Intermediate mass strangelets are positively charged

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Abstract

For a limited range of parameters, stable strange quark matter may be negatively charged in bulk due to one gluon exchange interactions. However, the reduction in strange quark occupation in the surface layer, which is responsible for surface tension, more than compensates this for intermediate mass strangelets, which therefore always have positive quark charge (e.g. for baryon number between 10^2 and 10^{18} assuming $\alpha_S = 0.9$). While details are sensitive to the choice of renormalisation, the general conclusion is not. This rules out a scenario where negatively charged strangelets produced in ultrarelativistic heavy ion colliders might grow indefinitely with potentially disastrous consequences.

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There has recently been some concern about the possibility that a negatively charged strangelet formed in ultrarelativistic heavy ion collisions might grow by absorbing nuclei to, in principle, swallow the Earth. Two investigations have described a number of reasons, theoretical as well as experimental, why such a scenario is exceedingly unlikely [1].

Many unlikely coincidences are required for a dangerous situation to develop. Positively charged quark matter repels ordinary nuclei, so at the very least, the small strangelets (baryon number $A \ll 4 \times 10^2$, where 4×10^2 is the total number of nucleons involved in a collision of two gold or lead nuclei) hypothetically formed in ultrarelativistic heavy ion collisions must have negative quark charge. And for growth to proceed, not only must the ultimate state of stable bulk strange quark matter be negatively charged, but so must the path of intermediate mass strangelets all the way from low A to $A \rightarrow \infty$.

The present study demonstrates, that even in the unlikely case where bulk strange quark matter is both stable and has negative quark charge, and where a long-lived, negatively charged strangelet is formed in a collision, such a strangelet *cannot* grow significantly by absorbing nuclei. The reason is, that the quark charge of intermediate mass strangelets near the lowest energy state is always positive regardless of the bulk charge. Thus the potentially dangerous strangelet growth path is blocked by Coulomb repulsion.

Following the original suggestions of Bodmer [2] and Witten [3], Farhi and Jaffe [4] presented a detailed study of strange quark matter properties in bulk, as well as finite size strangelets. Concerning the issue of the electric charge of bulk strange matter they showed, that the quark charge of stable strange matter is always positive for small strong interaction constant, α_S [5]. This is easy to understand physically, because of the fortuitous cancellation of charge for a gas with equal numbers of up, down and strange quarks. For finite strange quark mass and up and down quark masses close to zero, the number of s-quarks will be reduced, and the net quark charge be positive, even though the actual numbers are small, with a typical charge-to-baryon number ratio of 10^{-3}–10^{-7}. One gluon exchange interactions are repulsive for massless up and down quarks, but attractive for massive, nonrelativistic quarks. Thus, for sufficiently high α_S Farhi and Jaffe showed, that there is a part of parameter space where this effect increases the abundance of s-quarks enough to give the system a total negative quark charge. Farhi and Jaffe noted, that details were quite sensitive to the choice of renormalisation point, and speculated that it might not be stable with respect to including higher orders in α_S. But negative quark charge for bulk strange quark matter seems a valid albeit unlikely possibility.

The energy and quark content of finite size lumps of strange quark matter (sometimes called strangelets, at least in the low-A regime) is modified relative to that of bulk strange quark matter. These finite size effects (surface tension and curvature energy) generally destabilize strangelets [4,6–10], but they also change the overall charge of the system, and as shown below they always lead to a wide range of intermediate mass strangelets with positive rather than negative charge. Physically because the main effect of the finite size is a depletion of s-quarks in the surface layer, which removes negative charge from the system relative to the bulk solution. The underlying mechanism is a simple consequence of quantum mechanics. Whereas a relativistic, low-mass quark may have nonzero density at the system boundary, the wave function (and therefore the density) must be zero in the nonrelativistic limit of a very massive quark. The massive s-quarks are therefore suppressed more than u and d near the surface [11].

It will be shown below, that finite size effects increase the total charge per baryon by an amount $\Delta Z/A \approx 0.3A^{-1/3}$. This number is to be compared with a negative charge per baryon (numerically) less than 10^{-6} for stable bulk quark matter with $\alpha_S = 0.9$, and 10^{-3} for $\alpha_S = 1.2$. Thus the total charge of strangelets becomes positive for $A < 10^{18}$ and $A < 10^7$ respectively.

