Feynman-Schwinger Representation method for bound states

Çetin Şavklı

Department of Physics, College of William and Mary, Williamsburg, Virginia 23187

In nuclear and particle physics one is often faced with problems where perturbation theory is not applicable. An example of this is the description of bound states. Therefore, an exact solution of field theory to all orders is an unavoidable and interesting problem. Path integrals provide a framework for exact solutions in field theory. In this talk I will present an economical method of evaluating path integrals using the Feynman-Schwinger representation (FSR).

1. Introduction

The basic idea in the Feynman-Schwinger representation is to replace path integrals over quantum fields with path integrals over particle trajectories. The step of going from field configurations to particle trajectories dramatically reduces the number of degrees of freedom.

In this talk we present analytic and numerical applications of Feynman-Schwinger representation [1–8] to nonperturbative problems. The first application is the calculation of 1-body propagator in massive scalar qed. The second application involves the calculation of 2-body bound state masses in scalar $\chi^2\phi$ interaction.

2. 1-body propagator in massive SQED

Massive scalar QED in 0+1 dimension is a simple interaction that enables one to obtain a fully analytical result for the dressed and bound state masses within the FSR approach. In 0+1 dimension contribution of the matter loops identically vanish and therefore quenched FSR calculations give the exact result [6]. In this section we compare the self energy result obtained by three different approaches; namely the simple bubble sum, the Dyson-Schwinger equation, and the Feynman-Schwinger representation.

The Minkowski metric expression for the scalar QED Lagrangian in Feynman gauge is given by

$$L_{SQED} = -m^2\chi^2 - \frac{1}{4}F^2 + \frac{1}{2}\mu^2A^2 - \frac{1}{2}(\partial A)^2 + (\partial_{\mu} - ieA_{\mu})\chi^*(\partial^\mu + ieA^\mu)\chi,$$

where $A$ represents the gauge field of mass $\mu$, and $\chi$ is the charged field of mass $m$. The field tensor $F$ is zero in 0+1 dimensions, and the dynamics is described by the gauge

*This work was supported in part by the US Department of Energy under grant No. DE-FG02-97ER41032. Author thanks F. Gross and J. Tjon for useful discussions.
Figure 1. Various interactions included in each approach are shown. The Feynman-Schwinger approach includes all diagrams. In 1-dimension the contribution of diagrams with loops of charged particles identically vanishes. [6]

fixing term \((\partial A)^2\). The kind of diagrams included in each method are displayed in Fig. 1. The main difference between the rainbow Dyson-Schwinger and the Feynman-Schwinger diagrams is the crossed diagrams. These diagrams involve photon lines that cross each other. The FSR approach also includes all possible four-point interaction contributions while the rainbow DSE only includes the tadpole type four-point interactions. In principle all four-point interactions can also be incorporated into the simple bubble sum and the rainbow DSE.

In Fig. 2 we display all dressed mass results. The bubble summation develops a complex mass pole beyond a critical coupling \(e_{\text{crit}}^2 = 0.4 \text{ (GeV)}^3\). At the critical point a \('collision\' takes place with another real solution, leading to two complex conjugated solutions with increasing \(e^2\). The result obtained from the Dyson-Schwinger Equation displays a similar characteristic. At low coupling strengths the rainbow DS and the bubble results are very close and they converge to the exact result given by the Feynman-Schwinger approach. Similar to the bubble result the DS result develops a complex mass pole at a critical coupling of \(e_{\text{crit}}^2 = 0.49 \text{ (GeV)}^3\). These results clearly show that the solution of the Dyson-Schwinger Equation in rainbow approximation is not a good approximation to the exact result. The second application of the FSR involves bound states in scalar \(\chi^2\phi\) interaction.

3. Two-body bound states in scalar \(\chi^2\phi\)

The application of the FSR to scalar particles have been considered in References [4,7,8]. The Euclidean Lagrangian for this theory is given by

\[
\mathcal{L}_E = \chi^* [m^2 - \partial^2 + g\phi] \chi + \frac{1}{2} \phi(\mu^2 - \partial^2)\phi.
\]
Figure 2. The 1-body mass calculated by the FSR approach, the Dyson-Schwinger equation, and the bubble summation for values of $m = \mu = 1$ GeV. Mass results obtained by bubble summation and the rainbow Dyson-Schwinger equations significantly deviate from the exact result provided by the FSR method.

Here we present the results for the 2-body bound states. The final result for the two-body propagator involves a quantum mechanical path integral that sums up contributions coming from all possible trajectories of particles

$$G = -\int_0^\infty ds \int_0^\infty d\bar{s} \int (\mathcal{D}z)_{xy} \int (\mathcal{D}\bar{z})_{x\bar{y}} e^{-S[Z]},$$

where $S[Z]$ term involves kinetic and interaction energy contributions. Further details on the treatment of scalar interactions can be found in Refs. [7,8]. The interaction kernel $\Delta(x)$ is defined by

$$\Delta(x, \mu) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{ip\cdot x}}{p^2 + \mu^2} = \frac{\mu}{4\pi^2|x|} K_1(\mu|x|).$$

where $\mu$ is chosen to be $\mu = 0.15 GeV$. The ultraviolet singularity in the kernel $\Delta(x, \mu)$ Eq. (4) can be regularized using a Pauli-Villars regularization prescription. The Pauli-Villars mass we choose is $M_{PV} = 3\mu$. In Figure 3 we present the comparison of the 2-body bound state masses obtained by the FSR to various bound state equations. The FSR
Figure 3. The coupling constant dependence of the 2-body bound state mass is shown. The Bethe-Salpeter equation in ladder approximation gives the poorest result (BSE), while the Gross equation (GR) gives the strongest binding among manifestly covariant equations. Inclusion of retardation effects push the Equal-time result (ET) significantly up (ET with retardation shown as MW).

calculation involves summation of all ladder and crossed ladder diagrams, and excludes the self energy contributions. According to Figure 3 all bound state equations underbind. Among the manifestly covariant equations the Gross equation gives the closest result to the exact calculation obtained by the FSR method. This is due to the fact that in the limit of infinitely heavy-light systems the Gross equation effectively sums all ladder and crossed ladder diagrams. Equal-time equation also produces a strong binding but the inclusion of retardation effects pushes the Equal-time results away from the exact results (Mandelzweig-Wallace equation). In particular the Bethe-Salpeter equation in the ladder approximation (BSE in Figure 3) gives the lowest binding. A comparison of the ladder Bethe-Salpeter, Gross, and the FSR results shows that the exchange of crossed ladder diagrams plays a very significant role.
4. Conclusion

Results presented in this talk clearly shows that *approximate calculations in field theory may lead to serious deviations from the exact results*. Therefore it is important to develop rigorous nonperturbative methods. The FSR is a promising candidate for doing nonperturbative calculations in field theory.

REFERENCES