Exclusive QCD

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I give a brief introduction to the physics of generalized parton distributions and distribution amplitudes. I then report on the status of the calculation of radiative corrections for the exclusive processes where these quantities occur.

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1 Introduction

In recent years a formalism has been developed which highlights the close connection between exclusive and inclusive strong interaction processes. The cornerstones of this formalism are the concepts of generalized parton distributions and generalized distribution amplitudes. These quantities contain valuable information on the non-perturbative transition between partons and hadrons, whose understanding remains one of the great outstanding tasks in QCD. They can be accessed in several exclusive reactions that are within the reach of current and planned experimental facilities. In this contribution I will first give a brief overview of the formalism and the physics behind it, and then report on the status of the calculation of radiative corrections in this context.

A key process in the development of the QCD improved parton model has been inclusive deep inelastic scattering, $e p \to e X$, which via the optical theorem is conveniently expressed in terms of the imaginary part of the forward Compton amplitude, $\gamma^*(q) + p(p) \to \gamma^*(q) + p(p)$. In the Bjorken region of large photon virtuality $Q^2 = -q^2$ and c.m. energy $(p + q)^2$, this amplitude factorizes into a perturbatively calculable scattering process at the level of quarks and gluons and process independent matrix elements

\[
\langle p(p)|O(\lambda)|p(p)\rangle.
\]

Here $O(\lambda)$ stands for quark or gluon operators $\bar{\psi}(0) n_\mu \gamma^\mu \psi(\lambda n), \bar{\psi}(0) n_\mu \gamma^\mu \gamma^5 \psi(\lambda n), n_\mu n_\nu F^{\mu\alpha}(0) F_{\nu\alpha}(\lambda n), n_\mu n_\nu \tilde{F}^{\mu\alpha}(0) \tilde{F}_{\nu\alpha}(\lambda n)$, whose fields are separated by a light-like distance $\lambda n$ (i.e., $n^2 = 0$). These matrix elements, represented by a blob in fig. 1a, are parameterized by parton distributions; they describe the transition between hadronic and partonic degrees of freedom.

This factorization into short- and long-distance dynamics is actually more general.

\[\]
It also holds for the nonforward Compton amplitude, $\gamma^*(q) + p(p) \rightarrow \gamma^*(q') + p(p')$, in a generalization of Bjorken kinematics, namely if the c.m. energy $(p + q)^2$ and at least one of the photon virtualities $|q^2|, |q'^2|$ are large while the invariant momentum transfer $(p - p')^2$ is small [1–3]. In the particular case where the outgoing photon is on shell, one speaks of deeply virtual Compton scattering, which can be accessed in the physical process $ep \rightarrow ep \gamma$, i.e., in exclusive electroproduction of a photon. The non-perturbative physics is now described by matrix elements with the same operators as before, but between \textit{different} hadron states,

$$\langle p(p')|O(\lambda)|p(p)\rangle.$$ (2)

The nonzero momentum transfer to the proton implies that the momenta of the two parton lines attached to the blob in fig. 1a must differ as well. A simple calculation shows that in particular their momentum fractions with respect to the hadrons cannot be equal. For this reason, the generalized parton distributions which parameterize the matrix elements (2) are often called “skewed”.

A completely analogous type of factorization occurs in the crossed channel, \emph{i.e.}, in $\gamma^*(q) + \gamma^*(q') \rightarrow p(p) + \overline{p}(p')$, if at least one of the photon virtualities is large, in particular compared with the invariant mass $(p + p')^2$ of the produced hadron pair [1,4]. A corresponding diagram is shown in fig. 1b. Matrix elements

$$\langle p(p)\overline{p}(p')|O(\lambda)|0\rangle$$ (3)

with again the same operators as before now parameterize the non-perturbative transition from a quark-antiquark or gluon pair to the final state hadrons. In addition to the $pp$ system one can consider a wide range of hadrons, say $\pi\pi$ or $KK$, which are not easily available as beam particles in Compton scattering. The production mechanism represented in fig. 1b is the same as in the process $\gamma^*\gamma \rightarrow \pi$, where the nonperturbative input is represented by the quark-antiquark distribution amplitude of the pion. Data on this process have in fact provided one of the best available constraints so far on this important quantity [5]. The matrix elements (3) are a direct generalization of usual distribution amplitudes, where $\langle p\overline{p}\rangle$ is replaced by a single-meson state.