In the ideal Fermi-gas approximation the energy of a system composed of quark flavors i is given by

$$E = \sum_i (\Omega_i + N_i \mu_i) + BV. \tag{1}$$

Here Ω_i, N_i and μ_i denote thermodynamic potentials, total number of quarks, and chemical potentials, respectively. B is the bag constant, V is the bag volume. (For simplicity, all thermodynamical equations are written without explicit electron/positron terms. Such terms were included in the numerical calculations).

In the multiple reflection expansion framework of Balian and Bloch [12], the thermodynamical quantities can be derived from a density of states of the form

$$\frac{dN_i}{dk} = 6\left\{\frac{k^2 V}{2\pi^2} + f_S\left(\frac{m_i}{k}\right) kS + f_C\left(\frac{m_i}{k}\right) C + ...\right\}, \tag{2}$$

where area $S = 4\pi R^2$ and curvature $C = 8\pi R$ for a sphere. The volume term is universal, whereas the functions f_S and f_C depend on the boundary conditions.

The number of quarks of flavor i is

$$N_i = \int_0^{k_{Fi}} \frac{dN_i}{dk} dk = n_{i,V} V + n_{i,S} S + n_{i,C} C, \tag{3}$$

with Fermi momentum $k_{Fi} = \mu_i (1 - \lambda_i^2)^{1/2}$; $\lambda_i \equiv m_i/\mu_i$.

The corresponding thermodynamical potentials are

$$\Omega_i = \Omega_{i,V} V + \Omega_{i,S} S + \Omega_{i,C} C, \tag{4}$$

where $\partial\Omega_i/\partial\mu_i = -N_i$, and $\partial\Omega_{i,j}/\partial\mu_i = -n_{i,j}$.

Minimizing the total energy at fixed N_i gives the pressure equilibrium constraint

$$B = -\sum_i \Omega_{i,V} - \frac{2}{R}\sum_i \Omega_{i,S} - \frac{2}{R^2}\sum_i \Omega_{i,C}. \tag{5}$$

Eliminating B from Eq. (1) then gives the energy for a spherical quark lump as

$$E = \sum_i (N_i \mu_i + \frac{1}{3}\Omega_{i,S} S + \frac{2}{3}\Omega_{i,C} C). \tag{6}$$

For a bulk gas at zero external pressure these are just the well-known conditions $0 = P = \sum_i -\Omega_{i,V} - B$; $E = \sum_i N_i \mu_i$.

In the following I will assume MIT bag model boundary conditions [13], but the main conclusion depends only on the fact that f_S is negative.

The volume terms including first order α_S corrections are given by

$$\Omega_{i,V} = -\frac{\mu_i^4}{4\pi^2}\left((1-\lambda_i^2)^{1/2}(1-\frac{5}{2}\lambda_i^2) + \frac{3}{2}\lambda_i^4 \ln\frac{1+(1-\lambda_i^2)^{1/2}}{\lambda_i}\right.$$
$$-\frac{2\alpha_S}{\pi}\left[3\left[(1-\lambda_i^2)^{1/2} - \lambda_i^2\ln(1+(1-\lambda_i^2)^{1/2})\right]^2\right.$$
$$-2(1-\lambda_i^2)^2 - 3\lambda_i^4\ln^2\lambda_i$$
$$\left.\left.+6\ln\left(\frac{\sigma}{\mu_i}\right)\left(\lambda_i^2(1-\lambda_i^2)^{1/2} - \lambda_i^4\ln\left(\frac{1+(1-\lambda_i^2)^{1/2}}{\lambda_i}\right)\right)\right]\right), \quad (7)$$

$$n_{i,V} = \frac{\mu_i^3}{\pi^2}\left((1-\lambda_i^2)^{3/2}\right.$$
$$-\frac{2\alpha_S}{\pi}\left[\frac{3}{2}\left[(1-\lambda_i^2)^{1/2} - \lambda_i^2\ln(1+(1-\lambda_i^2)^{1/2})\right]\right.$$
$$\times\left[(1-\lambda_i^2)^{1/2} + (1-\lambda_i^2)^{-1/2}\right.$$
$$\left.-\lambda_i^4(1-\lambda_i^2)^{-1/2}(1+(1-\lambda_i^2)^{1/2})^{-1}\right]$$
$$-2(1-\lambda_i^2) + \frac{3}{2}\lambda_i^4\ln\lambda_i$$
$$-\frac{3}{2}\left[\lambda_i^2(1-\lambda_i^2)^{1/2} - \lambda_i^4\ln\left(\frac{1+(1-\lambda_i^2)^{1/2}}{\lambda_i}\right)\right]$$
$$\left.\left.+\frac{3}{2}\ln\left(\frac{\sigma}{\mu_i}\right)(-\lambda_i^4 - \lambda_i^3 + 2\lambda_i^2)(1-\lambda_i^2)^{-1/2}\right]\right). \quad (8)$$

