Fate of the Sterile Neutrino

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ABSTRACT

In light of recent Super-Kamiokande data and global fits that seem to exclude both pure $\nu_\mu \rightarrow \nu_s$ oscillations of atmospheric neutrinos and pure $\nu_e \rightarrow \nu_s$ oscillations of solar neutrinos (where $\nu_s$ is a sterile neutrino), we reconsider four-neutrino models to explain the LSND, atmospheric, and solar neutrino oscillation indications. We argue that the solar data, with the exception of the $^{37}$Cl results, are suggestive of $\nu_e \rightarrow \nu_s$ oscillations that average to a probability of approximately $\frac{1}{2}$. In this interpretation, with two pairs of nearly degenerate mass eigenstates separated by order 1 eV, the day-night asymmetry, seasonal dependence, and energy dependence for $^8$B neutrinos should be small. Alternatively, we find that four-neutrino models with one mass eigenstate widely separated from the others (and with small sterile mixings to active neutrinos) may now be acceptable in light of recently updated LSND results; the $^{37}$Cl data can be accommodated in this model. For each scenario, we present simple four-neutrino mixing matrices that fit the stated criterion and discuss future tests.

1 Introduction

Accelerator, atmospheric, and solar neutrino data give evidence for neutrino oscillations and thus for neutrino masses and mixing. The LSND accelerator experiment finds a small $\nu_\mu \rightarrow \nu_e$ appearance probability and a mass-squared difference $\delta m^2_{\text{LSND}} > 0.2$ eV$^2$ [1, 2]. The atmospheric experiments measure the $\nu_\mu \rightarrow \nu_\mu$ survival probability versus both path-length $L$ and neutrino energy $E_\nu$. The amplitude for $\nu_\mu \rightarrow \nu_x$ ($x \neq e$) oscillations is inferred to be maximal or near-maximal with $\delta m^2_{\text{ATM}} \sim 3 \times 10^{-3}$ eV$^2$ [3, 4, 5]. The combined solar neutrino experiments determine the $\nu_e \rightarrow \nu_e$ survival probability to be $\sim 0.3$ to 0.7, depending on the neutrino energy [6, 7, 8, 9] and $\delta m^2_{\text{SOLAR}} \lesssim 10^{-3}$ eV$^2$ is required for consistency with the null result for the $\bar{\nu}_e \rightarrow \bar{\nu}_e$ probability measurement in the CHOOZ experiment [10]. Thus, taken together, the data require three distinct $\delta m^2$. With three neutrinos, there are only two independent $\delta m^2$, so a fourth sterile neutrino ($\nu_s$) needs to be invoked in addition to $\nu_e$, $\nu_\mu$, and $\nu_\tau$. Having no weak interactions, the sterile neutrino escapes the $N_\nu \simeq 3$ constraint from the invisible decay width of the $Z$-boson [11].
Successful global descriptions of the oscillation data have been made in a four-neutrino framework \[12, 13, 14, 15, 16\]. Compatibility of the LSND result with null results of accelerator \[17\] and reactor \[18\] disappearance experiments was found \[12, 13\] to favor a 2+2 mass spectrum, with the mass-squared differences $\delta m^2_{\text{SOLAR}}$ and $\delta m^2_{\text{ATM}}$ separated by the larger difference $\delta m^2_{\text{LSND}}$, over a 1+3 spectrum, with one mass eigenstate widely separated from the others. In the simplest of the 2+2 models, the oscillations are $\nu_{\mu} \rightarrow \nu_{\tau}$ (atmospheric) and $\nu_e \rightarrow \nu_s$ (solar), or alternatively $\nu_{\mu} \rightarrow \nu_s$ (atmospheric) and $\nu_e \rightarrow \nu_\tau$ (solar). More generally, sterile neutrinos may be involved in both atmospheric and solar oscillations. In the 2+2 mixing schemes, the sterile flavor content must be significant in the solar or atmospheric oscillations or in both.

New results from Super-Kamiokande (SuperK) \[4, 7\] impact critically on oscillations to sterile neutrinos. The zenith angle dependence of high-energy atmospheric events, along with the neutral current $\pi^0$ production rate, exclude pure $\nu_{\mu} \rightarrow \nu_s$ oscillations at 99% C.L. The electron energy dependence of solar neutrino events, along with the absence of a significant day-night effect, exclude pure $\nu_e \rightarrow \nu_s$ oscillations at 99% C.L. when all of the solar data, including the $^{37}$Cl results, are taken into account. Thus a dominant involvement of $\nu_s$ in either atmospheric or solar oscillations is brought into question. Also, LSND has reported new results \[2\] with a somewhat lower average oscillation probability than before and the KARMEN experiment \[19\] excludes part of the LSND allowed region. Consequently, previous exclusions \[12, 13\] of the 1+3 schemes need to be reassessed. In the context of this new data, we reconsider four-neutrino oscillation models and the implications for the existence of a sterile neutrino.

