CHIRAL EXTRAPOLATIONS AND EXOTIC MESON SPECTRUM

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We examine the chiral corrections to exotic meson masses calculated in lattice QCD. In particular, we ask whether the non-linear chiral behavior at small quark masses, which has been found in other hadronic systems, could lead to large corrections to the predictions of exotic meson masses based on linear extrapolations to the chiral limit. We find that our present understanding of exotic meson decay dynamics suggests that open channels may not make a significant contribution to such non-linearities whereas the virtual, closed channels may be important.
1. Introduction. One of the biggest challenges in hadronic physics is to understand the role of gluonic degrees of freedom. Even though there is evidence from high energy experiments that gluons contribute significantly to hadron structure, for example to the momentum and spin sum rules, there is a pressing need for direct observation of gluonic excitations at low energies. In particular, it is expected that glue should manifest itself in the meson spectrum and reflect on the nature of confinement.

In recent years a few candidates for glueballs and hybrid mesons have been reported. For example, there are two major glueball candidates, the $f_0(1500)$ and $f_0(1710)$, scalar-isoscalar mesons observed in $\bar{p}p$ annihilation, in central production as well as in $J/\psi (f_0(1710))$ decays [1]. However, as a result of mixing with regular $q\bar{q}$ states, none of these is expected to be a purely gluonic state. This is a general problem with glueballs – they have regular quantum numbers and therefore cannot be unambiguously identified as purely gluonic excitations. On the other hand, the existence of mesons with exotic quantum numbers (combinations of spin, parity and charge conjugation, $J^{PC}$, which cannot be attributed to valence quarks alone), would be an explicit proof that gluonic degrees of freedom can indeed play an active role at low energies.

Current estimates based on lattice QCD [2, 3], as well as QCD-based models [4], suggest that the lightest exotic hybrid should have $J^{PC} = 1^{-+}$ and mass slightly below 2 GeV. On the experimental side, two exotic candidates with these quantum numbers have recently been reported by the E852 BNL collaboration. One, in the $\eta\pi^-$ channel of the reaction $\pi^-(18\text{GeV})p \rightarrow X p \rightarrow \eta\pi^- p$, occurs at $M_X \approx 1400\text{MeV}$ [5], while the other, in the $\rho^0\pi^-$ and $\eta'\pi^-$ channels, has a mass $M_X \approx 1600\text{MeV}$ [6, 7]. Although the lighter state could be obscured by final state interaction effects as well as leakage from the strong $X = a_2$ decay, the heavier one seems to have a fairly clean signal.

In the large $N_c$ limit it has been shown that exotic meson widths obey the same $N_c$ scaling laws as the regular mesons [8]. The lack of an overwhelming number of exotic resonance candidates may therefore be the result of a small production cross-section in the typical reactions employed in exotic searches – e.g., high energy $\pi N$ or $K N$ scattering. Indeed, low lying exotic mesons are expected to have valence
quarks in a spin-1 configuration and their production would therefore be suppressed in peripheral pseudoscalar meson-nucleon scattering. By the same arguments one would expect exotic production to be enhanced in real photon-nucleon scattering [9, 10, 11]. This is particularly encouraging for the experimental studies of exotic meson photoproduction which have recently been proposed in connection with the JLab 12 GeV energy upgrade [12].

In view of the theoretical importance of the topic and the exciting new experimental possibilities for exotic meson searches it is of prime importance to constrain the theoretical predictions for exotic meson masses – especially from lattice QCD. One of the important practical questions in this regard is the effect of light quarks on the predicted spectrum. All calculations that have been performed so far involve quarks that are much heavier than the physical $u$ and $d$ quarks and, with the exception of the initial study by the SESAM collaboration [3] (based on full QCD with two degenerate Wilson fermions), are performed in the quenched approximation. This is directly related to the present limitations on computer power. It is well known that, as a consequence of dynamical chiral symmetry breaking, hadron properties are non-analytic functions of the light quark masses. This non-analyticity can lead to rapid, non-linear variations of masses as the light quark masses approach zero. Whether or not this could make a sizable difference to the lattice QCD predictions for exotic state masses is the question we address.