The expression for $\Omega_{i,V}$ is taken from [4] apart from correction of a sign misprint. Farhi and Jaffe discuss at length the choice of renormalisation scale, σ. In accordance with their work I have picked a value of $\sigma = 313$MeV in the following. Details of the numerical results are sensitive to this choice, especially at high α_S, but calculations for other choices of σ assure that the conclusions are invariant.

The surface contribution follows from $f_S(m/k) = -\left[1-(2/\pi)\tan^{-1}(k/m)\right]/8\pi$ [6]

$$\Omega_{i,S} = \frac{3}{4\pi}\mu_i^3\left[\frac{(1-\lambda_i^2)}{6} - \frac{\lambda_i^2(1-\lambda_i)}{3}\right.$$
$$-\frac{1}{3\pi}\left(\tan^{-1}\left[\frac{(1-\lambda_i^2)^{1/2}}{\lambda_i}\right] - 2\lambda_i(1-\lambda_i^2)^{1/2}\right.$$
$$\left.\left.+\lambda_i^3\ln\left[\frac{1+(1-\lambda_i^2)^{1/2}}{\lambda_i}\right]\right)\right]; \quad (9)$$

$$n_{i,S} = -\frac{3}{4\pi}\mu_i^2\left[\frac{(1-\lambda_i^2)}{2} - \frac{1}{\pi}\left(\tan^{-1}\left[\frac{(1-\lambda_i^2)^{1/2}}{\lambda_i}\right]\right.\right.$$
$$\left.\left.-\lambda_i(1-\lambda_i^2)^{1/2}\right)\right]. \quad (10)$$

For massless quarks $\Omega_{i,S} = n_{i,S} = 0$. For massive quarks $\Omega_{i,S}$ (which is always positive, except for $\lambda_i = 0$ or 1) has the properties of a surface tension. The corresponding change in quark number per unit area, $n_{i,S}$, is always negative, approaching 0 for $\lambda_i \to 0$ (massless quarks) or 1 (infinitely massive quarks, i.e. no i-quarks present).

Curvature terms ($\Omega_{i,C}$ and $n_{i,C}$) are decisive for understanding the average physical properties of small $A < 100$ strangelets, but are not very important for the results at higher

A. They were selfconsistently included in the numerical calculations, using the expressions from [10]. Coulomb energies are negligible because of the low charge-to-mass ratio.

Figures 1–4 show results of applying the above equations to strange quark matter at $\alpha_S = 0.9$, varying B and m_s, and assuming massless u- and d-quarks. Figure 1 shows contour curves for equal energy per baryon of bulk strange quark matter in chemical equilibrium, i.e. with $\mu_d = \mu_s$ maintained mainly by $u+d \leftrightarrow s+u$, and $\mu_d = \mu_u + \mu_e$, normally maintained by reactions like $d \rightarrow u + e^- + \bar{\nu}_e$. Typical quark chemical potentials are 300 MeV, whereas μ_e is a few MeV. A special situation occurs when $-m_e < \mu_e < m_e$, since here the weak reactions involving electrons or positrons are blocked, and the total quark charge of the system is zero with neither electrons nor positrons present. For $\mu_e > m_e$ local charge neutrality is maintained by a small fraction of electrons, whereas positrons take over for $\mu_e < -m_e$. Dotted curves in Fig. 1 are contours for fixed number of electrons per baryon (positrons counted as negative number of electrons), which equals the net quark charge per baryon, Z/A. A characteristic property of strange quark matter is a very low Z/A, except for very high m_s, where s-quarks are energetically unfavorable. For $-m_e < \mu_e < m_e$, Z/A becomes 0, but for low α_S this happens only for unrealistically low m_s, and negative bulk charge does not occur at all. For high α_S, the regime of zero charge propagates to intermediate m_s values, and a region of negative bulk Z/A shows up. For $\alpha_S = 0.9$ the bulk Z/A is still numerically very small, reaching only values of -10^{-6}, whereas $Z/A \approx -10^{-3}$ may be reached for a high $\alpha_S = 1.2$. It is these regimes of negative (or to a lesser extent zero) charge which could in principle lead to disastrous consequences if a negative strangelet were formed in ultrarelativistic heavy ion collisions and began to grow.