2 2+2 Models

The 2+2 models have two nearly degenerate pairs of neutrino mass eigenstates giving $\delta m^2_{\text{ATM}}$ and $\delta m^2_{\text{SOLAR}}$ separated by the LSND scale $\delta m^2_{\text{LSND}} \sim 1 \text{ eV}^2$. Maximal $\nu_3, \nu_2$ mixing describes the atmospheric data and maximal $\nu_0, \nu_1$ mixing describes the solar data. The mass hierarchy may be normal ($m_3 \gg m_1$) or inverted ($m_3 \ll m_1$). The $\delta m^2_{\text{SOLAR}}$ splitting must be smaller than $10^{-3} \text{ eV}^2$ to avoid the CHOOZ reactor constraint on $\bar{\nu}_e \rightarrow \bar{\nu}_e$ oscillations and larger than the $\sim 10^{-10} \text{ eV}^2$ of just-so vacuum oscillations\[20\] so that the oscillations average to give an approximately energy-independent solar $\nu_e$ survival probability.

2.1 Solar Neutrinos: maximal $\nu_e \rightarrow \nu_s$ oscillations

The new SuperK solar neutrino data show a remarkably flat recoil electron energy distribution from 5 MeV to 14 MeV, with average value \[7\]

$$\text{data}/\text{SSM} = 0.465^{+0.016}_{-0.014},$$

(1)

where SSM is the Standard Solar Model prediction \[9\]. The $^8$B flux normalization in the SSM is somewhat uncertain, and the above result is suggestive of rapid oscillations with maximal amplitude that give an average oscillation probability, $\langle P(\nu_e \rightarrow \nu_\tau) \rangle$, of approximately $\frac{1}{2}$.

Regeneration of $\nu_e$ in the Earth, due to the effects of coherent forward scattering of the neutrinos with matter \[21\], causes a day-night variation in the measured neutrino flux.
The smallness of the day-night effect $2(D - N)/(D + N) = -0.034 \pm 0.022 \pm 0.013$ [7] in the SuperK data (1.3$\sigma$ from zero) puts strong constraints on solar solutions with matter effects [7, 22, 23, 24]. It has been noted that there can be a day-night effect even for maximal mixing [23, 24], but calculations for active neutrinos show that it is small ($\sim 1$–2 $\%$) for $\delta m_{\text{solar}}^2 \gtrsim 5 \times 10^{-5} \text{ eV}^2$ or $\delta m_{\text{solar}}^2 \lesssim 10^{-7} \text{ eV}^2$ [7]. In $\nu_e \to \nu_s$ oscillations, the difference in coherent scattering amplitudes in matter is $\sqrt{2}G_F(N_e - N_n)/2 \approx \sqrt{2}G_FN_e/2$ (where $N_e$ and $N_n$ are the electron and neutron number densities, respectively, and $N_e \sim N_n$ in the Earth). The corresponding amplitude difference for $\nu_e \to \nu_\mu$ (or $\nu_e \to \nu_\tau$) is $\sqrt{2}G_FN_e$. Therefore, for maximal mixing the day-night effect should be smaller with a sterile neutrino than with an active neutrino, but it would not be zero.

The regeneration effect in the earth for neutrinos detected at night raises the value of $\langle P(\nu_e \to \nu_e) \rangle$ for maximal mixing with active neutrinos from 0.50 to about 0.54 [23]. For sterile neutrinos, we roughly estimate that $\langle P(\nu_e \to \nu_e) \rangle \approx 0.52$ in SuperK for maximal mixing with regeneration. Regeneration also causes a weak dependence on energy of the solar neutrino suppression. For maximal mixing of $\nu_e$ with an active neutrino, $\langle P(\nu_e \to \nu_e) \rangle$ is slightly higher at higher neutrino energies [23], consistent with the weak trend of the SuperK data; less energy dependence should be present with $\nu_e$ oscillations to $\nu_s$.