There has been considerable activity concerning the appropriate chiral extrapolation of lattice results for hadron properties in the past few years, ranging from magnetic moments [13] to charge radii [14] and structure functions [15] as well as masses [16]. The overall conclusion from this work is that for current quark masses in the region above 50–60 MeV, where most lattice results are available, hadron properties are smoothly varying functions of the quark masses – much like a constituent quark model. However, as one goes below this range, so that the corresponding pion Compton wavelength is larger than the source, one finds rapid, non-linear variations which are a direct result of dynamical chiral symmetry breaking. These variations can change the mass of hadrons extracted by naive linear extrapolation by a hundred MeV or more. (Certainly this was the case for the $N$ and $\Delta$ [16], while for the $\rho$ the
A variation of this order of magnitude for the predictions of exotic meson masses would clearly be phenomenologically important and our purpose is to check whether this seems likely.

2. Chiral corrections from pion loops. Our investigation of the possible non-linearity of the extrapolation of exotic meson masses to the chiral limit, as a function of quark mass, will follow closely the earlier investigations for the $N$ and $\Delta$ baryons and the $\rho$ meson. The essential point is that rapid non-linear variations can only arise from coupling to those pion-hadron channels which give rise to the leading (LNA) or next-to-leading non-analytic (NLNA) behavior of the exotic particle’s self-energy. In a decay such as $E \rightarrow \pi H$, this means that $H$ must be degenerate or nearly degenerate with the exotic meson $E$. In addition, the relevant coupling constant should be reasonably large. For example, in the case of the $\rho$ meson the relevant channels are $\omega \pi$ (LNA) and $\pi \pi$ (NLNA) [17] and the extrapolation of the $\rho$ meson mass was carried out using:

$$m_\rho = a + bm_\pi^2 + \sigma_{\omega \pi} + \sigma_{\pi \pi}, \quad (1)$$

where $\sigma_{ij}$ are the self-energy contributions from the channels $ij = \omega \pi$ or $\pi \pi$.

Our first step in studying the extrapolation of exotic meson masses is therefore to look at the channels to which the exotics can couple which involve a pion and to check what is known about the corresponding coupling constants. The matrix element describing a decay of the $J^{PC} = 1^{-+}$, isovector, exotic mesons with mass $m_X$, is given by:

$$\langle 1^{-+}; P'|V|AB, k, P \rangle = (2\pi)^3 \delta^3(P' - P)m_X \sum_{LS} \sum_{M_L M_S} Y_{LM_L} \left( \hat{k}_X \right) \left( \frac{k(m_X)}{m_X} \right)^L \times g_{LS}(k(m_X)) \langle s_A \lambda_A, s_B \lambda_B | \Sigma M_S, L M_L | 1 M_X \rangle \langle I_A I_A^3, I_B I_B^3 | 1, I_X^3 \rangle, \quad (2)$$

where $k(m_X) = |k|$ is the break-up momentum of the $AB$ meson pair, from a decay of a state of mass $m_X$,

$$k(m_X) = \left( \frac{(m_X^2 - (m_A - m_B)^2)(m_X^2 - (m_A + m_B)^2)}{4m_X^4} \right)^{\frac{1}{2}} \equiv \lambda(m_X, m_A, m_B), \quad (3)$$

produced with angular momentum $L$ and spin $S$, with $L + S = 1$. 