## 2 Some physics aspects

As a consequence of the finite momentum transfer to the proton, generalized parton distributions admit two different kinematical regimes. In the first, they describe the emission of a parton with momentum fraction $x + \xi$ from the parent hadron and its reabsorption with momentum fraction $x - \xi$, see fig. 2a. In the limit where $p = p'$ one has $\xi = 0$ and recovers the usual parton distributions. In a second regime, which does not exist for $p = p'$, one has the coherent emission of a quark-antiquark or gluon
pair from the parent hadron of momentum $p$, leaving the hadron with momentum $p'$, see fig. 2b. One is thus sensitive to aspects of the proton structure that cannot be accessed by the ordinary parton densities. The second regime is reminiscent of a distribution amplitude, shown in fig. 2c, where there is no hadron left behind after emission of the partons. We will encounter an important manifestation of this similarity in section 4.1.

It has long been known that the usual parton distributions can be represented in terms of light-cone wave functions, which completely specify the structure of a hadron in terms of quark and gluon configurations [6]. In this representation, depicted in fig. 3a, the wave functions appear squared, which reflects the crucial parton model feature that parton densities can be understood as classical probabilities for the emission of a parton from a hadron. The wave function representation of generalized parton distributions [7] provides a key to their physical interpretation: they are not probabilities but interference terms of wave functions for different parton configurations in a hadron. In this sense they contain characteristic information on the quantum fluctuations of QCD bounds states, going beyond the classical probability picture of the parton model.

Generalized parton distributions have a rich structure in spin, since the helicities of the partons and the hadrons can be varied independently. Of particular interest are those combinations where the helicity difference on the hadron side is not compensated on the parton side: in that case angular momentum conservation must be ensured by a transfer of orbital angular momentum, which is possible if there is a finite transfer of transverse momentum. Thus, generalized distributions carry information on the orbital angular momentum of partons—information that is hard to access otherwise. If one takes moments of the generalized distributions in the momentum fraction $x$, the operators in the matrix elements (2) are transformed into local operators, i.e., one obtains form factors of various local currents. A sum rule due to Ji [2] states that the second moment of a particular combination of generalized distributions gives a form factor whose value at zero momentum transfer is the total angular momentum.
of quarks in the proton, consisting both of a spin and an orbital part.

The generalized distribution amplitudes of fig. 1b are intimately connected with generalized parton distributions by crossing symmetry. Their moments are in fact related by an analytic continuation in the Mandelstam invariant, i.e., \( t = (p - p')^2 \) for distributions and \( s = (p + p')^2 \) for distribution amplitudes. On the other hand, generalized distribution amplitudes contain physics quite distinct from that of parton distributions: they involve not only the partonic structure of a single hadron, but also the interactions between hadrons. They parameterize what one might call the “exclusive limit” of fragmentation, i.e., of the transition between parton and hadron degrees of freedom. It is interesting to note that one can make a connection with phenomenologically successful pictures of hadronization such as the Lund string model [8].

3 Processes

As we saw in the introduction, generalized parton distributions can be accessed in deeply virtual Compton scattering, measurable by electroproduction \( ep \rightarrow e p \gamma \). Another class of processes where they occur is exclusive electroproduction of a meson instead of a photon, \( ep \rightarrow e p \rho^0 \), \( ep \rightarrow e n \pi^+ \), etc. Example diagrams are shown in fig. 4. Notice that for vector mesons both quark and gluon distributions contribute at leading order in \( \alpha_s \). This is in contrast to Compton scattering, where gluons only appear at the level of one-loop corrections, as they do in inclusive deep inelastic scattering.

