Probing MSSM Higgs Sector CP Violation at a Photon Collider *

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Abstract

We study the phenomenological implications of the Higgs sector CP violation at a photon collider. In our model, the CP violation is radiatively induced by the non–trivial CP phases of the third–generation scalar–quark sector in the MSSM. We re–evaluate the $s$–channel resonance production cross sections and the polarization asymmetries of the neutral Higgs bosons based on the calculation of the mass matrix of the neutral Higgs bosons which is valid for any values of the relevant SUSY parameters. The CP properties of the Higgs bosons can be precisely probed through their $s$–channel resonance productions at a photon linear collider by exploiting circularly and/or linearly polarized backscattered laser photons.

I. INTRODUCTION

The soft CP–violating Yukawa interactions in the minimal supersymmetric standard model (MSSM) cause the CP–even and CP–odd neutral Higgs bosons to mix via loop corrections [1–5]. Although the mixing is a radiative effect, the induced CP violation in the MSSM Higgs sector can be large enough to affect the Higgs phenomenology significantly at present and future colliders [1,3,5–13].

In the light of the possible large CP–violating mixing, we study the effects of the CP–violating mixing on the $s$–channel resonance production cross sections and the polarization asymmetries of the neutral Higgs bosons at a photon collider. We studied the effects of the CP–violating mixing at a photon collider via their $s$–channel resonance productions in Ref. [10] based on the mass matrix derived by Pilaftsis and Wagner [3]. The mass matrix, however, is not applicable for large squark mass splitting. In this work, we re–evaluate the production cross sections and the polarization asymmetries with the mass matrix [4]

which is valid for any values of the soft–breaking parameters. Our study clearly shows that the CP properties of Higgs bosons can be precisely probed through their s–channel resonance productions via photon–photon collisions by use of circularly and/or linearly polarized backscattered laser photons at a TeV–scale linear $e^+e^-$ collider.

This paper is organized as follows. In Sec. II we give a brief review of the calculation [4] of the loop–induced CP–violating mass matrix of the three neutral Higgs bosons. In Sec. III, we investigate in detail the dependence of the total production rates and the three polarization asymmetries on the CP–violating phases. Finally, we summarize our findings in Sec. IV.

II. CP VIOLATION IN THE MSSM HIGGS SECTOR

The loop–corrected mass matrix of the neutral Higgs bosons in the MSSM can be calculated from the effective potential [14,15]

$$V_{\text{Higgs}} = \frac{1}{2} m_1^2 (\phi_1^2 + a_1^2) + \frac{1}{2} m_2^2 (\phi_2^2 + a_2^2) - |m_{12}| (\phi_1 \phi_2 - a_1 a_2) \cos(\xi + \theta_{12})$$

$$+ |m_{12}| (\phi_1 a_2 + \phi_2 a_1) \sin(\xi + \theta_{12}) + \frac{g^2}{8} D^2 + \frac{1}{64\pi^2} \text{Str} \left[ M^4 \left( \log \frac{M^2}{Q^2} - \frac{3}{2} \right) \right],$$

(1)

with $D = \phi_2^2 + a_2^2 - \phi_1^2 - a_1^2$, $\tilde{g}^2 = (g^2 + g'^2)/4$, and $\phi_i$ and $a_i$ ($i = 1, 2$) are the real fields of the neutral components of the two Higgs doublets:

$$H_i^0 = \frac{1}{\sqrt{2}} (\phi_i + i a_i), \quad H_2^0 = \frac{e^{i\xi}}{\sqrt{2}} (\phi_2 + i a_2).$$

The parameters $g$ and $g'$ are the SU(2)$_L$ and U(1)$_Y$ gauge couplings, respectively, and $Q$ denotes the renormalization scale. All the tree–level parameters of the effective potential (1) such as $m_1^2$, $m_2^2$ and $m_{12}^2 = |m_{12}| e^{i\theta_{12}}$, are the running parameters evaluated at the scale $Q$. The potential (1) is then almost independent of $Q$ up to two–loop–order corrections. The super–trace is to be taken over all the bosons and fermions that couple to the Higgs fields.