However, the inclusion of finite size effects rules out such a disaster scenario, because they reduce the number of negatively charged s-quarks in the surface layer sufficiently to render the total Z/A positive, as illustrated in Figures 2–4.

Quantitatively, stable strangelets are electrically positive for radii from a few fm up to 10^6 fm ($10^2 < A < 10^{18}$) for $\alpha_S = 0.9$, and for $10^2 < A < 10^7$ for $\alpha_S = 1.2$. This is seen from the detailed numerical calculations, but can be understood qualitatively from the expression for $n_{s,S}$, which is the main contributor to the change in charge for $A > 10^2$. Thus $\Delta Z \approx -(1/3) n_{s,S} S \approx 0.18(\mu_s R)^2$ (the number assuming maximum depletion of s-quarks, but the actual depletion is close to this for a wide range of λ_s). With $A \approx V\mu_d^3/\pi^2$ and inserting typical numbers for chemical potentials, this leads to

$$\Delta Z/A \approx 0.3 A^{-1/3} \approx 0.3 R_{\mathrm{fm}}^{-1}, \tag{11}$$

in close agreement with the results of the full numerical calculations.

The numerical results depend on the choice of renormalisation, but qualitatively similar results are found for other choices of σ. A perturbative expansion to first order is questionable at high α_S, and one might also worry that the finite size terms being known only to zeroth order could be inconsistent, but since these terms are perturbations on the bulk solution itself, this omission should matter only at higher order. The general qualitative result, demonstrated here in quantitative detail to first order in α_S within the MIT bag model, namely that surface depletion of s-quarks removes sufficient negative charge to make strangelets positive below some high A-value, seems quite robust. It depends only on the fact that f_s is negative, which leads to a negative $n_{s,S}$, and a positive surface tension. This would seem a natural condition in any model for stable strangelets [11].

Thus the conclusion is that even within the limited range of parameters where bulk strange quark matter is stable and negatively charged, intermediate mass strangelets have positive quark charge because of surface depletion of s-quarks. Even if very small strangelets might still be created with negative charge in ultrarelativistic heavy ion collisions due perhaps to stabilizing shell effects [14] (such effects would play no practical role at high A), they would not be able to bridge the positively charged regime of intermediate A on their way to the putative state of bulk, negatively charged strange quark matter. This rules out the disaster scenarios discussed recently.

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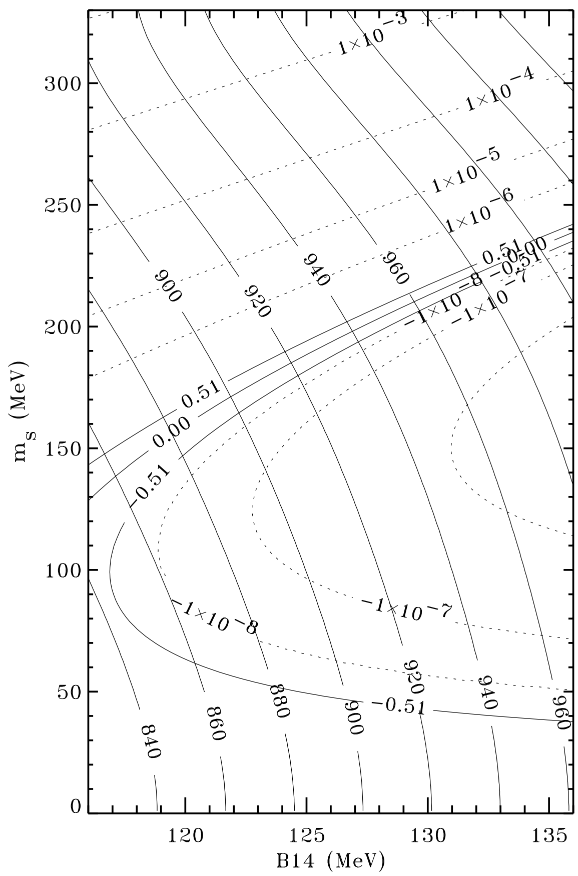


FIG. 1. Solid curves going from upper left to lower right show constant energy per baryon of bulk strange quark matter in MeV as a function of $B^{1/4}$ and m_s for $\alpha_S = 0.9$. B is bounded from below by requiring that $E/A > 930$ MeV for up-down quark matter ($m_s \rightarrow \infty$). Dotted curves give total number of electrons per baryon in the system (equal to the net quark charge per baryon). Solid curves labeled 0.51, 0, and -0.51 indicate limits where the electron chemical potential equals m_e, 0, and $-m_e$. For $-m_e < \mu_e < m_e$ bulk quark matter is kept electrically neutral by quarks alone. Below $-m_e$ the net quark charge becomes negative (as indicated by negative numbers along the dotted curves), but is neutralised by positrons.