To date, we have no experimental determinations of generalized parton distributions. However, first measurements of the above processes in the relevant kinematical domain have been performed, in particular by HERMES, H1, and ZEUS at DESY. Further and more precise investigations are planned Jefferson Lab and at CERN (COMPASS), and several future accelerator projects would be well suited for in-depth
Figure 4: Born level diagrams for the exclusive production of a meson from a deeply virtual photon. The large blobs denote generalized gluon or quark distributions, and the small blobs the meson distribution amplitude. Diagram (a) only contributes for mesons with negative charge conjugation parity.

studies of these processes.

Generalized distribution amplitudes can be accessed in $\gamma^*\gamma^*$ or $\gamma^*\gamma$ processes at $e^+e^-$ colliders, and cross section estimates [9] indicate that the production of pion pairs $\gamma^*\gamma \rightarrow \pi\pi$ should be well in the reach of BABAR, BELLE, and CLEO.

4 Radiative Corrections

As in all applications of QCD factorization, radiative corrections in our context manifest themselves in two ways. The process independent hadronic matrix elements depend on a factorization scale via evolution equations, whose kernels can be calculated in perturbation theory. On the other hand, there are radiative corrections to the hard scattering subprocesses for each individual reaction.

4.1 Evolution

The evolution of generalized distribution amplitudes is exactly the same as the one of the usual distribution amplitudes for mesons, described by the ERBL equations [15]. This is not surprising, since evolution arises from the renormalization of the bilocal operators $O(\lambda)$ and is insensitive to whether the matrix elements (3) involve a single meson or a two-particle state with the same quantum numbers.

The evolution of general parton distributions on the other hand, is more complex and of considerable theoretical interest. It is in fact this aspect on which the earliest studies of these quantities have focused on [1,14]. In the regime of fig. 2a, the evolution is similar to the standard DGLAP evolution of parton densities, with a kernel that depends on the parameter $\xi$ describing the longitudinal momentum transfer to the
partons (see fig. 2). In the regime of fig. 2b, evolution acts in a similar way as ERBL evolution, which highlights the close analogy between figs. 2b and c. The evolution equations for ordinary parton distributions and for distribution amplitudes, although acting in quite different ways, are thus intimately related, which stems from the fact that these quantities are defined through the same bilocal operators $O(\lambda)$. The evolution kernels for generalized parton distributions, called “extended ERBL kernels”, contain the usual DGLAP and ERBL kernels as limiting cases; in this sense the evolution of generalized parton distributions interpolates between the two extremes of DGLAP and ERBL evolution.

The extended ERBL kernels have been calculated to LO by many groups. They can be found to NLO accuracy in [16], where conformal and supersymmetric constraints were employed in order to reconstruct them from the known NLO DGLAP kernels. A numerical study (limited to parton helicity independent distributions) showed that the effect of NLO evolution was moderate compared with LO evolution [17]. For the model distributions used there, the difference between NLO and LO evolution was a few percent for non-singlet distributions and not more than 10 to 30% in the singlet sector.

4.2 The two-photon processes

The one-loop corrections to deeply virtual Compton scattering have been independently calculated by three groups [10,11]. In addition to diagrams like the one in fig. 1a they involve diagrams with the generalized gluon distributions in the proton. In [10] one finds the NLO results for the general nonforward amplitude $\gamma^* (q) + p(p) \to \gamma^* (q') + p(p')$; in the limit $q = q'$ their imaginary parts reduce to the well-known expressions for unpolarized and polarized deep inelastic scattering. In [11] a numerical study for $\gamma^* p \to \gamma p$ was performed, making an ansatz for the yet unknown generalized quark and gluon distributions. It was found that the NLO corrections can be large, up to about 50%, and depend sensitively on the value of the Bjorken variable $x_B = Q^2 / (2 p \cdot q)$.

By an analytic continuation of the hard scattering kernel, the one-loop corrections for $\gamma^* \gamma \to \pi \pi$ have been obtained from those for the general nonforward Compton amplitude [12]. Numerical studies show that the size of the corrections is very sensitive to the relative size of the two-gluon and the quark-antiquark distribution amplitudes [12,13]. In other words, this process may offer an interesting way to investigate how strongly the two-pion system couples to $gg$ in comparison with $q\bar{q}$.