The matrix $M$ in Eq. (1) is the field–dependent mass matrix of all modes that couple to the Higgs bosons. The dominant contributions in the MSSM come from third generation quarks and squarks because of their large Yukawa couplings. The field–dependent masses of the third generation quarks are given by

$$m_b^2 = \frac{1}{2} |h_b|^2 (\phi_1^2 + a_1^2), \quad m_t^2 = \frac{1}{2} |h_t|^2 (\phi_2^2 + a_2^2),$$

(3)

where $h_b$ and $h_t$ are the bottom and top Yukawa couplings, respectively. The corresponding squark mass matrices read:

$$M_i^2 = \begin{pmatrix} m_Q^2 + m_t^2 - \frac{1}{8} (g^2 - \tilde{g}^2) D & -h_t \left[ A_i (H_2^0)^* + \mu H_1^0 \right] \\ -h_t \left[ A_i H_2^0 + \mu^* (H_1^0)^* \right] & m_U^2 + m_i^2 - \frac{g^2}{6} D \end{pmatrix},$$

$$M_b^2 = \begin{pmatrix} m_Q^2 + m_b^2 + \frac{1}{8} (g^2 + \tilde{g}^2) D & -h_b \left[ A_b (H_1^0)^* + \mu H_2^0 \right] \\ -h_b \left[ A_b H_1^0 + \mu^* (H_2^0)^* \right] & m_D^2 + m_b^2 + \frac{g^2}{12} D \end{pmatrix},$$

(4)
where $m_{Q}^2$, $m_{U}^2$ and $m_{D}^2$ are the real soft SUSY-breaking squark-mass parameters, $A_t$ and $A_b$ are the complex soft SUSY-breaking trilinear parameters, and $\mu$ is the complex supersymmetric Higgsino mass parameter.

The mass matrix of the Higgs bosons (at vanishing external momenta) is then given by the second derivatives of the potential evaluated at its minimum point

$$
(\phi_1, \phi_2, a_1, a_2) = (\langle \phi_1 \rangle, \langle \phi_2 \rangle, \langle a_1 \rangle, \langle a_2 \rangle) = (v \cos \beta, v \sin \beta, 0, 0),
$$

where $v = (\sqrt{2} G_F)^{-1/2} \simeq 246$ GeV. The massless state $G^0 = a_1 \cos \beta - a_2 \sin \beta$ is the would-be-Goldstone mode to be absorbed by the $Z$ boson. We are thus left with a mass-squared matrix $\mathcal{M}_H^2$ for three physical states, $a (= a_1 \sin \beta + a_2 \cos \beta)$, $\phi_1$ and $\phi_2$. The mass matrix is real and symmetric, i.e. it has 6 independent entries. The diagonal entry for the pseudoscalar component $a$ reads:

$$
\mathcal{M}_H^2|_{aa} = m_A^2 + \frac{3}{8\pi^2} \left\{ \frac{m_t^2 m_i^2}{\sin^2 \beta} g(m_i^2, m_i^2) \Delta_i^2 + \frac{|h_b|^2 m_b^2}{\cos^2 \beta} g(m_b^2, m_b^2) \Delta_b^2 \right\},
$$

where $m_A$ is the loop-corrected pseudoscalar mass in the CP invariant theories. The CP-violating entries of the mass matrix, which mix $a$ with $\phi_1$ and $\phi_2$, are given by

$$
\mathcal{M}_H^2|_{a\phi_1} = \frac{3}{16\pi^2} \left\{ \frac{m_t^2 \Delta_i}{\sin \beta} \left[ g(m_i^2, m_i^2) \left( X_i \cot \beta - 2 |h_t| R_i \right) - \hat{g}^2 \cot \beta \log \frac{m_i^2}{m_t^2} \right] + \frac{m_b^2 \Delta_b}{\cos \beta} \left[ -g(m_b^2, m_b^2) \left( X_b + 2 |h_b| R_b \right) + \left( \hat{g}^2 - 2 |h_b|^2 \right) \log \frac{m_b^2}{m_i^2} \right] \right\},
$$

