Two-photon annihilation into pion pairs

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Abstract. We discuss pion pair production in two-photon collisions in two different kinematical regimes. When both photons are real and at moderately large center-of-mass energy \( \sqrt{s} \) we elaborate on partonic transverse momentum and Sudakov corrections within the hard scattering approach. We also point out the difference between our approach and that of other authors. When one of the photons is highly virtual the produced pion pair can be described in terms of a two-pion distribution amplitude, for which we derive the perturbative limit at large \( s \).

Due to the pointlike structure of the photon exclusive hadron production in two-photon collisions provides a very useful field for the test of perturbative QCD. In the limit of large \( \sqrt{s} \), the amplitude of \( \gamma^*(\gamma \rightarrow \pi\pi \) factorises into a perturbatively calculable hard photon-parton scattering, which in lowest order can simply be obtained from one-gluon exchange diagrams, and soft parts that are expressed in terms of distribution amplitudes describing the transition of partons to pions [1].

At large c.m. energies \( \sqrt{s} \), transverse momenta of the partons relative to the pion are negligible and the conventional collinear hard scattering formula can be applied [2]. At moderately large \( \sqrt{s} \) of a few GeV, however, the collinear approach is known to suffer severely from substantial endpoint contributions where the strong coupling \( \alpha_s \) becomes large, such that perturbation theory is not applicable [3]. These problems can be overcome by including transverse momenta and Sudakov corrections [4,5]. The correspondingly modified hard

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scattering approach leads to perturbative predictions, which in most cases are not sufficient to account for the experimental data [6–8]. Hadronic form factors and Compton scattering, for example, are dominated by soft contributions at presently accessible c.m. energies [9–11]. In this talk we will discuss Sudakov suppressions in $\gamma\gamma \to \pi^+\pi^-$ in the few GeV region and point out the difference between our approach and that of Ref. [12].

Another interesting, more theoretically motivated application of the hard scattering approach is the process $\gamma^*\gamma \to \pi^+\pi^-$ at large photon virtuality $Q^2$ and large $s$. In this kinematical regime, we will briefly outline the calculation of the perturbative limit of the two-pion distribution amplitude (2π-DA) [13].

**SUDAKOV SUPPRESSION IN $\gamma\gamma \to \pi^+\pi^-$**

In close analogy to the calculation of hadronic form factors [5–7] in the modified perturbative approach we can express the helicity amplitude $M_{\lambda\lambda'}$ of the process $\gamma\gamma \to \pi^+\pi^-$ in transverse configuration space as the convolution

$$
M_{\lambda\lambda'}(s, \Theta) = \int dx dy d^2 b_\perp d^2 b'_\perp \frac{4\pi}{4\pi} \hat{\Psi}_\pi(x, b_\perp) \hat{\Psi}_\pi(y, b'_\perp) \hat{T}_{H,\lambda\lambda'}(x, y, b_\perp, b'_\perp; s, \Theta, \mu_R) \exp[-S(x, y, b_\perp, b'_\perp; \mu_R)],
$$

where $\Theta$ is the scattering angle in the center-of-mass system of the produced pions and $\lambda$, $\lambda'$ are the photon helicities. The hat denotes the Fourier transform of a function w.r.t. the transverse momenta $k_\perp$, $k'_\perp$ of the partons relative to the pions. The Fourier conjugated variables $b_\perp$, $b'_\perp$ are the transverse separations of the quark-antiquark pairs and $x$, $y$ describe how they share the pions’ longitudinal momenta.

Using a phenomenological ansatz for the wave function of the pion’s valence Fock state we write

$$
\Psi_\pi(x, k_\perp) = \frac{\sqrt{6\pi}}{f_\pi} \exp \left[ -\frac{k_\perp}{8\pi f_\pi^2 x (1 - x)} \right],
$$

with $f_\pi = 131$ MeV being the pion decay constant. Integrating Eq. (2) over transverse momenta leads to the asymptotic form of the pion distribution amplitude $\phi_\pi$. The use of the asymptotic form is justified through the phenomenology of the $\pi$-$\gamma$-transition form factor [15] and the parameters of expression (2) are fixed by various pion decay processes [14]. The Gaussian $k_\perp$-dependence describes well soft contributions [6,10,11].