4.3 Power corrections

The factorized description discussed so far is valid in the limit of infinitely large photon virtuality $Q^2$, and at finite $Q^2$ there are as usual corrections suppressed by powers of $1/Q$, up to logarithmic terms.
An estimation of $1/Q^2$ corrections to deeply virtual Compton scattering and to pion electroproduction has been made in [18] with the help of the renormalon technique, resumming vacuum polarization insertions in the gluon lines of figs. 1a and 4b. The corrections, evaluated at $Q^2 = 4 \text{ GeV}^2$, were found to grow with $x_B$ and to be important (of order 10 to 50%) for the Compton process. For pion production they came out substantially larger (100% and more), with a strong dependence on the ansatz made for the generalized quark distributions.

The structure of the $1/Q$ corrections to the Compton process [19] and its crossed counterpart in $\gamma^*\gamma$ collisions [20] has been completely classified in the framework of the operator product expansion. These corrections factorize into a hard scattering subprocess and generalized parton distributions of twist 3, in contrast to the twist-2 distributions discussed so far. The evolution equations of the twist-3 distributions are known [21], in part to LO and in part to NLO, whereas the NLO corrections to the hard scattering have not been calculated as yet. It is worth mentioning that the $1/Q$ suppressed terms contribute only to amplitudes where the helicities of the two photons differ by 1, whereas the leading contributions only feed amplitudes where the photon helicities are equal or differ by 2. The corresponding helicity amplitudes can be separated using appropriate angular distributions [9,22]; the two-photon processes might therefore provide a window on twist-3 effects that are not masked by large twist-2 pieces.

4.4 Meson production

For meson electroproduction, the one-loop corrections to the hard scattering kernels have not yet been evaluated. In the case of pion production, they are closely connected with the one-loop corrections to the pion form factor in the hard-scattering formalism of Brodsky and Lepage. In fact, the Feynman diagrams for the latter can be obtained from those for pion electroproduction (see fig. 4b) by replacing the quark distribution in the proton with the pion distribution amplitude. The NLO corrections to the pion form factor can but need not be important, depending crucially on the choice of renormalization scale in $\alpha_s$ [23].

For the production of vector mesons, the additional calculation of the one-loop corrections to the gluon exchange diagrams (see fig. 4a) is necessary for a complete NLO evaluation. It would be very interesting to know the size of these corrections. Frankfurt et al. [24] have studied the tree level diagrams, including in the hard scattering process the transverse momentum $k_T$ of the quark-antiquark pair in the vector meson, i.e., replacing the meson distribution amplitude in fig. 4a with the $k_T$ dependent light-cone wave function. This inclusion of this finite $k_T$ effects led to a very strong suppression of the amplitude when a meson wave function was taken that decreases as slowly as $1/k_T^2$ at large transverse momentum. This large-$k_T$ falloff is, however, mediated by hard gluon exchange between the quark and antiquark forming
the meson, and as such should be included not in the meson wave function but in the hard scattering process, where it is a part of the NLO corrections.

5 Summary

Generalized parton distributions and distribution amplitudes provide novel tools to study the interplay between partons and hadrons in QCD. They connect several well-studied concepts such as parton densities, distribution amplitudes, form factors, and light-cone wave functions, and contain information beyond what can be learned from each of these.

These novel quantities can be studied in certain exclusive processes at large momentum transfer, whose investigation is in reach of present-day and future experiments. The description of these processes relies on factorization theorems and thus has a solid basis in QCD.

A quantitatively reliable extraction of generalized parton distributions and distribution amplitudes will require a sufficient understanding of and control over radiative corrections. The logarithmic evolution of these quantities is well studied and the kernels are known to NLO. As to the corrections to the hard scattering subprocess, they are known to NLO in the case of Compton scattering and of $\gamma^*\gamma$ collisions, but a deeper understanding of when and why they are large is still to be achieved. Not much is known about the NLO corrections to meson production, but some pieces of evidence exist that they may be important in certain kinematical situations.

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References