$$
\mathcal{M}_H^2|_{a\phi_2} = \frac{3}{16\pi^2} \left\{ \frac{m_t^2 \Delta_i}{\sin \beta} \left[ -g(m_i^2, m_i^2) \left( X_i + 2 |h_t|^2 R_i \right) + \left( \hat{g}^2 - 2 |h_t|^2 \right) \log \frac{m_i^2}{m_t^2} \right] + \frac{m_b^2 \Delta_b}{\cos \beta} \left[ g(m_b^2, m_b^2) \left( X_b \tan \beta - 2 |h_b|^2 R_b \right) - \hat{g}^2 \tan \beta \log \frac{m_b^2}{m_i^2} \right] \right\},
$$

where $g(x, y) = 2 - [(x + y)/(x - y)] \log(x/y)$. The size of these CP-violating entries is determined by the re-phasing invariant quantities

$$
\Delta_i = \frac{3 m(A_t \mu e^{i \xi})}{m_{t2}^2 - m_{t1}^2}, \quad \Delta_b = \frac{3 m(A_b \mu e^{i \xi})}{m_{b2}^2 - m_{b1}^2},
$$

which measure the amount of CP violation in the top and bottom squark-mass matrices. In the CP-conserving limit, both $\Delta_i$ and $\Delta_b$ vanish, leading to $|m_{t2}^2| \sin(\xi + \theta_{12}) = 0$. The definition of the mass-squared $m_A^2$ and the dimensionless quantities $X_{t,b}$, $R_{t,b}$ and $R'_{t,b}$, as well as the other CP-preserving entries of the mass matrix squared $\mathcal{M}_H^2$, can be found in Ref. [4]. The real and symmetric matrix $\mathcal{M}_H^2$ can now be diagonalized with an orthogonal matrix $O$:

$$
\begin{pmatrix}
a \\
\phi_1 \\
\phi_2
\end{pmatrix}
= O
\begin{pmatrix}
H_1 \\
H_2 \\
H_3
\end{pmatrix}.
$$
Our convention for the three mass eigenvalues is \( m_{H_1} \leq m_{H_2} \leq m_{H_3} \).

The loop–corrected neutral–Higgs–boson sector depends on various parameters from the other sectors of the MSSM; \( m_A \), which becomes the mass of the CP–odd Higgs boson if CP is conserved, and \( \tan \beta \) fix the tree–level Higgs potential; and \( \mu \), \( A_t \), \( A_b \) and the soft–breaking third generation sfermion masses \( m_{\tilde{Q}} \), \( m_{\tilde{U}} \), and \( m_{\tilde{D}} \), which fix the third generation squark mass matrices. After minimization of the potential the rephasing invariant sum \( \theta_{12} + \xi \) of the radiatively induced phase \( \xi \) and the phase \( \theta_{12} \) of the soft breaking parameter \( m_{12}^2 \) is no longer an independent parameter. \(^\dagger\) The physically meaningful CP phases in the Higgs sector are thus the phases of the re–phasing invariant combinations \( A_t \mu e^{i\xi} \) and \( A_b \mu e^{i\xi} \) appearing in Eq. (9). The neutral–Higgs–boson mixing also depends on the complex gluino–mass parameter \( M_{\tilde{g}} \) through one–loop corrections to the top and bottom quark masses [16].

Noting that the size of the radiative Higgs sector CP violation is determined by the rephasing invariant combinations \( A_t \mu e^{i\xi} \) and \( A_b \mu e^{i\xi} \), see Eq. (9), we take for our numerical analysis the following set of parameters:

\[
\begin{align*}
|A_t| &= |A_b| = 1 \text{ TeV}, \\
|\mu| &= 2 \text{ TeV}, \\
m_{\tilde{Q}, \tilde{U}, \tilde{D}} &= |M_{\tilde{g}}| = 0.5 \text{ TeV}, \\
\arg(M_{\tilde{g}}) &= 0, \quad (11)
\end{align*}
\]

under the constraint:

\[
\Phi \equiv \arg(A_t \mu e^{i\xi}) = \arg(A_b \mu e^{i\xi}). \quad (12)
\]

We vary the common phase \( \Phi \) as well as \( m_A \) and \( \tan \beta \) in the following numerical studies. Our choice of relatively large magnitudes of \( |A_t \mu| = |A_b \mu| \) enhances the effects of the CP violation in the MSSM Higgs sector.

The CP–violating phase could weaken the LEP lower limit on the lightest Higgs boson mass significantly [17,18]. In our analysis we show our results when the lightest Higgs–boson mass is above 70 GeV.

