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SYNCHROTRON TUNE AND BEAM ENERGY AT LEP2

A.S. Muller, J. Wenninger

Abstract

The synchrotron tunes depend on energy loss and total accelerating voltage. This dependency can be used to extract the beam energy in a very precise way. Since the energy calibration of LEP 2 requires the highest possible accuracies detailed systematic studies of the parameters involved in the analysis are necessary. This paper presents method, systematic studies and results of the energy determination from the synchrotron tune.

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SYNCHROTRON TUNE AND BEAM ENERGY AT LEP2

A.-S. Müller, CERN and Mainz University, J. Wenninger, CERN

Abstract

The synchrotron tune depends on energy loss and total accelerating voltage. This dependency can be used to extract the beam energy in a very precise way. Since the energy calibration of LEP2 requires the highest possible accuracies detailed systematic studies of the parameters involved in the analysis are necessary. This paper presents method, systematic studies and results of the energy determination from the synchrotron tune.

1 INTRODUCTION

The precise measurements of the Z and the W boson masses require an excellent knowledge of the centre-of-mass energy at the interaction points. The most precise average beam energy measurement is based on the technique of resonant depolarisation. For the energy calibration in the LEP1 phase relative accuracies of 10^{-5} have been achieved. However, this technique works only in the low energy region (40 – 60 GeV), for the LEP2 phase extrapolation methods have to be used to obtain the beam energy [1]. The study and analysis of the dependence of the synchrotron tune Q_s on the total accelerating voltage provides a powerful way to extract the beam energy making use of existing LEP equipment. Details on method and measurements can be found in [2].

2 SYNCHROTRON OSCILLATIONS AT LEP

For a given machine optics the synchrotron tune depends mainly on the total RF voltage V_{RF} and the beam energy. The synchrotron tune is particularly sensitive to the energy for low RF voltages since the relation between Q_s and the total accelerating voltage V_{RF} is given by

$$Q_s^2 = \left(\frac{\alpha_c h}{2\pi E}\right)\sqrt{e^2\, V_{RF}^2 - U_0^2} \qquad (1)$$

where $U_0 = (C_\gamma/\rho)\, E^4$ is the energy loss, α_c the momentum compaction factor, h the harmonic number and ρ the average magnetic radius. Equation (1) is not suited for a high precision energy determination since it assumes that the RF voltage is homogeneously distributed along the ring, that the synchrotron oscillation amplitudes are small and since it neglects a damping term related to the synchrotron radiation. At LEP2 however these assumptions no longer hold true: the RF cavities are concentrated in the four even straight sections and synchrotron radiation losses of the order of a percent of the beam energy are non-negligible. Obviously a more detailed description of Q_s is needed which takes all these effects into account. The contributions and corrections to eq.(1) needed for an appropriate description of the synchrotron tune are discussed in the following.

2.1 Impact of the RF System

The energy determinations from Q_s are based on measurements of the synchrotron tune as function of the total RF voltage V_{RF}. The effective voltage however can be significantly different from the sum of all individual cavity voltages due to voltage calibration, phasing and longitudinal alignment errors. A crucial correction to eq.(1) consists therefore in introducing a "voltage correction factor" g which translates $V_{RF} \rightarrow g\, V_{RF}$. The energy measurement is strongly correlated to g as can be seen in figure 1 when the energy extracted from the fit is plotted as function of the voltage calibration g. For a proper energy determination the value of g has to be determined from separate datasets at beam energies which have been previously determined very precisely with resonant depolarisation.

The MAD program [3] was used to study the dependence of Q_s on the distribution of the RF voltage. Figure 2 shows Q_s generated for a beam energy of 50 GeV with different RF configurations: a realistic case with the standard LEP RF distribution, a case where the same total voltage is concentrated in one point and the limit of a homogeneous distribution where the voltage is distributed over the whole ring. To account for the RF distribution, a term $M\, V_{RF}^4$ is added to eq.(1) beneath the square root. The weight factor M is taken from a fit to the MAD dataset. This factor includes the corrections of the model needed to account for the approximations in the derivation of the analytical model of eq.(1).