3
In terms of the couplings, $g_{LS}$, the partial decay widths are given by

$$\Gamma_{LS} = \frac{m_X}{32\pi^2} \left( \frac{k}{m_X} \right)^{2L+1} g_{LS}^2(k).$$  \tag{4}$$

These couplings have been calculated in Refs. [19, 20], in two models based on the flux tube picture of gluonic excitations but assuming different $q\bar{q}$ production mechanisms. In general the models agree on predicting sizable couplings to the so called “S+P” channels, where one of the two mesons in the final state has quarks in relative S-wave and the other in relative P-wave, e.g., $\pi b_1$. In Table 1 we summarize the results of these calculations, listing only the decay channels containing a pion (S-wave quarks). Since the overall normalization of the decay matrix elements in these models is somewhat arbitrary, we have rescaled the original predictions given in Ref. [20] to match the total width (170 MeV) and mass ($m_X = 1.6$ GeV) of the exotic $\rho\pi$ state found by the E852 experiment [6].

Two features deserve particular attention. The couplings to ground state meson multiplets are generally smaller than to excited meson multiplets. This may be artificial; it is associated with the underlying structure of the model for pair creation which, when applied to exotic decays, strongly suppresses final states with mesons which have similar orbital wave functions. These models are based on a simple quark model picture in which (say) the $\pi$, $\rho$ and $\eta$ have very similar orbital wave functions. This is obviously an oversimplification. Secondly, the PSS and IKP models are dra-
Decay channel | wave | $g^2_L(k(m_X))/4\pi$ | $\Gamma_L$ [MeV] |
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<tr>
<td>$\rho\pi$</td>
<td>P</td>
<td>24.59</td>
<td>$O(100)$</td>
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<tr>
<td>$b_1(1235)\pi$</td>
<td>S</td>
<td>7.12</td>
<td>$O(100)$</td>
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Table 2: Isovector Hybrid couplings $g^2_L(k(m_X))/4\pi$

ecially different when it comes to predicting ratios of branching ratios. In general, PSS predicts that higher partial waves should be strongly suppressed.

An alternative approach was presented in Ref. [11]. There, it was assumed that the decay of the observed exotic is dominated by just two channels: $\rho\pi$ – the one in which it has been seen – and the rest, say $b_1\pi$ (in an S-wave). The couplings obtained this way are shown in Table II.

The shift in the mass of the exotic meson associated with the pion loop, $\Sigma$, can be calculated in second order perturbation theory (from $\Sigma = \langle P'|VGV|P'\rangle/\langle P'|P\rangle$) and is given by:

$$\Sigma = \sum_L \frac{m_X^2}{64\pi^3} \mathcal{P} \int_{m_A+m_B} \frac{dM}{M} \left( \frac{k(M)}{m_X} \right)^{2L+1} \frac{g^2_L(k(M))}{m_X-M}$$

$$= \sum_L \frac{m_X}{64\pi^3} \mathcal{P} \int_0 ^{dk} \frac{k^2 k^2}{\sqrt{k^2 + m_A^2} \sqrt{k^2 + m_B^2}} \left( \frac{k}{m_X} \right)^{2L} \frac{g^2_L(k)}{m_X - \sqrt{k^2 + m_A^2 - k^2 + m_B^2}}$$

with $k(M) = \lambda(M, m_A, m_B)$. The leading non-analytic (LNA) behavior of this self-energy is obtained by extracting the piece, independent of the ultra-violet cut-off (or form factor, $g_L(k)/g_L(k(m_X))$), which exhibits the strongest non-analytic variation as a function of the quark mass as one approaches the chiral limit. (This is the term with the lowest odd power of $m_\pi$ or the lowest power of $m_\pi$ multiplying $\ln m_\pi$.) Setting $m_B = m_\pi$ the form of the LNA behavior depends on the relation between $m_X - m_A$ and $m_\pi$ and can be easily extracted analytically from Eq. (5) in two limiting cases. Consider first the limit $m_X - m_A << m_\pi$, corresponding to an off-shell transition between the exotic meson and a nearly degenerate meson plus pion. In this case the self energy contribution reduces to

$$\Sigma = -\sum_L \frac{1}{64\pi^3} \mathcal{P} \int_0 ^{dk} \frac{k^2}{m_X} \left( \frac{k}{m_X} \right)^{2L} \frac{g^2_L(k)}{k^2 + m_\pi^2},$$
and the LNA behavior is given by

\[ \Sigma_{LNA} = \sum_L (-1)^L \frac{m_X g_L^2(k_\pi)}{4\pi} \left( \frac{m_\pi}{m_X} \right)^{2L+1}. \]  

