) and (2) one finds, within every fractional power \( v^\frac{h}{2} \), the following compatibility conditions

\[
P_i \sum_{\Delta, l} a_{\Delta, l}^j \mathcal{H}_{\Delta, l}(x_{1,2,3,4}) = P_j \sum_{\Delta, l} a_{\Delta, l}^i \mathcal{H}_{\Delta, l}(x_{1,2,3,4})
\]

which hold for all pairs. Here the sums are taken over operators which have the same \( h \). Thus, eqs. (4) imply non-trivial relations between the CPWAs.
of primary operators belonging to the same supersymmetry multiplet(s) with anomalous dimension $h$. Only one of these primary operators is the superconformal primary operator, i.e., it generates under supersymmetry the whole multiplet, while the others are its descendents.

Now we see that a superconformal primary operator appears only in the singlet of $SU(4)$. Indeed, let us choose in (4) the irrep $j$ to be the singlet. The polynomial $P_1$ is distinguished from the other $P_i$’s by the presence of a constant term. Suppose that a superconformal primary operator with a canonical dimension $\Delta_0$ contributes to the OPE and transforms in some irrep $i$ which is not a singlet. Due to the constant in $P_1$, the lowest-order monomials on the r.h.s. of (4) would have $T = 2n + m = \Delta_0$. Clearly, all the other $P_i$’s always raise the $T$-grading by at least unity. The lowest dimension operator with canonical dimension $\Delta'_0$ in the singlet would have the lowest terms with at least $T = \Delta_0 - 1$ (or lower) to saturate (4). Hence, $\Delta'_0$ is always lower then $\Delta_0$, and therefore the corresponding operator cannot be a supersymmetry descendent of an operator in the irrep $i$. This shows that anomalous superconformal primary operators are occure in the singlet of the $R$-symmetry group.

References


