MAXIMUM–ENTROPY RECONSTRUCTION OF THE DISTRIBUTION OF
MASS IN CLUSTER MS1054-03 FROM WEAK LENSING DATA

P.J. Marshall
Astrophysics Group, Cavendish Laboratory, Madingley Road, Cambridge CB3 0HE, England

Weak gravitational lensing studies of clusters of galaxies provide complimentary information
to that from X-ray data; the measured signal depends only on the cluster’s projected mass,
independent of dynamical assumptions. Here we apply a maximum–entropy method algorithm
for reconstructing the two-dimensional density distribution to two sets of shear data for the

1 Weak Lensing Introduction

The mass in a cluster of galaxies produces a net distortion of the shapes of images of galaxies lying
behind the cluster. The galaxies appear stretched tangentially around the mass concentrations.
This distortion is often described by the reduced shear field $g$:

$$ g = \frac{\gamma}{1 - \kappa} $$

where the convergence $\kappa$ is the projected mass density of the cluster relative to a critical density
$\Sigma_{\text{crit}}$ (which is a function of $z_{\text{cluster}}, z_{\text{galaxies}}, \Omega_m, \Omega_\Lambda$ and $H_0$). The shear $\gamma$ is related to the
convergence by a convolution over the whole image plane:

$$ \gamma(\theta) = \frac{1}{\pi} \int D(\theta - \theta') \kappa(\theta')d^2\theta' $$

If we parameterise the (elliptical) galaxy image shapes by

$$ \epsilon = \epsilon_1 + i\epsilon_2 = \left(1 - \frac{b}{a}\right) \left(1 + \frac{b}{a}\right)^{-2i\phi} $$

then averaging over many galaxies (to remove the intrinsic shapes) can be shown to give an unbiased estimate of the
reduced shear:

$$ \langle \epsilon \rangle = g $$

We want to infer $\kappa$ from a measured $g$ field...
2 Maximum Entropy Weak Lens Reconstruction

We aim to find the convergence map that maximises the posterior probability:

\[ \Pr(\kappa|\text{data}) = \frac{\Pr(\text{data}|\kappa)\Pr(\kappa)}{\Pr(\text{data})} \]

The data are the measured values of \( g_1 \) and \( g_2 \) in the pixels of the background galaxy averaging grid; \( \kappa \) is reconstructed on a larger grid, since mass lying outside the observing region will produce a shear signal within it.

Assuming Gaussian errors on the \( g \) values, the likelihood is

\[ \Pr(\text{data}|\kappa) \propto e^{-\frac{\chi^2}{2}} \]

where

\[ \chi^2 = \sum_i \frac{(g_{\text{observed}}^i - g_{\text{predicted}}^i)^2}{\sigma_i^2} \]

The prior probability has to be specified – an entropic prior is appropriate:

\[ \Pr(\kappa) \propto e^{\alpha S} \]

The entropy \( S(\kappa) \) becomes more negative as \( \kappa \) departs from the default model – taken to be \( \sim \) zero mass density; it acts to suppress over-fitting to the noise in the data, and to control the reconstruction of mass outside the observed field where the constraints from the data are weak. Minimising \( F = \chi^2/2 - \alpha S \) gives the best-fit convergence distribution \( \hat{\kappa} \).

The uncertainties in \( \hat{\kappa} \) are estimated by approximating \( \Pr(\kappa|\text{data}) \) by a multivariate Gaussian:

\[ \Pr(\kappa|\text{data}) \approx \exp \left[ -\frac{1}{2} (\kappa - \hat{\kappa})^T \nabla_{\kappa} \nabla_{\kappa} F(\kappa - \hat{\kappa}) \right] \]

So we have the covariance matrix for the convergence pixel values, the (square root of the) diagonal elements of which provide an estimate of the uncertainty in each pixel value.

To produce the plots in this poster the reconstructed convergence distributions have been smoothed; the “signal to noise contours” were obtained by dividing the reconstructed convergence in each pixel by its uncertainty, and then smoothing on the same scale.