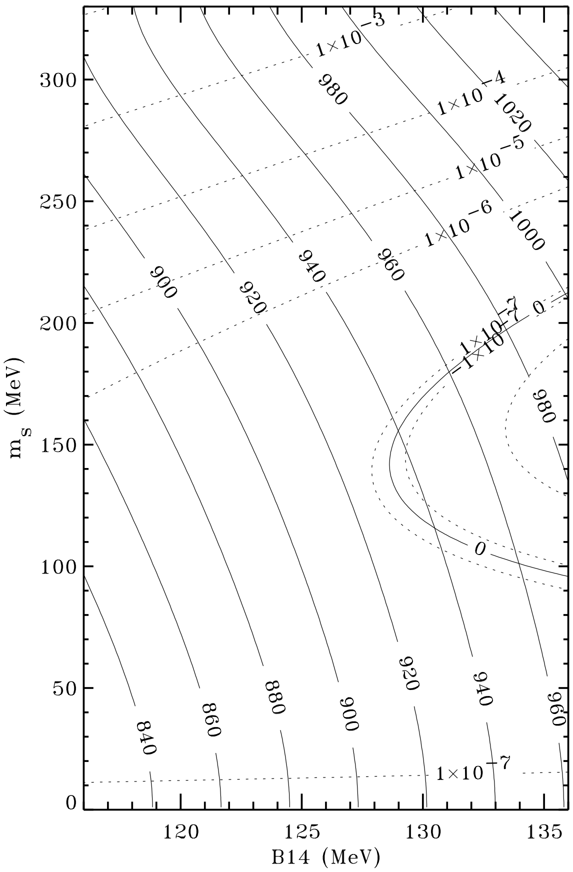


FIG. 2. As Figure 1, but for a strangelet with radius 10^6fm, corresponding to $A \approx 10^{18}$. Negative quark charge is now limited to the small region in the middle right part of the diagram. For systems smaller than this, finite size effects make the total quark charge positive for all stable strangelets ($E/A < 930$ MeV).

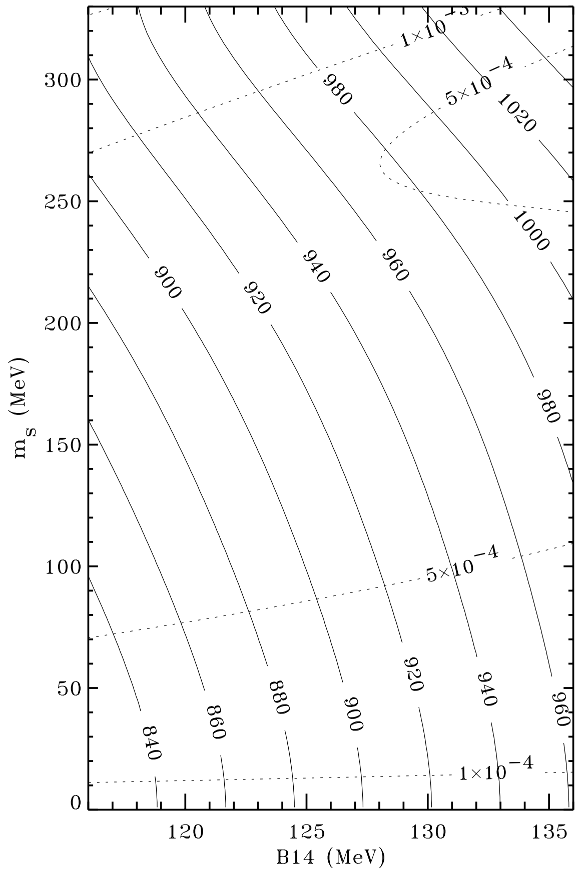


FIG. 3. As Figure 2, but for $R = 1000$ fm ($A \approx 10^9$). The total quark charge is positive throughout the diagram, typically with $Z/A \approx 5 \times 10^{-4}$.