Assuming $\langle P(\nu_e \to \nu_e) \rangle = 0.52$ for maximal $\nu_e \to \nu_s$ oscillations, the SuperK result in Eq. (1) would be reproduced by a $^8\text{B}$ flux normalization of $n = 0.89$. The SSM predictions (in SNU) for the Gallium experiments are (70, $pp$), (34, $^7\text{Be}$), (3, $p\text{ep}$), (10, CNO), and (12, $^8\text{B}$), giving a total of 129 SNU; the observed values are $75.4_{-7.4}^{+7.8}$ (SAGE), $77.5_{-7.5}^{+7.5}$ (GALLEX) and $65.8_{-10.2}^{+10.8}$ (GNO). With the above $^8\text{B}$ normalization and $\langle P(\nu_e \to \nu_e) \rangle = \frac{1}{2}$, the predicted rate in the Gallium experiments is 63.8 SNU. Thus all of the Gallium measurements are consistent within 2$\sigma$ of the value that would be found for maximal amplitude $\nu_e \to \nu_s$ oscillations that average to $\frac{1}{2}$. The new data from GNO are especially suggestive of this interpretation. A least-squares fit to the SuperK and Gallium rates with the $^8\text{B}$ flux normalization $n$ as a free parameter gives a best fit value of $n = 0.90$ with $\chi^2 = 5.5$ for 3 degrees of freedom, which corresponds to a 14% goodness of fit. In this calculation we have not taken into account any regeneration effect for the Gallium data, which may improve the fit.

The discrepant measurement in the above interpretation is the $^{37}\text{Cl}$ value of data$/$SSM = 0.33 $\pm$ 0.03 from the Homestake mine experiment, which would require a significant energy dependence of the solar $\nu_e$ flux suppression. Most global fits to neutrino oscillation data include the $^{37}\text{Cl}$ data (and often disregard the LSND data) and then $\nu_e \to \nu_s$ oscillations are excluded. We instead suggest the possibility that the solar data$/$SSM flux ratio is relatively flat over the entire 0.233 MeV to 14 MeV energy range and is described by $\nu_e \to \nu_s$ oscillations with maximal mixing.

An important test of approximate constant suppression of the solar neutrino spectrum will be the BOREXINO [25] experiment, which can measure the $^7\text{Be}$ component of the solar neutrino flux. Nearly complete suppression of the $^7\text{Be}$ component is needed for the $^{37}\text{Cl}$ data to be consistent with the suppression of the $^8\text{B}$ component measured by SuperK. However, a suppression of the $^7\text{Be}$ flux similar to the data$/$SSM measured by SuperK would favor maximal $\nu_e \to \nu_s$ mixing of solar neutrinos. The SNO [26] and ICARUS [27] experiments will also provide critical tests of the $^8\text{B}$ flux suppression, and SNO will test maximal $\nu_e \to \nu_s$ oscillations through the NC/CC ratio (which should be the same as the value without oscillations).
2.2 Mixing matrix for the 2 + 2 model

We assume a 2 + 2 scenario in which one pair of nearly degenerate mass eigenstates has maximal $\nu_e \rightarrow \nu_s$ mixing for solar neutrinos and the other pair has maximal (or nearly maximal) $\nu_\mu \rightarrow \nu_\tau$ oscillations for atmospheric neutrinos. Small off-diagonal mixings between $\nu_e, \nu_s$ and $\nu_\mu, \nu_\tau$ can accommodate the LSND results. Neglecting $CP$-violating phases for the present, an approximate mixing with these properties is

$$
\begin{pmatrix}
\nu_s \\
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
\approx
\begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \epsilon & \epsilon \\
0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix}
\begin{pmatrix}
\nu_0 \\
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix},
$$

(2)

where $|\delta m^2_{10}| \ll |\delta m^2_{32}| \ll |\delta m^2_{21}|$. In the notation of Ref. [28], we have chosen the mixing angles $\theta_{01} = \theta_{23} = \pi/4$, $\theta_{02} = \theta_{03} = 0$, and $\theta_{12} = \theta_{13}$, with $\epsilon = \sin \theta_{13}$. The oscillation probabilities are given in Table 1. The parameter $\epsilon$ is determined from the LSND data to be approximately

$$
\epsilon \approx \left(\frac{0.016 \text{ eV}^2}{|\delta m^2_{\text{LSND}}|}\right)^{0.91},
$$

(3)

where $\delta m^2_{\text{LSND}}$ is restricted to the range 0.2 to 1.7 eV$^2$ by the BUGEY and KARMEN experiments; for $\delta m^2_{\text{LSND}} \simeq 6 \text{ eV}^2$, $\epsilon \simeq 0.022$ is marginally possible.