The Sudakov corrections are incorporated in the factor $e^{-S}$, where $S$ is the Sudakov function [4] (see also [16]). Since it suppresses large quark-antiquark separations it serves as a natural infrared cut-off and thus no external regulator is needed to avoid the singularity of $\alpha_s$. 
In leading order QCD, the hard photon-parton scattering amplitude $T_H$ is to be calculated from 20 one-gluon exchange diagrams, four representatives of which are shown in Fig.1. Following the authors of Ref. [5] we choose the renormalisation scale $\mu_R$ to be the largest mass scale appearing in the gluon virtualities. Owing to the structure of the hard scattering amplitude its analytical Fourier transform cannot be calculated exactly and we have to resort to approximations. As the longitudinal momentum fractions occur quadratically in the gluon propagators we keep transverse momentum corrections there if not otherwise stated.

Our results for the differential cross section at $\Theta = 90^\circ$ using different approximations for the quark propagators are shown in Fig.2. The dot-dashed line shows the result obtained by replacing the quark propagators by their collinear limits. In the solid curve we take into account transverse momenta in quark propagators in those integration regions where they have singularities. We see that the effect in the few GeV region is dramatic, which means that one can generally not ignore $k_\perp$-corrections in quark propagators, as has been done in Ref. [17], for instance. For comparison we also show the result of the collinear hard scattering approach, i.e. completely neglecting $k_\perp$-corrections in quark as well as in gluon propagators, where we have frozen $\alpha_s$ below 1 GeV (dashed line). We note that our main result, given by the solid curve, approaches the collinear approximation for $s \gtrsim 20$ GeV$^2$, i.e. for c.m. energies above 4-5 GeV. In brief, the $k_\perp$-corrections of the quark propagators effect the transition amplitude such that it reduces its absolute magnitude while receiving a large phase.

In Fig. 3 we therefore only compare our upper estimate, obtained by ignoring transverse momenta in the quark propagators and given by the dot-dashed line, with the data of the combined cross section $\sigma(\gamma\gamma \rightarrow \pi^+\pi^-, K^+K^-)$ of Ref. [18], where we have accounted for the contributions from kaons by a relative factor $(f_K/f_\pi)^4 \simeq 2.2$. For comparison we again show the collinear approximation. As we can see, the curves are already far below the data, so that the inclusion of $k_\perp$-corrections in the quark propagators would further increase the discrepancy.

In Ref. [2] it was shown that, in the collinear approximation, essential parts of the hard scattering amplitudes are accidentally proportional to the pion form factor. Using this relation and a phenomenological value for $F_\pi$ the authors of [12] obtained reasonable agreement with the data. However, we would like to emphasise that the assumed value for the pion form factor, $Q^2 F_\pi(Q^2) = 0.3$ GeV$^2$, is rather large for a perturbative calculation. With a renormalisation scale of the order of the typical virtuality of the exchanged gluon and using the asymptotic form of the pion distribution amplitude, the pion form factor in the collinear approach reads [1] $Q^2 F_\pi(Q^2) = 8\pi f_\pi\alpha_s(Q^2)$ and ranges between 0.17 and 0.1 GeV$^2$ for $1$ GeV $\lesssim Q \lesssim 4$ GeV. Since the pion form factor enters the cross section for $\gamma\gamma \rightarrow \pi^+\pi^-$ quadratically that accounts for the difference between our result for the collinear approximation
and that of Ref. [12].

Finally, we would like to point out that with the inclusion of $k_\perp$-corrections in the hard scattering amplitude the simple relation between the cross section and the pion form factor does not longer hold. In particular, our predictions are independent of any phenomenological value for the pion form factor.

**THE PERTURBATIVE LIMIT OF THE 2\(\pi\)-DA**

We now turn to the kinematical regime where one of the photons has a large virtuality $Q^2$. In Refs. [19] it was shown that for $s \ll Q^2$ the helicity amplitude of $\gamma^*\gamma \rightarrow \pi\pi$ factorises in a hard part and a generalised distribution amplitude $\Phi_{2\pi}$:

$$M_{\lambda\lambda'}(\zeta, s) = \frac{1}{2} \delta_{\lambda\lambda'} \sum_q e_0^2 e_q^2 \int_0^1 dz \frac{2z - 1}{z(1 - z)} \Phi_{2\pi}^q(z, \zeta, s), \quad (3)$$

where the light-cone fractions $z = k^+/P^+$ and $\zeta = p^+/P^+$ respectively describe how the partons and the pions share the light-cone plus component of the total momentum $P = p + p'$ of the pions and the sum runs over all quark flavours $q$. The 2-$\pi$-DA, first discussed in [20], represents the collinear hadronisation of two partons into a pion pair. The helicity selection rule, expressed through the Kronecker delta, immediately follows from the collinear scattering of massless quarks. Note that apart from logarithmic corrections the leading order expression (3) is completely independent of $Q$.