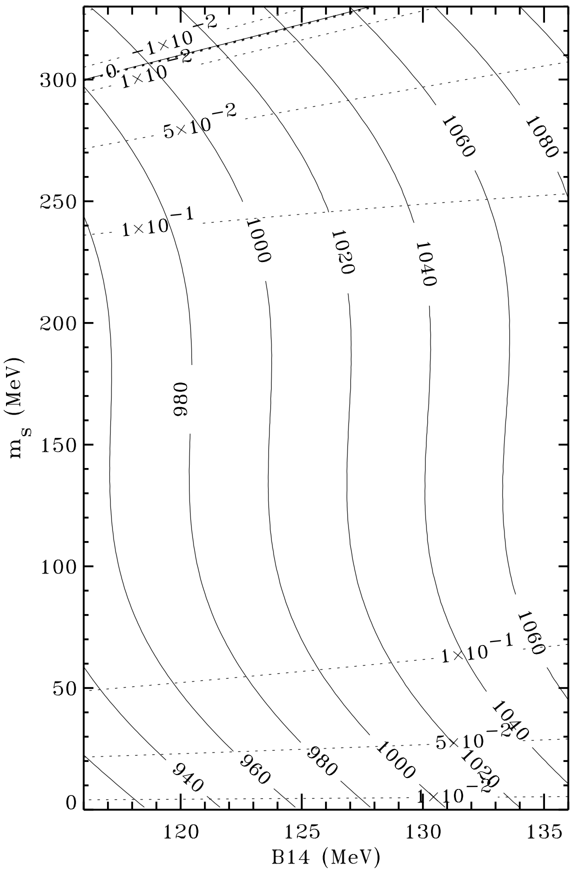
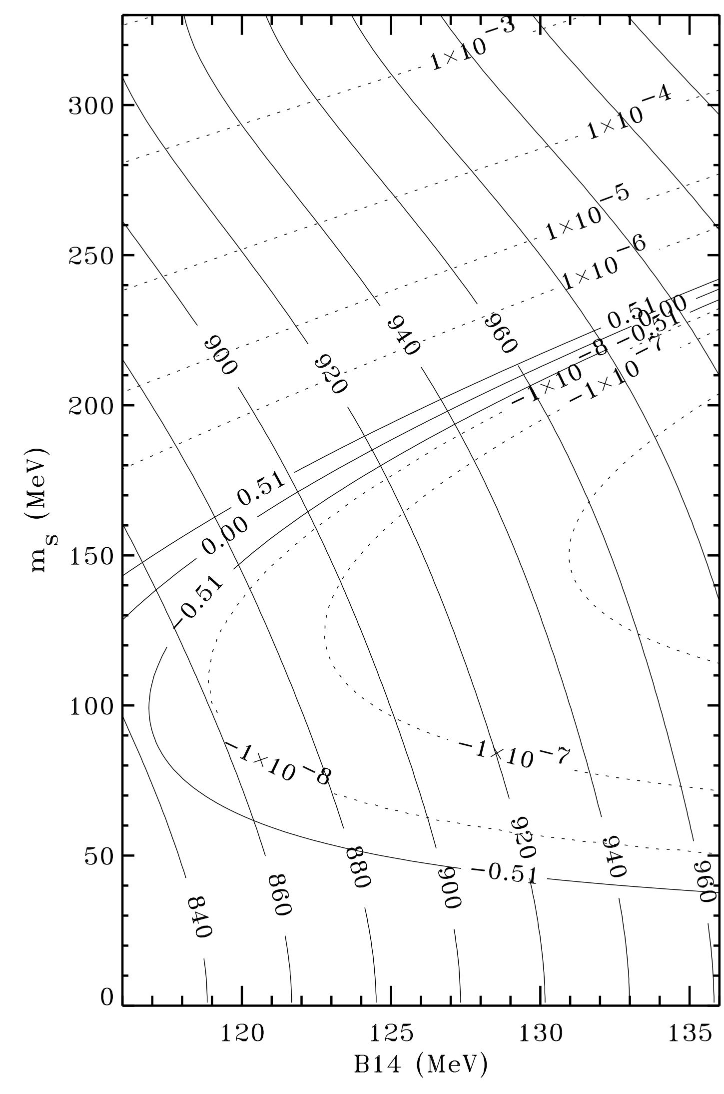