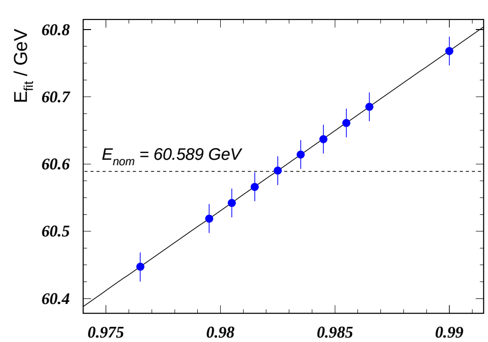

Figure 1: *The fitted energy as function of the input parameter g for a test dataset taken at 60.589 GeV. The χ^2 of the fit increases strongly for extreme values of the voltage calibration factor g.*

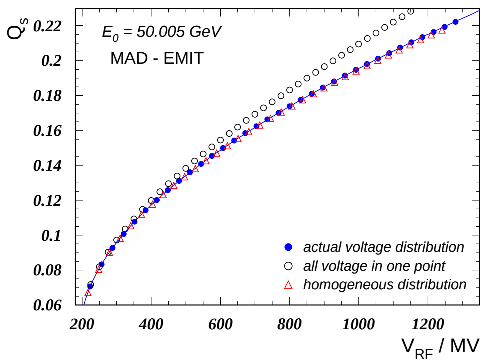

Figure 2: *Synchrotron tune as function of total RF voltage as calculated with* MAD *for different RF configurations. The curve is a fit to the "realistic" RF distribution using the fit model eq.(4) with appropriate input parameters.*

2.2 Total Energy Loss

Besides the main dipoles which are responsible for most of the energy loss, other smaller sources cannot be neglected. These additional losses originate from synchrotron radiation due to beam position offsets in quadrupoles, synchrotron radiation in corrector magnets, parasitic mode losses caused by the resistive part of the longitudinal impedance, synchrotron radiation due to finite beam sizes and from energy offsets for non-central orbits. The energy loss due to synchrotron radiation in sextupole magnets is negligible. The total energy loss U_0 used in eq.(1) has to be written as

$$\tilde{U}_0 = \frac{C_\gamma}{\rho} E^4 + K \qquad (2)$$

where K is the sum of all additional energy losses. The energy loss in a quadrupole is related to a position offset (x_0, y_0) and a finite beam size by $\Delta E \propto k^2\, E^4\, (x_0^2 + y_0^2 + \sigma_x^2 + \sigma_y^2)$ with (σ_x, σ_y), for any density distribution, the RMS size of the transverse beam profile.

Momentum offsets $\Delta p/p$ introduced by a difference between the operation frequency f_{RF} and the central frequency f_{RF}^c (tidal deformations of the earth,...) [4] have to be taken into account:

$$E \;\rightarrow\; E_c \;=\; E\left(1 - \frac{1}{\alpha_c}\,\frac{(f_{RF} - f_{RF}^c)}{f_{RF}^c}\right) \qquad (3)$$

For a of non-zero momentum offset the energy is not matched to the dipole energy and the beam moves off-centre in the quadrupoles. Since the position offset is proportional to $\Delta p/p$, one expects a quadratic dependence of the energy loss on $\Delta p/p$. Figure 3 shows how the relative energy loss changes with momentum deviation for two different energies. The deviation at low and high beam energies is sufficiently similar to find a common parameterisation on which the calculation of corrections can be based.