If we demand that $s, -t, -u \gg \Lambda^2$, where $\Lambda$ is a typical hadronic scale of the order of 1 GeV, while keeping the constraint that the photon virtuality is the dominant scale, $s \ll Q^2$, we can use the conventional hard scattering approach [1] to calculate the helicity amplitude (3) in terms of the hard scattering amplitude $T_H$ and two single pion DAs $\phi_{\pi}$:

$$M_{\lambda\lambda'}(s, t, u) = \frac{f_{\pi}^2}{24} \int_0^1 dx dy \phi_{\pi}(\bar{y}) \phi_{\pi}(x) T_{H,\lambda\lambda'}(x, y, s, t, u). \quad (4)$$

Using light-cone gauge and organising the result in powers of $s/Q$ one can show [13] that the leading contributions are independent of $Q$, reflecting the correct scaling behaviour, and come from the diagrams of the group B in Fig. 1. Moreover, the helicity selection rule of Eq. (3) is reproduced. Reexpressing Eq. (4) through the light-cone fractions $z$ and $\zeta$ for each diagram, we can then read off the large-$s$ limit of the 2-$\pi$-DA for a flavour $q = u$ by comparison of Eqs. (3) and (4):

$$\Phi_{u2\pi}^q(z, \zeta, s) = \frac{8\pi f_{\pi}^2}{9} \left\{ \Theta(\zeta - z) \frac{\zeta}{\zeta - z} \phi_{\pi}\left(\frac{z}{\zeta}\right) I(\bar{z}, \bar{\zeta}, s; \phi_{\pi}) \right. \right.$$

$$- \Theta(z - \zeta) \frac{\bar{\zeta}}{\bar{z} - \zeta} \phi_{\pi}\left(\frac{\bar{z}}{\zeta}\right) I(z, \zeta, s; \phi_{\pi}) \right\}, \quad (5)$$
where the integral $I$ is given by

$$I(z, \zeta, s; \phi) = \int_0^1 dx \frac{\alpha_s}{s} \frac{z^+ \zeta}{x-\zeta} \phi_n(x).$$

The $2\pi$-DAs for u- and d-quarks are related by $\Phi^u_{2\pi}(z, \zeta, s) = -\Phi^d_{2\pi}(\bar{z}, \bar{\zeta}, s)$ and since higher Fock states are suppressed by powers of $\alpha_s/s$ there is no s-quark contribution. The $1/s$ scaling of Eq. (5) is a characteristic feature of the hard scattering approach [1,2].

Our result manifestly fulfills the charge conjugation relation $\Phi^q_{2\pi}(z, \zeta, s) = -\Phi^{\bar{q}}_{2\pi}(\bar{z}, \bar{\zeta}, s)$ and it can be shown to comply with a general polynomiality condition [21]. It possesses integrable logarithmic singularities at $z = \zeta$, which reflect the above mentioned endpoint problems of the collinear hard scattering approach when the exchanged gluon becomes soft.

**CONCLUSIONS**

Two-photon annihilation into pion pairs allows for a sensitive test of perturbative QCD. Using a self-consistent approach, where there are no large endpoint contributions spoiling the applicability of perturbation theory, we have shown that the hard contributions are not sufficient to explain the experimental data of $\gamma\gamma \rightarrow \pi^+\pi^-$. Therefore considerable soft contributions have to be expected. New data are desirable to determine the onset of the perturbative regime, which seems not to start below c.m. energies of 4-5 GeV. When one of the photons is far off-shell and at large $s$, where transverse momenta become irrelevant, the collinear hard scattering approach can be applied to calculate the perturbative limit of the $2\pi$-DA in terms of the conventional pion distribution amplitudes.

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**REFERENCES**

**FIGURE 2.** Differential cross section at c.m. angle $\Theta = 90^\circ$ with different approximations.

**FIGURE 3.** The combined cross section $\sigma(\gamma\gamma \rightarrow \pi^+\pi^-, K^+K^-)$ as a function of the c.m. energy $W = \sqrt{s}$. 