(7)

and \( k_\pi \approx 0 \). In the case \( m_X - m_A \gg m_\pi \), corresponding to a physical decay process to two light mesons (one of them being the pion), one obtains,

\[ \Sigma = \sum_L \frac{m_X}{64\pi^3} \int_0^\beta dk k^2 \frac{g_L^2(k)}{m_A(m_X - m_A) \sqrt{k^2 + m_\pi^2}}, \]  

(8)

leading to

\[ \Sigma_{LNA} = \sum_L (-1)^L \frac{m_X^3}{16\pi^2 m_A(m_X - m_A)} \frac{(2L + 1)!! g_L^2(k_\pi)}{4\pi} \left( \frac{m_\pi^2}{m_X^2} \right)^{L+1} \ln m_\pi. \]  

(9)

Even though, in the present case, neither of these is the exact, four-dimensional, pion loop contribution, they should give a reliable guide as to the non-linearity that one may expect in the extrapolation of lattice results to small quark mass.

In order to estimate the full self energy contribution, we need to know the off-shell dependence of the coupling constant. In the models of Ref. [11, 19, 20] the momentum dependence of the couplings arises from the Fourier transform of quark wave function overlaps and is typically of a gaussian form,

\[ g_L(k) = g_L(k(m_X)) e^{-k^2/2\beta^2 + k^2(m_X)/2\beta^2} \]  

(10)

with the scale parameter expected to be in the range \( \beta \approx 0.2 - 1 \) GeV. To be consistent with the approximation of a heavy source, corresponding to Eqs. (6) and (8), we need to set \( k^2(m_X) \approx 0 \) or \( k^2(m_X) \approx (m_X - m_A)^2 \) respectively. Thus the momentum dependence of the form factor near \( k^2 = k_\pi^2 \) is not expected to significantly renormalize the value at \( k^2 = k_\pi^2 \) and in Eqs. (6) and (8) we can simply use the on-shell couplings from Tables 1 and 2. However, the off-shell transition matrix element may have a more significant momentum dependence. This could happen, for example, because of nodes present in the radial wave functions of excited mesons (e.g. the \( \rho(1450) \) ). Such nontrivial momentum dependence of the form factor could possibly introduce a factor of \( 2 - 3 \) uncertainty in our predictions. Keeping this
possibility in mind, using Eq. (7) we find that for the maximal strength $P$-wave decays with couplings of the order of 20,

$$\Sigma \approx -0.1m_{\pi}^3,$$

with both $\Sigma$ and $m_{\pi}$ in GeV, and all couplings of order 1 are irrelevant. For the $D$-waves with coupling as large as 300 (IKP model, D-wave in the $b_1\pi$ channel) we obtain,

$$\Sigma \approx +0.5m_{\pi}^5,$$

from Eq. (7) or $\Sigma \approx +0.4m_{\pi}^6\ln m_{\pi}$ from Eq. (9). In the case of the $b_1\pi$ or $f_1\pi$ decay channels the second approximation is probably more accurate.

To be more precise we also check how the full formula for the energy shift compares with the simpler expression for a heavy source given above. In Fig. 1 the dashed line shows $\Sigma$ as a function of the pion mass, calculated including the three largest $P$ wave open channels from Table 1 ($\rho(1450)\pi$, $\eta_u(1295)\pi$ and $\rho\pi$) using the PSS (larger) couplings. The magnitude of $\Sigma$ is determined by the scale parameter chosen as $\beta = 400$ MeV in Fig. 1(a) and $\beta = 700$ MeV in Fig 1(b). For small $\beta$ the dominant contribution comes from the lightest open channel. This is because there is a large mismatch between $k(m_X)$ and $\langle k \rangle \sim \beta$ which leads to enhancement from
Figure 2: Pion loop contribution from the open, $D$-wave channels. The shaded region corresponds to $\beta$ in the range from 400 to 700 MeV.