3 The MS1054-03 Data

MS1054-03 is a high redshift (\( z = 0.83 \)) galaxy cluster; X-ray and dynamical measurements suggest a high mass (\( T_X \approx 12.3 keV \), \( \sigma \approx 1150 \text{ km s}^{-1} \)). 2 sets of weak lensing data have been analysed:

- Clowe et al 2000 – Single Keck pointing \( \rightarrow \) \( \epsilon \) measured for 2723 background galaxies in a 50 square arcminutes field
- Hoekstra et al 2000 – Irregularly shaped mosaic of 6 HST WFPC2 images \( \rightarrow \) \( \epsilon \pm \delta \epsilon \) measured for 2446 background galaxies (in a region of area \( \sim 30 \text{ arcmin}^2 \)). A higher density of galaxy images combined with a much smaller point spread function means that the data is of higher quality – a higher resolution reconstruction can be performed.
The LensEnt Reconstructions

- Keck data – The cluster is visible as the central North-East/South-West elongated blob; the highest contour is $3\sigma$. Other features are present, possibly due to noise.
- HST data – The shape of the cluster is shown to apparently higher resolution, but again there are many noise peaks. As for the cluster, how many sub-clumps does it contain? 1, 2 or 3?

4.1 How Much Structure Do We Believe?

The plotted signal-to-noise contours show the significance levels of structure if we accept both the data and the reconstruction at face value – if present, outliers in the data could introduce spurious signals.

As a test, the shears were rotated by $\frac{\pi}{4}$ to reconstruct the “imaginary” convergence – the cluster (having real mass!) disappears but spurious features in the North (Keck data) and South-East (HST data) remain, presumably due to noise spikes in the data.
We can illustrate our new knowledge of the cluster’s shape by plotting samples drawn from the posterior probability distribution – approximately $\frac{2}{3}$ of the sample images will lie within $\pm 1\sigma$ of the best-fit reconstruction. A wide range of structure is consistent with the shear data!
4.2 Mass Estimation

Given the reconstructed convergence distribution $\hat{\kappa}_i$ (with associated covariance matrix $C_{ij}$), and a value for $\Sigma_{\text{crit}}$ (calculated by Hoekstra et al.\textsuperscript{3} and so only really appropriate for that dataset), we can estimate the projected mass within a circular aperture:

$$M = \sum_i a_i \kappa_i \pm \sigma_M = \sum_{ij} a_i a_j C_{ij}$$

where $a_i$ is the proportion of the area of the $i^{th}$ pixel lying within the aperture.

5 Is This The Best We Can Do?

So far we have assumed that the convergences in each pixel are independent; this is not the case for physical cluster structure on larger scales than one pixel. An intrinsic correlation function (ICF),\textsuperscript{7} that maps some “hidden” convergence distribution (which has an entropic prior) onto the “visible” distribution used to fit the data, eases the introduction of large-scale structure into the reconstruction. One step further is to use a multiple-scale ICF (still under development). The reconstruction from the HST data is smoother, with the cluster appearing as a possible two-component extended feature. This map has been centred on the CD galaxy, with North upwards and East to the left. Contours are signal-to-noise as before.

The inclusion of an ICF does, to some extent, add unwanted smoothness – a small scale convergence distribution (fed back from the previous results) is reconstructed with some increase in smoothness. The converse is also true: the smooth output from the multi-resolution ICF reconstruction is made “grainier” by the single scale reconstruction code in the presence of high noise. So which reconstruction should we prefer? The one that maximises the probability of getting the data, given a hypothesis of the degree of smoothness of the convergence distribution – the EVIDENCE, $Pr(data)$. This does indeed turn out to be higher for the multi-scale ICF reconstruction.
6 Conclusions

- Reconstruction of the mass distribution in MS1054-03 from both Keck and HST data show the non-spherical cluster shape found by previous authors.

- Evaluation of the reconstruction uncertainties show that the cluster substructure is present at relatively low significance; sample convergence maps show a wide range of possible cluster configurations consistent with the weak lensing data.

- Given the (small) statistical and (larger) systematic errors involved, a mass estimate of $1 \times 10^{15} h_{50}^{-1} M_\odot$ would be a reasonable consensus value for MS1054-03 – as found by Hoekstra et al.

- The inclusion of an ICF in the algorithm appears promising, particularly with regard to determining the level of substructure really present in clusters of galaxies; much more investigation is required before drawing any definite conclusions!

The LensEnt reconstruction code used for the first part of this analysis is available from

http://www.mrao.cam.ac.uk/projects/lensent/

with the ICF extensions to follow in the very near future.

References