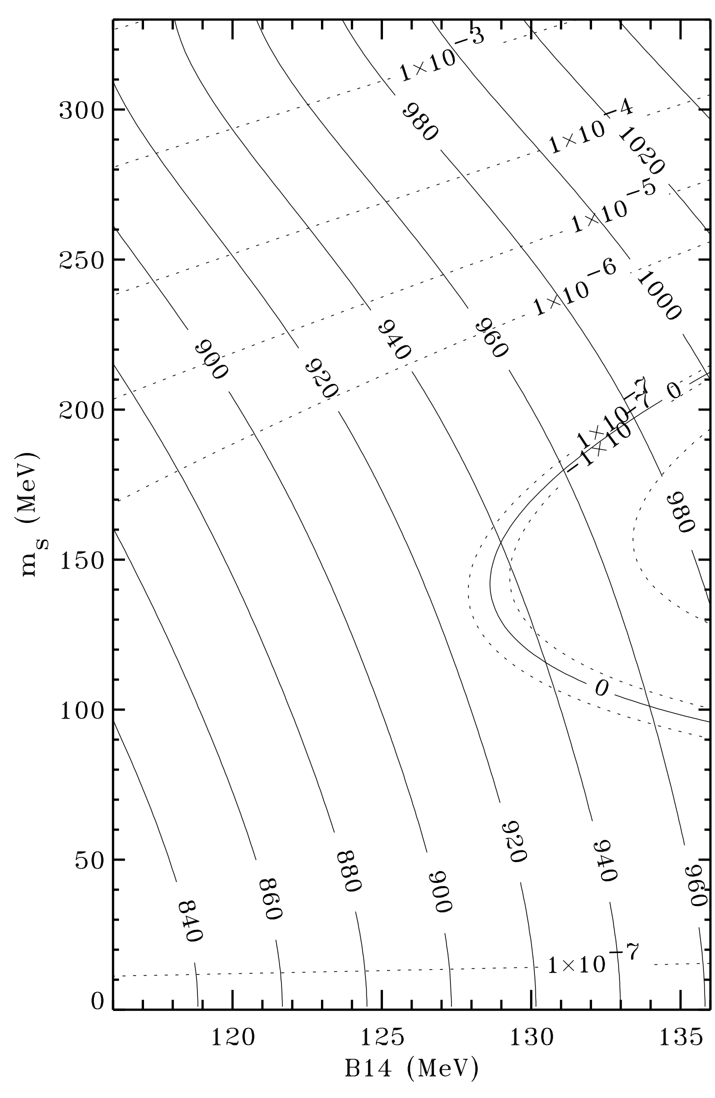