$g_L(k)$ (Eq. (10)). As $\beta$ increases it is the channels with largest on-shell coupling that dominate. In Fig. 2 the contribution to $\Sigma$ from the two largest $D$-wave couplings ($f_1\pi$ and $b_1\pi$ in the IKP model) is shown for $\beta$ in the range between 400 MeV and 700 GeV. In all cases we find that the mass shifts from the open channels do not exceed 50 MeV. The magnitude of the self energy is naturally sensitive to the scale parameter $\beta$, in particular for higher partial waves. Nevertheless, the weak $m_\pi$ dependence for all $\beta$, in particular for the two limiting values corresponding to the upper and lower bounds, decreasing for higher partial waves, is consistent with Eqs. (11) and (12).

These are small shifts as compared, for example, to the case of the pion contribution to the nucleon mass, where the LNA term is of order $-5.6m_\pi^3$. To be more specific, the $\pi NN$ vertex, written in the notation of Eq. (2), becomes

$$g_{\pi NN}u(p', \lambda, I_3')\gamma_5 \tau^5 u(p, \lambda, I_3) = m_N \bar{g}_{\pi NN} \langle \frac{1}{2} I_3, 1 I_3' | \frac{1}{2} \lambda, 1 M_L | \frac{1}{2} \lambda' \rangle g_{\pi NN} m_N \tilde{Y}_{1M_L}(q_\pi)$$

where

$$\tilde{g}_{\pi NN} = \left( \sqrt{3} \times \sqrt{3} \frac{\sqrt{4\pi}}{\sqrt{3}} g_{\pi NN} \right)$$

Eq. (7) then gives,

$$-\frac{m_N}{32\pi} \frac{3 \times 4\pi g_{\pi NN}^2}{4\pi} \left( \frac{m_\pi}{m_X} \right)^3 = -\frac{3}{32\pi} g_{\pi NN}^2 m_N^3 = -\frac{3 g_{\pi NN}^2}{32\pi f_\pi^2 m_\pi^3}$$

Comparing $\tilde{g}_{\pi NN}^2/4\pi = 12\pi \times 14.4 \approx 540$ to the typical $P$-wave coupling from Table 1 or 2, it becomes clear why the open channels give small corrections to the exotic meson mass.
However, just as in the case of the nucleon, one expects the largest LNA behavior to come from virtual transitions to nearly degenerate states; in this case from transitions between an isovector exotic and a pion plus an isoscalar exotic meson. The couplings in Tables 1 and 2 only account for real decays and therefore cannot be used to estimate such transitions. To the best of our knowledge there are no microscopic calculations of such matrix elements, however, these can in principle be derived from PCAC. In the soft pion limit one has,

\begin{equation}
\langle 1^{-+}, \lambda', \mathbf{p}', I = 1, a|q_\mu A'^{\mu b}(0)|1^{-+}, \lambda, \mathbf{p}, I = 0 \rangle = f_{\pi}\langle 1^{-+}, \mathbf{p}', \lambda', I = 1, a|V|\pi^b, \mathbf{q}; 1^{-+}, \lambda, \mathbf{p}, I = 0 \rangle/(2\pi)^3 \delta^3(\mathbf{p}' - \mathbf{p}).
\end{equation}

(16)

If one assumes the flux tube does not affect the axial charge, then in the static limit the valence quark contribution to the lhs is given by $2m_X\delta_{ab}\bar{\varepsilon}^\lambda(\lambda') \cdot \big[\bar{\varepsilon}^\lambda(\lambda) \times \hat{q}\big]$. After comparing with Eq. (2) this yields

\begin{equation}
g_1^{-+}(I=1)\rightarrow 1^{-+}(I=0),\pi = 2\sqrt{8\pi/m_X^3} f_{\pi}.
\end{equation}