III. PHOTON LINEAR COLLIDER

One of the cleanest determinations of the neutral Higgs sector CP violation in the MSSM can be achieved by observing the CP properties of all three neutral Higgs particles directly. In this light, the \( s \)–channel resonance production of neutral Higgs bosons in \( \gamma \gamma \) collisions \(^\dagger\)

\(^\dagger\)As discussed in [4], \( \xi \) and \( \theta_{12} \) are not separately physical parameters. For example, one or the other can be set to zero in certain phase conventions for the fields. Similar remarks hold for the phases of \( A_t \), \( A_b \) and \( \mu \). Altogether there are only three rephasing invariant (i.e. physical) phases, which we write as \( \theta_{12} + \xi \), \( \arg(A_t \mu e^{i\xi}) \) and \( \arg(A_b \mu e^{i\xi}) \). The minimization of the potential fixes one of these combinations, leaving two independent physical phases as free input parameters.
[19] has long been recognized as an important instrument to study the CP properties of Higgs particles [20,21] at a linear $e^+e^-$ collider (LC) by use of polarized high energy laser lights obtained by Compton back-scattering of polarized laser light off the electron and positron beams [22]. In this section, we demonstrate that the polarized back-scattered laser photons at a TeV–scale LC enable us to investigate the CP violation of the Higgs sector in the MSSM through s-channel Higgs–boson production via $\gamma\gamma$ collisions in detail including its dependence on the relevant SUSY parameters [10].

In the presence of the CP–violating neutral Higgs–boson mixing, the amplitude for the two–photon fusion process $\gamma\gamma \to H_i$ ($i = 1, 2, 3$) can be written in terms of two (complex) form factors $S_i^\gamma(s)$ and $P_i^\gamma(s)$ as

$$M(\gamma\gamma \to H_i) = \sqrt{s} \left\{ S_i^\gamma(s) \left( \epsilon_1 \cdot \epsilon_2 - \frac{2}{s} k_1 \cdot \epsilon_2 k_2 \cdot \epsilon_1 \right) - P_i^\gamma(s) \frac{2}{s} \epsilon_{i\mu\nu\rho} \epsilon_{2\mu} \epsilon_{2\nu} \epsilon_{1\rho} \right\}, \quad (13)$$

where $s$ is the c.m. energy squared of two colliding photons. In the two–photon c.m. coordinate system with one photon momentum $\vec{k}_1$ along the positive $z$ direction and the other one $\vec{k}_2$ along the negative $z$ direction, the wave vectors $\epsilon_{1,2}$ of two photons are given by

$$\epsilon_1(\lambda) = \epsilon_2(\lambda) = \frac{1}{\sqrt{2}} (0, -\lambda, -i, 0). \quad (14)$$

where $\lambda = \pm 1$ denote the right and left photon helicities, respectively. In the MSSM with radiative CP–violating Higgs mixing, the scalar and pseudoscalar form factors are given by

$$S_i^\gamma(s) = 2NC \sum_{f=t,b} \epsilon_i^2 \left\{ g_{sf}^i \frac{\sqrt{s}}{m_f} F_{sf}(\tau_{sf}) + \frac{1}{4} \sum_{j=1,2} g_{f_j}^i \frac{\sqrt{s}}{m_{f_j}^2} F_0(\tau_{sf_j}) \right\}$$

$$+ \frac{g \sqrt{s}}{2m_W} \left( c_\beta O_{2,i} + s_\beta O_{3,i} \right) F_1(\tau_{sW}) + \frac{v \sqrt{s} \lambda}{2M^2_{H^\pm}} F_0(\tau_{sH^\pm}),$$

$$P_i^\gamma(s) = 2NC \sum_{f=t,b} \epsilon_i^2 g_{pf}^i \frac{\sqrt{s}}{m_f} F_{pf}(\tau_{sf}), \quad (15)$$

with $\tau_{sx} = s/4m_x^2$ and $N_C = 3$. The definitions of the four form factors $F_0$, $F_{sf}$, $F_{pf}$, and $F_1$ and the couplings of the neutral Higgs bosons to fermions, sfermions, and the charged Higgs–boson and $W$–boson pairs can be found in Ref. [10]. The amplitude for the production of the lightest Higgs boson, mass of which is bounded below about 130 GeV in the MSSM, is dominated by the contribution from $W$–boson loop through the scalar form factor $S_1^\gamma(M_{H_1}^2)$. Inserting the wave vectors (14) into Eq. (13) we obtain the production helicity amplitude for the photon fusion process as follows