Besides the synchrotron radiation in the bending dipoles the so-called parasitic mode losses are the largest contribution to the total energy loss. The energy loss per turn

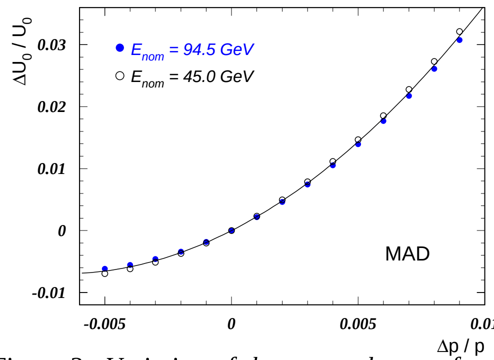

Figure 3: *Variation of the energy loss as function of momentum deviation as calculated with* MAD.

of revolution period $T_{\rm rev}$ and particle is given by $U_{\rm pm} = e\,T_{\rm rev}\,I_b\,\kappa_{||}$ with the longitudinal loss factor

$$\kappa_{||} \;\propto\; \int_0^\infty d\omega\; ReZ_{||}(\omega)\; h(\omega,\sigma)$$

where $h(\omega,\sigma)$ is the spectral power density of the bunch of RMS length σ and $ReZ_{||}(\omega)$ the longitudinal resistive impedance. In order to determine the effective parasitic mode losses, measurements of Q_s as function of RF voltage at an energy of 60.589 GeV were repeated for five bunch currents. Figure 4 shows these measurements. The difference in energy loss due to parasitic mode losses for the different bunch currents is clearly visible. A global (simultaneous) fit to all five datasets allows to extract an effective loss factor $\kappa_{||} = (18.5 \pm 2.0)$ MeV/mA. The residuals of this simultaneous fit shown in fig. 5 reflect the good description of the measurements by the final model eq.(4).

3 BEAM ENERGY MEASUREMENTS

Taking into account all effects described previously Q_s can be expressed as

$$Q_s^4 = \left(\frac{\alpha_c h}{2\pi}\right)^2 \left\{\frac{g^2 e^2 V_{RF}^2}{E_c^2} + M g^4 V_{RF}^4 - \frac{1}{E_c^2}\,\tilde{U}_0^2\right\} \qquad (4)$$

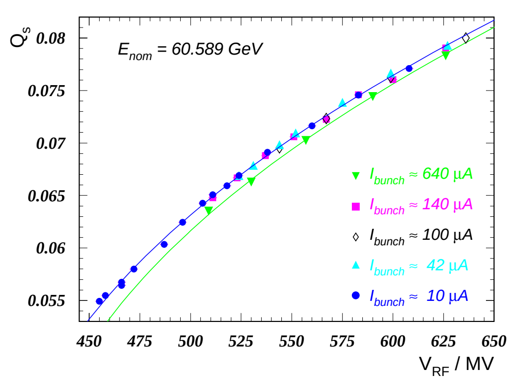

Figure 4: Q_s *as function of total RF voltage. The two curves are individual fits to the 640 μA and 10 μA datasets. The difference in energy loss (parasitic mode losses) is clearly visible.*

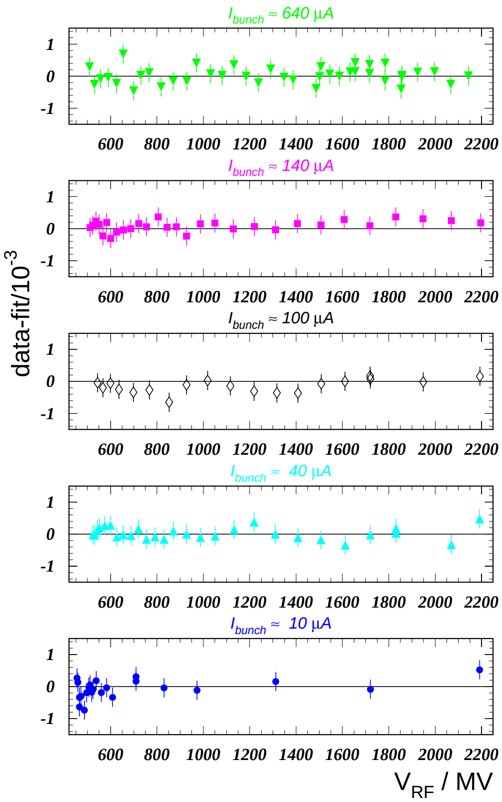

Figure 5: *Differences between data and fit for a global (simultaneous) fit to all five datasets of fig. 4. The residuals of the individual datasets are shown in different plots.*