(17)

The lack of any contribution from the flux-tube makes this coupling identical to the one for ordinary mesons, e.g. the $\rho\omega\pi$ coupling (with $m_X = m_\rho \sim m_\omega$). The phenomenological value for the $g_{\rho\omega\pi}$, in our notation, is $g_{\rho\omega\pi}/m_\rho = \sqrt{8\pi/3}\tilde{g}_{\rho\omega\pi}$ with $\tilde{g}_{\rho\omega\pi} \sim 15$ GeV$^{-1}$, so that the simple quark model overestimates the coupling by approximately 50%. The flux-tube may contribute to the axial current if it couples to the spin of the quarks. In particular, since it is expected that the ground state corresponds to a flux-tube in a $P$-wave with respect to the valence $Q\bar{Q}$ pair, the overlap of the spin and orbital wave functions may modify the numerical factor in Eq. (17). However, it is not expected to alter the $\sqrt{8\pi/3m_X/f_\pi}$ enhancement arising from the soft pion emission. Thus, from Eqs. (7) and (17) we estimate

\begin{equation}
\Sigma = -\frac{1}{12\pi f_\pi^2} m_\pi^3 \approx -3m_\pi^3.
\end{equation}

(18)

The magnitude of this correction is almost as large as that found in the nucleon case. The contribution to $\Sigma$ from the virtual transitions calculated using the expression in Eq. (6), is shown in Fig. 1 with the dotted line. Its magnitude is
strongly dependent on the cutoff parameter $\beta$, and can be as large as $O(100\text{MeV})$ for $\beta = 700 \text{MeV}$. It also has significant variation as a function of $m_\pi$, as expected from Eq. (18). The total self energy shift arising from the real and virtual corrections is shown in Fig. 1 with the solid line. This simple analysis of virtual corrections also applies to the $\rho(1600)\pi$ channel which, depending on the relative phase, could enhance the overall LNA behavior by a factor of two. Finally if the other light exotics $J^{PC} = 0^{-+}, 2^{-+}$ are not too far from the $J^{PC} = 1^{-+}$, they could significantly enhance the $d$-wave ($\propto m_\pi^2$) behavior (cf. Fig.2).

3. Conclusions. We have explored the self-energy corrections to the mass of the $1^{-+}$ exotic meson which are most likely to introduce some non-linearity in the chiral extrapolation of lattice estimates of its mass. As in earlier work involving the $N$ and $\Delta$ baryons and the $\rho$ meson, the guiding principle was to find those coupled channels involving a pion which yield the leading and next-to-leading non-analytic behavior of the self-energy. Using several models for the coupling constants to these decay channels available in the literature we find that the effects of the open channels are rather too small to lead to any significant non-linearity.

On the other hand, the virtual corrections can potentially be as large as those found for the nucleon, $\Delta$ or $\rho$ meson. Thus current estimates of the masses of exotic mesons, based on linear chiral extrapolation of lattice results obtained at relatively large input quark masses, may not be sufficiently accurate and nonlinear behavior similar to that found for the nucleon and $\Delta$ seems likely. A careful chiral extrapolation, constrained by new calculations at lower quark mass (which are needed urgently), may well lead to a physical mass for the $1^{-+}$ of order 100 MeV or more lower than that found by naive linear extrapolation. To estimate the strength of the virtual transition we used the soft pion limit theorem and in the future more precise calculations of the axial current matrix elements involving exotic mesons should be performed to test this approximation. Finally, it will be essential to revisit these conclusions as we learn more about the open channel exotic decay modes. In particular, if the existing models turn out to significantly underestimate these pion couplings, then one would also need to revise their contribution to the lattice extrapolation procedure.

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