$$M_{\lambda_1\lambda_2} = -\sqrt{s} \frac{\alpha}{4\pi} \left\{ S_i^\gamma(s) \delta_{\lambda_1\lambda_2} + i\lambda_1 P_i^\gamma(s) \delta_{\lambda_1\lambda_2} \right\}, \quad (16)$$

with $\lambda_{1,2} = \pm$. For the s–channel resonance production of the neutral Higgs boson $H_i$, the c.m. energy squared $s$ is to be replaced with $m_{H_i}^2$. And the absolute polarized amplitude squared is given by
where \( \{ \zeta_i \} \) are the Stokes parameters describing the polarization transfer from the laser light to the high energy photons; \( \zeta_2 \) is the degree of circular polarization and \( \{ \zeta_3, \zeta_1 \} \) the degree of linear polarization transverse and normal to the plane defined by the electron direction and the direction of the maximal linear polarization of the initial laser light. To acquire the high sensitivity to CP violation, it is necessary to control both the energy of the initial laser light and its degrees of the circular and transverse polarization \([22,10]\). The unpolarized amplitude squared \( |\mathcal{M}|^2 \) is given by

\[
|\mathcal{M}|^2 = |\mathcal{M}_0|^2 \left\{ 1 + \zeta_2 \zeta_2 + A_1 \left[ \zeta_2 + \zeta_2 \right] + A_2 \left[ \zeta_1 \zeta_3 + \zeta_3 \zeta_1 \right] - A_3 \left[ \zeta_1 \zeta_1 - \zeta_3 \zeta_3 \right] \right\},
\]  

(17)

and three polarization asymmetries \( A_i \) \( (i = 1, 2, 3) \) are defined in terms of the helicity amplitudes and expressed in terms of the form factors \( S_i^\gamma \) and \( P_i^\gamma \) as

\[
A_1 = \frac{|\mathcal{M}_+|^2 - |\mathcal{M}_-|^2}{|\mathcal{M}_+|^2 + |\mathcal{M}_-|^2} = \frac{2I \left[ S_i^\gamma(M_{H_i}^2)P_i^\gamma(M_{H_i}^2) \right]}{S_i^\gamma(M_{H_i}^2)^2 + P_i^\gamma(M_{H_i}^2)^2},
\]

\[
A_2 = \frac{2I(\mathcal{M}_-^\ast \mathcal{M}_+)}{|\mathcal{M}_+|^2 + |\mathcal{M}_-|^2} = \frac{2R \left[ S_i^\gamma(M_{H_i}^2)P_i^\gamma(M_{H_i}^2) \right]}{S_i^\gamma(M_{H_i}^2)^2 + P_i^\gamma(M_{H_i}^2)^2},
\]

\[
A_3 = \frac{2R(\mathcal{M}_-^\ast \mathcal{M}_+)}{|\mathcal{M}_+|^2 + |\mathcal{M}_-|^2} = \frac{|S_i^\gamma(M_{H_i}^2)|^2 - |P_i^\gamma(M_{H_i}^2)|^2}{|S_i^\gamma(M_{H_i}^2)|^2 + |P_i^\gamma(M_{H_i}^2)|^2}.
\]

(19)

In the CP–invariant theories, the two form factors \( S_i^\gamma \) and \( P_i^\gamma \) can not coexist. In other words, non–zero \( A_{1,2} \) and/or \( |A_3| < 1 \) indicate the CP violation.