Other forms of the mixing matrix (see, e.g., Refs. [12, 13, 14, 15, 16, 28]) are also acceptable, provided that the $\nu_e-\nu_s$ mixing is nearly maximal. Future short and long-baseline experiments will be useful in determining the mixing matrix. For example, there are no $\nu_e \rightarrow \nu_\tau$ oscillations at short baselines for the mixing in Eq. (2), but there could be if the $U_{e2}$ and $U_{e3}$ mixing matrix elements are not equal in magnitude or have different phases (see, e.g., Refs. [14, 28, 29, 30]), in which case unitarity requires $U_{e0} \neq 0$ and/or $U_{e1} \neq 0$. Also, both $\nu_e \rightarrow \nu_\mu$ and $\nu_e \rightarrow \nu_\tau$ oscillations are possible at long baselines, and there can be $CP$ violation in these channels if the four-neutrino mixing matrix is not real (see, e.g., Refs. [28, 30, 31, 32]).

3 1 + 3 Models

Another way to circumvent the SuperK exclusion of sterile neutrinos is to assume that the solar and atmospheric oscillations are approximately described by oscillations of three active neutrinos; then the LSND result can be explained by a coupling of $\nu_e$ and $\nu_\mu$ through small mixings with a sterile neutrino that has a mass eigenvalue widely separated from the others. This 1 + 3 model was previously disfavored by incompatibility of the LSND result with null results of the CDHS and BUGEY experiments [12, 13]. However, in newly updated results from the LSND experiment [2], the LSND allowed region is slightly shifted, and this opens a small window for the 1+3 models. The solar neutrino data can be explained by oscillations of three active neutrinos in the usual way; the results of the $^{37}$Cl experiment can be reconciled with those of SuperK in part because oscillations of $\nu_e$ to active neutrinos in SuperK also show up as neutral current events (with a rate of about 1/6 of the charge-current rate), thereby giving a higher data/SSM in SuperK than in the $^{37}$Cl experiment.
By way of example, consider the approximate mixing

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau \\
\nu_s
\end{pmatrix}
\sim
\begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \epsilon \\
-\frac{1}{2} & \frac{1}{2} & \frac{\delta}{\sqrt{2}} & 0 \\
\frac{1}{2} - \frac{\epsilon}{\sqrt{2}} & -\frac{1}{2} - \frac{\epsilon}{\sqrt{2}} & \frac{\delta}{\sqrt{2}} & 1
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3 \\
\nu_0
\end{pmatrix},
$$

(4)

where \(\epsilon\) and \(\delta\) are small and both the flavor and mass eigenstates are reordered to reflect the fact that \(\nu_0\) (which is predominantly \(\nu_s\)) is the heaviest state. In the notation of Ref. [28], we have chosen the mixing angles \(\theta_{01} = \theta_{12} = \pi/4\), \(\theta_{02} = \theta_{23} = 0\), \(\sin \theta_{03} = \epsilon\), and \(\sin \theta_{13} = \delta\). Here the \(3 \times 3\) submatrix that describes the mixing of the three active neutrinos has the bimaximal form [33]. The oscillation probabilities are given in Table 2. For this mixing matrix the leading oscillation amplitudes at the \(\delta m^2_{\text{LSND}}\) scale in the \(1 + 3\) scheme are (when \(\epsilon, \delta \ll 1\))

$$
A_{\text{BUGEY}} \simeq 4\epsilon^2
$$

(5)
for \(\bar{\nu}_e\) disappearance in the BUGEY experiment,

$$
A_{\text{CDHS}} \simeq 4\delta^2
$$

(6)
for \(\nu_\mu\) disappearance in the CDHS experiment, and

$$
A_{\text{LSND}} = 4\epsilon^2\delta^2
$$

(7)
for \(\nu_\mu \rightarrow \nu_e\) appearance in the LSND experiment. Including the subleading oscillation at the \(\delta m^2_{\text{atm}}\) scale, and assuming that the leading oscillation averages,

$$
P(\nu_\mu \rightarrow \nu_\mu) \simeq (1 - 2\delta^2)(1 - \sin^2 1.27\delta m^2_{\text{atm}} L/E)
$$

