Simple Brane World Scenario with Positive Five Dimensional Cosmological Constant

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Abstract

We present a simple brane-world model in five dimensions. In this model we do not need any fine-tuning between the five dimensional cosmological constant and the brane tension to obtain four dimensional flat Minkowski space. The space-time of our solution has no naked singularities. Further the compactification scale of the fifth direction is automatically determined.

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1 Introduction

The idea we are living on a brane in a higher dimensional bulk has been considered since early 1980’s [1, 2]. After the discovery of D-brane in string theories on which some Yang-Mills fields and matter fields are localized, brane world scenario has been intensely discussed. Among various models, Randall and Sundrum gave simple ones in a five dimensional space-time with a negative bulk cosmological constant [3, 4]. In paper [3] they realized a large hierarchy using exponential warp factor, while in [4], they realized four dimensional gravity on a brane even in an infinitely large extra dimension. Moreover brane world scenario opens the possibility to approach the cosmological constant problem. Higher-dimensional approach to this problem was considered in paper [5]. Recently the authors of [6, 7] showed flat four dimensional space time can be realized regardless of the Standard Model contribution to the cosmological constant and this mechanism is sometimes called self-tuning mechanism. These models have changed dramatically the commonly assumed properties of Kaluza-Klein models.

There are much interesting points in brane world scenario, but some difficulties still remain. In the original Randall-Sundrum models [3, 4] brane tensions must be fine-tuned in order to get a flat four dimensional space-time. Moreover, in [3] the extra dimension is compactified by $S_1/Z_2$, but the size of the compactification scale is not determined by the dynamics of the model and we need some extra mechanism such as so-called Goldberger-Wise mechanism [8]. On the other hand, in the self-tuning mechanism [6, 7] we need not fine-tune on the brane tension. However it is usually difficult to avoid the appearance of naked singularities in the universe [9]. Some warped compactification models in six dimensions succeeded to avoid naked singularities [10, 11].

In this paper we consider a negative tension brane in a five dimensional bulk with a free scalar field. The cosmological constant in the bulk is positive. We show that it is possible to realize the four dimensional Minkowski space without fine tuning. Moreover there is no naked singularities in the metric.

2 The Model

We consider five dimensional space time with positive cosmological constant. We introduce a free scalar and a brane whose tension is $\sigma$. The action is

$$S = \int d^5x\sqrt{-g}\left(\frac{1}{2}R - \Lambda - \partial\phi \cdot \partial\phi\right) + \int d^5x\delta(r)\sqrt{-g_4}(-\sigma), \quad (1)$$
where $\Lambda$ is positive. This set up in absence of a brane has been considered in paper [12] and the authors discussed an oscillating metric.

We assume the metric has the following form,

$$ds^2 = e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu + dr^2,$$

and $\phi$ depends on $r$ only. Then equations of motion are

$$\left(\sqrt{-g}\phi'\right)' = 0,$$

$$A'' = \frac{2}{3}\phi^2 - \frac{2}{3}\sigma\delta(r),$$

$$A'^2 = \frac{1}{6}\phi^2 - \frac{1}{3}\Lambda,$$

where $'$ denotes the derivative of $r$. The solution in the bulk is

$$A(r) = \frac{1}{4}\ln\left[\frac{c_{\pm}}{\sqrt{2\Lambda}}\cos(-4\sqrt{\frac{\Lambda}{3}}(r - a_{\pm}))\right],$$

$$\phi'(r) = c_{\pm}\sqrt{2\Lambda}e^{-4A(r)}.$$  

There are integration parameters $a_{\pm}$ and $c_{\pm}$, where $\pm$ labels independent constants in two regions $r < 0$ and $r > 0$ for each. Those parameters should be partly determined by junction conditions at $r = 0$. The junction condition for $A$ can be obtained from eq.(4).