The unpolarized cross section for the \( s \)–channel Higgs–boson production is given by

\[
\sigma(\gamma \gamma \rightarrow H_i) = \frac{\pi}{M_{H_i}} |\mathcal{M}|^2 \delta \left( 1 - \frac{M_{H_i}^2}{s} \right) \equiv \sigma_0(\gamma \gamma \rightarrow H_i) \delta \left( 1 - \frac{M_{H_i}^2}{s} \right).
\]

(20)

Figure 1 shows the unpolarized cross section of the lightest Higgs boson, \( \sigma_0(\gamma \gamma \rightarrow H_1) \), in units of fb as a function of \( \Phi \) for four (\( \tan \beta = 4 \)) and five (\( \tan \beta = 10 \)) values of \( m_{H_1} \): \( m_{H_1} = 80 \text{ GeV} \) (solid line), 90 GeV (dashed line), 100 GeV (dotted line), 110 GeV (dash–dotted line), and 120 GeV (thick solid line). We take the parameter set Eq. (11) with \( \tan \beta = 4 \) (left) and \( \tan \beta = 10 \) (right). The unpolarized cross section for the lightest Higgs boson strongly depends on the CP phase \( \Phi \) as well as \( m_{H_1} \). The cross section is larger for larger values of \( m_{H_1} \). Note that this cross section is highly suppressed around \( \Phi = 100^\circ (90^\circ) \) and \( 260^\circ (270^\circ) \) for \( \tan \beta = 4 (10) \). This is because the lightest Higgs boson contains a large admixture of CP–odd state \( a \) in this region and the \( H_1W^\pm W^\mp \) coupling is significantly suppressed.

Figure 2 shows the unpolarized cross sections of the heavier two Higgs bosons, \( \sigma_0(\gamma \gamma \rightarrow H_{2,3}) \), in units of fb as a function of each Higgs–boson mass for five values of \( \Phi \): \( \Phi = 180^\circ \)
(thick solid line), 150° (solid line), 130° (dashed line), 60° (dotted line), and 20° (dash–dotted line). We take the parameter set Eq. (11) with \( \tan \beta = 4 \) (left) and \( \tan \beta = 10 \) (right). The upper two frames are for the intermediate Higgs boson and the lower ones for the heaviest Higgs boson. One can observe that these cross sections strongly depends on the CP phase \( \Phi \). The behavior of the cross sections can be understood by taking into account the \( \Phi \) dependence of the couplings of the corresponding Higgs boson to the fermion bilinear, the diagonal sfermion pair, and the \( W \)–boson and the charged Higgs–boson pairs. For example, let’s closely look into the left–upper frame of Fig. 2 with \( \tan \beta = 4 \) for the production of the second lightest Higgs boson \( H_2 \). For \( \Phi = 180° \), \( H_2 \) is CP–odd. In this case the couplings of \( H_2 \) to the diagonal sfermion pairs, the charged Higgs–boson and \( W \)–boson pairs vanish and the production cross section gets contributions only from the fermionic loops dominated by the top quark for \( \tan \beta = 4 \). The mass dependence of the cross section comes from the form factor \( F_{pf}(m_{H_2}^2/4 m_t^2) \) peaked at \( m_{H_2} = 2 m_t \). When the CP phase \( \Phi \) differs from 180°, \( H_2 \) becomes to contain the CP–even states. In other words, the cross section starts to get contributions from \( W \)–boson and sfermion loops. The \( W \)–boson–loop contribution is peaked at \( m_{H_2} = 2 m_W \) and the sfermion contributions, which is dominated by the lightest top squark \( \tilde{t}_1 \), peaked at \( m_{H_2} = 2 m_{\tilde{t}_1} \). Note that the mass of the lightest top squark also depends on \( \Phi \). The lightest top–squark mass becomes lighter when \( \Phi \) decrease from 180°. Usually, the contribution from the charged Higgs–boson loops is suppressed compared to the other three kinds of contributions due to \( m_{H^\pm} \sim m_{H_{2,3}} \).

Figures 3 and 4 shows three polarization asymmetries as functions of each Higgs–boson mass for \( \tan \beta = 4 \) and \( \tan \beta = 10 \), respectively. Noting that these polarization asymmetries satisfy the relations

\[
A_{1,2}(\Phi) = -A_{1,2}(360° - \Phi), \quad A_3(\Phi) = +A_3(360° - \Phi),
\]

(21) 

we choose five values of \( \Phi \) less than 180° to show the dependence of the asymmetries on the CP–violating phase: \( \Phi = 180° \) (thick solid line), 140° (solid line), 100° (dashed line), 60° (dotted line), and 20° (dash–dotted line). In the CP–conserving limit (\( \Phi = 180° \)), the polarization asymmetries \( A_{1,2} \) vanish and the asymmetry \( A_3 \) takes one of the values +1 (CP–even) or -1 (CP–odd) depending on the CP–parity of the Higgs bosons in this limit. For \( \tan \beta = 10 \), the polarization asymmetry \( A_1 \) of the lightest Higgs boson is quite sensitive to the CP–phase \( \Phi \). The other polarization asymmetries of the lightest Higgs boson are not much different from those in the CP–conserving limit except \( \Phi = 100° \). But, we note that all of three polarization asymmetries of the two heavier Higgs bosons significantly differ from those of the CP–conserving theory even for small CP violation with \( \Phi = 20° \) and 140° independently of \( \tan \beta \).