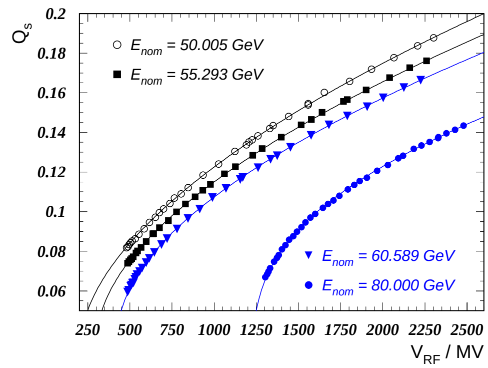

Figure 6: *Q_s as function of total accelerating voltage for measurements at different energies. The curves are fits to the individual datasets.*

with the relations from equations (3,2). A fit of this model to simulation data shows a good agreement between the extracted fit energy and the input energy. The momentum compaction factor α_c and the weight factor M are taken from MAD. All other parameters are allowed to vary in the fit. External knowledge is incorporated in the fit by introducing constraints of the type $(a - a_{\rm nom})^2/\sigma_a^2$ for all parameters where a stands for a fit parameter and σ_a for its uncertainty. The value the parameter is constrained to is denoted by $a_{\rm nom}$. The parameters for additional energy losses, voltage calibration and the weight factor M are constrained in the described way. A Monte Carlo technique is used to estimate the systematic contributions of the parameter constraints to the error of the fit energy. The final error on the beam energy is composed of a "statistical-type" uncertainty $\pm\Delta E$ due to the uncertainties of Q_s, g and K and a "bias-type" uncertainty $+\delta E$ due to the momentum compaction factor and the dipole fringe fields. This "bias-type" uncertainty $+\delta E$ is of the order of 18 MeV and is expected to be reduced significantly by further studies. Figure 6 shows measurements of Q_s as function of voltage at several energies. In table 1 the fit results are compared to the nominal machine energy (obtained from the magnet calibration curves) and to the energies measured with resonant depolarisation in the following fill or estimated with magnetic measurements [1]. All energies given are in GeV. For all measurements the fitted energies agree within their errors with the energies obtained from other methods. The absolute error is essentially energy independent and of the order of 20 MeV.

$E_{\rm nom}$	$E_{\rm pol}$	$E_{\rm nmr}$	$E_{\rm fit}$ ± stat. ± sys.
50.005	50.020	50.015	50.044 ± 0.014 ± 0.029
60.589	60.597	60.594	60.586 ± 0.012 ± 0.021
80.000	–	80.012	79.995 ± 0.011 ± 0.018
80.000	–	80.001	80.004 ± 0.015 ± 0.012
90.419	–	90.458	90.444 ± 0.007 ± 0.017

Table 1: Results of the fits using the model of eq.(4). All energies are given in GeV.

4 SUMMARY

The presented method to extract the beam energy directly from a fit to the measured synchrotron tune as function of total accelerating voltage provides a precise cross-check for the energy calibration of LEP2. The systematics and uncertainties of the method are understood and under control. The error on the beam energy is of the required magnitude for LEP energy calibration and can potentially be reduced by a better understanding of the momentum compaction factor and the bending radius of the LEP dipoles.

5 ACKNOWLEDGEMENTS

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6 REFERENCES

[1] A. Blondel et al., Eur. Phys. J., C 11, 1999.

[2] A.-S. Müller, Ph.D. thesis, U. Mainz (2000), Shaker Verlag and references therein.

[3] H. Grote and F. C. Iselin, SL Note 90-13 (AP) Rev. 4, CERN, March 1995.

[4] L. Arnaudon et al., NIM A357, 1995.