$$A'(+\epsilon) - A'(-\epsilon) = -\frac{2}{3}\sigma.$$  

There is still another condition for $\phi$, which says $\phi'(0)$ must be continuous. We can see these conditions allow only an $Z_2$ symmetric solution with the fixed point $r = 0$; $a_- = -a_+ = a$, and $c_- = c_+ = c$. Without loss of generality we can set $a < 0$. Eq.(8) leads to

$$A'(r)|_{-\epsilon} = \sqrt{\frac{\Lambda}{3}}\tan(4\sqrt{\frac{\Lambda}{3}}a) = \frac{\sigma}{3}, \quad (-\frac{\pi}{2} < 4\sqrt{\frac{\Lambda}{3}}a < 0).$$

It is easy to see from above that $\sigma$ must be negative. The bulk solution has another $Z_2$ symmetry at $r = \pm a$, so now we identify both points $r = a$ and $r = -a$. Here we get the extra dimension compactified as $S_1/Z_2$. Thus for any negative brane tension the above condition can be satisfied and $a$ is determined. Therefore we have obtained the regular solution (see figure 1) which realizes a four dimensional Minkowski space without any fine tuning.

Now we perform naive order analysis. When the absolute value of brane tension is small or equal to $\Lambda$, we take this in the following analysis, from eq. (9) we can obtain rough relation

$$a \sim \frac{\sigma}{\Lambda M_*^3},$$

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where $1/M^2_*$ is the five dimensional Newton constant. The four dimensional Planck constant $M_p$ is obtained by the integration of fifth dimension in the action (1) as follows,

$$M_p^2 \sim M^3_*|a| \sim \sigma/\Lambda. \quad (11)$$

Thus when we take

$$\Lambda^{1/2} \sim M_* \sim |\sigma|^{1/4} \sim 10^{18}\text{GeV}, \quad (12)$$

the Plank constant can be correctly obtained as $M_p \sim 10^{18}\text{ GeV}$. So every physics is similar to the flat $S_1/Z_2$ compactification.

![Figure 1: Figures for $A(r)$ and $\phi'(r)$. $r$ direction is compactified by $S_1/Z_2 \sim R/(Z_2 \times Z_2)$. $Z_2$ symmetries are imposed at $r = a$ and at $r = 0$. $A(r)$ and $\phi'(r)$ are even under these $Z_2$ actions. The brane sits at $r = 0$ and its image points.](image)

**3 Discussion**

We have constructed a simple brane model in five dimensions with a positive bulk cosmological constant. The fifth direction is compactified by $S_1/Z_2$ and its radius is determined by the brane tension which must be negative. However we do not need to relate the brane tension with five dimensional cosmological constant; no fine tuning is needed. Then it gives one solution for the cosmological constant problem.

The fixed radius of the extra dimension implies that our model has no massless radion mode. Five dimensional brane world scenario has fifth dimensional diffeomorphism $r \to r + \xi(x^\mu, r)$ in the bulk. With two branes, just like in Randall-Sundrum (RS) model [3], $\xi(x^\mu, r)$ is not just a gauge parameter anymore, but includes a physical mode; a distortion...
of relative position between the two branes. In RS model, this mode is massless and called radion. It is a reflection that the size of the proper distance between the two branes is a modulus in RS model. On the other hand in our model, the background figure 1 manifestly has no such modulus, and we can conclude there is no massless radion in our model.

One comment is in order; in the self-tuning mechanism [6, 7, 9] flat four dimensional space-time can be realized whatever the brane tension is. In such scenario, there is a serious problem that it is difficult to avoid the appearance of singularity/ies within a finite length from the brane. However, our model does not suffer from this problem i.e. the metric is regular everywhere because there is a brane in the middle of the bulk before reaching a would-be singularity, and the brane now becomes the fixed point of $Z_2$ orbifold.

It is interesting to investigate perturbations of our model from many point of view, especially to confirm the above naive discussion about a radion mode, or to discuss the stability of our model. Although this investigation seems complicated, it’s being achieved elsewhere.

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