**IV. CONCLUSIONS**

Based on the calculation of the mass matrix of the neutral Higgs bosons which is valid for any values of the relevant SUSY parameters, we have re–evaluated the \( s \)–channel resonance production cross sections and the polarization asymmetries of the neutral MSSM Higgs bosons in the presence of the non–trivial CP–violating mixing among them. The cross section
of the lightest Higgs boson which is dominated by the $W$–boson loop can be highly suppressed when the lightest Higgs boson is almost CP–odd. For the heavier Higgs bosons, the cross sections strongly depends on the CP–violating mixing. The polarization asymmetries of two heavier Higgs bosons are very sensitive to the non–trivial CP phases. Our detailed analysis has clearly shown that collisions of polarized photons can provide a significant opportunity for detecting CP violation in the MSSM Higgs sector induced at the loop level.

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\[ \sigma_0^\wedge (\gamma\gamma \rightarrow H_1) \ [ \text{fb} ] \]

\[ \tan \beta = 4 \]

\[ \tan \beta = 10 \]

FIG. 1. The unpolarized cross section of the lightest Higgs boson in units of fb as a function of $\Phi$ for four ($\tan \beta = 4$) and five ($\tan \beta = 10$) values of $m_{H_1}$: $m_{H_1} = 80$ GeV (solid line), 90 GeV (dashed line), 100 GeV (dotted line), 110 GeV (dash–dotted line), and 120 GeV (thick solid line). We take the parameter set Eq. (11) with $\tan \beta = 4$ (left) and $\tan \beta = 10$ (right).
\[ \tan \beta = 4 \]

\[ \sigma_0 (\gamma \gamma \rightarrow H_2) \text{ [ fb ]} \]

\[ M_{H_2} \text{ [ GeV ]} \]

\[ \Phi = 180^\circ \]

\[ \tan \beta = 10 \]

\[ \sigma_0 (\gamma \gamma \rightarrow H_2) \text{ [ fb ]} \]

\[ M_{H_2} \text{ [ GeV ]} \]

\[ \tan \beta = 4 \]

\[ \sigma_0 (\gamma \gamma \rightarrow H_3) \text{ [ fb ]} \]

\[ M_{H_3} \text{ [ GeV ]} \]

\[ \tan \beta = 10 \]

\[ \sigma_0 (\gamma \gamma \rightarrow H_3) \text{ [ fb ]} \]

\[ M_{H_3} \text{ [ GeV ]} \]

FIG. 2. The unpolarized cross sections of the \( H_2 \) (upper) and \( H_3 \) (lower) in units of fb as a function of each Higgs–boson mass for five values of \( \Phi \): \( \Phi = 180^\circ \) (thick solid line), \( 150^\circ \) (solid line), \( 130^\circ \) (dashed line), \( 60^\circ \) (dotted line), and \( 20^\circ \) (dash–dotted line). We take the parameter set Eq. (11) with \( \tan \beta = 4 \) (left) and \( \tan \beta = 10 \) (right).
FIG. 3. The polarizatin asymmetries $A_1$ (left column), $A_2$ (middle column), and $A_3$ (right column) as functions of each Higgs–boson mass for $\tan\beta = 4$ for five values of $\Phi$: $\Phi = 180^\circ$ (thick solid line), $\Phi = 140^\circ$ (solid line), $\Phi = 100^\circ$ (dashed line), $\Phi = 60^\circ$ (dotted line), and $\Phi = 20^\circ$ (dash–dotted line). The upper 3 frames are for $H_1$ and the middle 3 ones for $H_2$, and the lower 3 ones for $H_3$. 
FIG. 4. The same as Figure 3 but with $\tan \beta = 10$. 