FIG. 4. As Figure 2, but for $R = 3$ fm ($A \approx 30$). Z/A is positive almost everywhere, but a region of negative strangelet charge appears for high m_s, caused mainly by the increasingly important curvature contributions. Notice the significant increase in energy per baryon typical of small strangelets because of surface and curvature energy contributions. These small strangelets are at most metastable.



m_s (MeV)
B14 (MeV)
1×10−3
1×10−4
1×10−5
1×10−6
0.51
0.00
−0.51
−1×10−8
−1×10−7
900
920
940
960
840
860
880



m_s (MeV)
B14 (MeV)
0
50
100
150
200
250
300
120
125
130
135
840
860
880
900
920
940
960
980
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1020
1×10^-3
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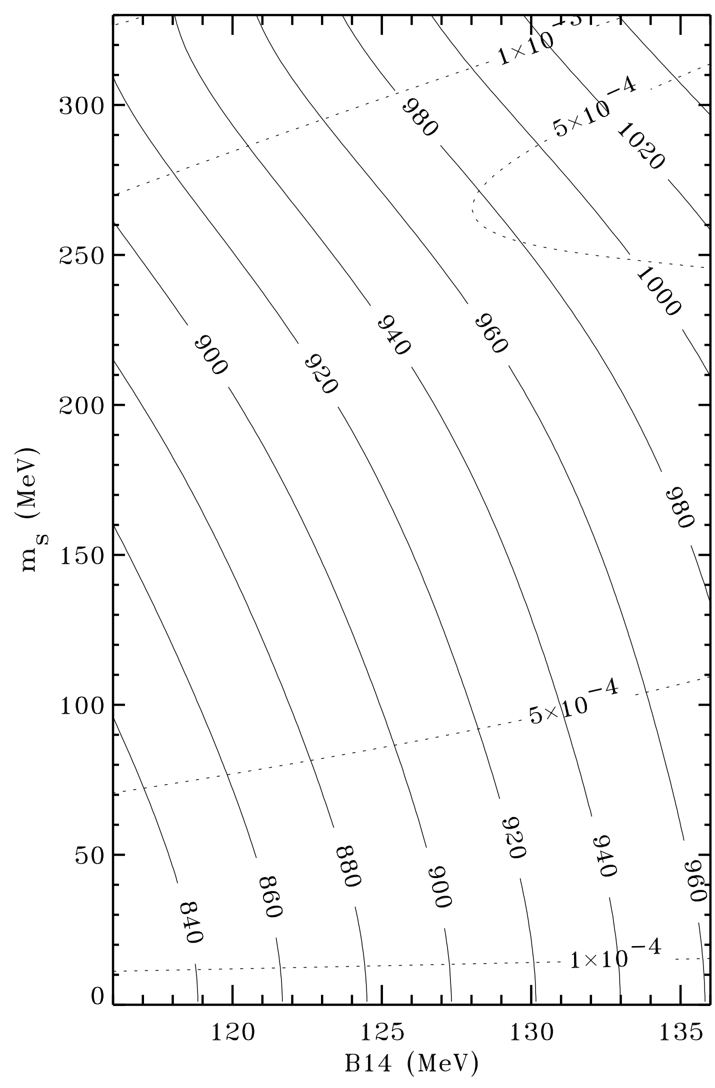
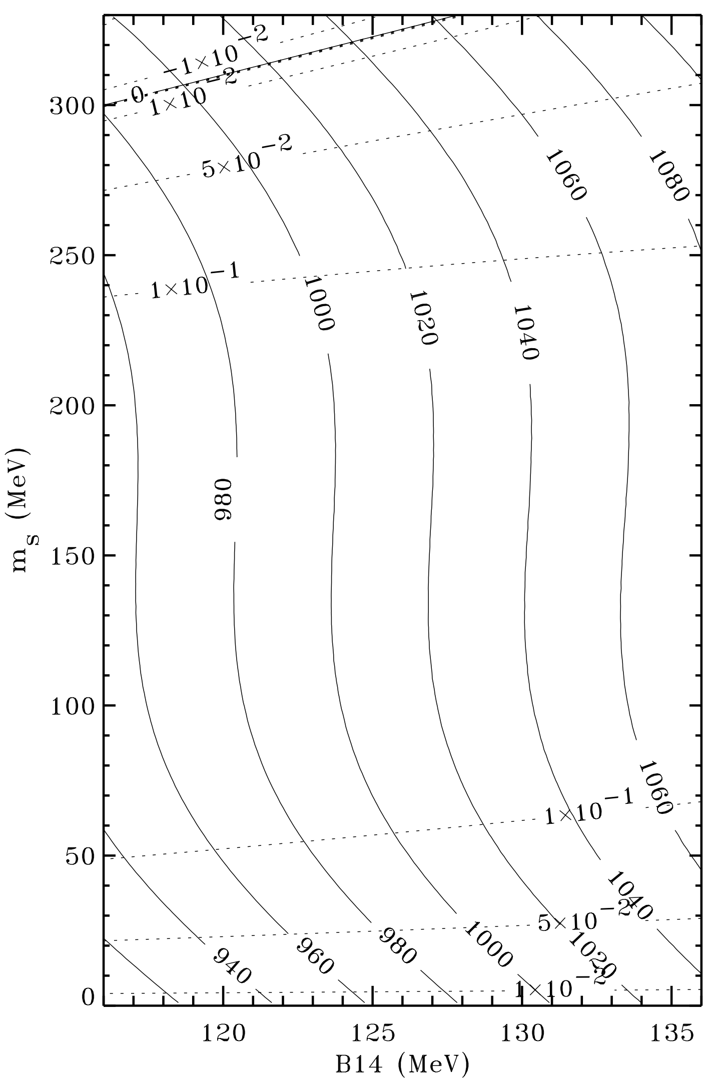
-1×10^-7
0



m_s (MeV)
B14 (MeV)
0
50
100
150
200
250
300
120
125
130
135
840
860
880
900
920
940
960
900
920
940
960
980
980
1000
1020
1×10^-4
5×10^-4
5×10^-4



m_s (MeV)
B14 (MeV)
0
50
100
150
200
250
300
120
125
130
135
940
960
980
1000
1020
1040
1060
1080
−1×10−2
0
1×10−2
5×10−2
1×10−1