(8)
for \(\nu_\mu \rightarrow \nu_\mu\) disappearance in atmospheric neutrino experiments, where \(\delta m^2_{\text{atm}}\) is in eV\(^2\), \(L\) in km, and \(E\) in GeV. The relative normalization of the atmospheric \(\nu_\mu\) and \(\nu_e\) fluxes is well known [34]; the predicted effective change in this normalization at the \(\delta m^2_{\text{atm}}\) oscillation scale is \((1 - 2\delta^2)/(1 - 2\epsilon^2)\). In the region where the CDHS constraint on \(\delta\) is weak or nonexistent \((\delta m^2_{\text{LSND}} \lesssim 0.4\text{ eV}^2)\), BUGEY constrains \(\epsilon\) to be less than about 0.1; conservatively assuming that the atmospheric data constrains the relative normalization to within 10%, we find \(\delta < 0.24\).

The upper limits on \(\epsilon\) (from \(A_{\text{BUGEY}}\)) and \(\delta\) (from \(A_{\text{atm}}\) and \(A_{\text{CDHS}}\)) and the allowed values of \(A_{\text{LSND}}\) vary with \(\delta m^2_{\text{LSND}}\). With the previously reported LSND data [1], there was no value of \(\delta m^2_{\text{LSND}}\) for which the constraints on \(\epsilon\) and \(\delta\) were consistent with the LSND 99% C.L. allowed range of \(A_{\text{LSND}}\); the maximum allowed values of \(\epsilon\) and \(\delta\) always implied an upper limit on \(A_{\text{LSND}}\) that was below the LSND measured value. However, with the recent shift of \(A_{\text{LSND}}\) to lower values, there are now three small \(\delta m^2_{\text{LSND}}\) islands where the LSND 99% C.L. region is compatible with the BUGEY, CDHS, and KARMEN constraints: \(\delta m^2_{\text{LSND}} \approx 0.9\text{ eV}^2\) and \(\delta m^2_{\text{LSND}} \approx 1.7\text{ eV}^2\), for which the BUGEY constraint is somewhat less restrictive, and \(\delta m^2_{\text{LSND}} \approx 6\text{ eV}^2\), for which both the BUGEY and KARMEN constraints are somewhat less restrictive. The E776 experiment at BNL [35] gives a somewhat tighter constraint than KARMEN at \(\delta m^2_{\text{LSND}} = 6\text{ eV}^2\), but a small region is still allowed here.
Except for a small region near $\delta m_{LSND}^2 \simeq 0.2$ eV$^2$ (see below), the combined data are still inconsistent with oscillations having $\delta m_{LSND}^2 < 0.9$ eV$^2$, and for all $\delta m_{LSND}^2$ when the LSND 90% C.L. allowed region is used. The above analysis is summarized in Table 3; acceptable values of $\delta m_{LSND}^2$ are those for which $(4\epsilon^2\delta^2)_{\text{max}}$ lies within the range of $A_{LSND}$ allowed by LSND and KARMEN.

There is no CDHS constraint for $\delta m_{LSND}^2 < 0.25$ eV$^2$, which suggests that $\delta m_{LSND}^2 \simeq 0.2$ eV$^2$ may also be allowed. However, at these $\delta m_{LSND}^2$ values $\delta$ must be more than 0.5 to reconcile the BUGEY limit with the LSND measurement, which is not consistent with the relative normalization of the $\nu_\mu$ and $\nu_e$ fluxes (see above).

The model of Eq. (4) has bimaximal mixing in the $3 \times 3$ active sector. Similar results for the 1 + 3 model can be obtained for any $3 \times 3$ submatrix that can describe the atmospheric and solar data for active neutrinos, as long as $\epsilon$ and $\delta$ are small. This is easily seen by realizing that the general expressions for the short-baseline amplitudes in the 1 + 3 model in Eqs. (5)–(7) depend only on the magnitudes of the mixing matrix elements $U_{e0}$ and $U_{\mu0}$ [\epsilon and $\delta$, respectively, in Eq. (4)]. If $U_{\tau0} \neq 0$, then there can be $\nu_\mu \to \nu_\tau$ and $\nu_e \to \nu_\tau$ oscillations at the $\delta m_{LSND}^2$ scale. If $U_{e3} \neq 0$, then there can be $\nu_e \to \nu_\mu$ and $\nu_e \to \nu_\tau$ oscillations at the $\delta m_{atm}^2$ scale.

Because the allowed windows are already severely constrained by accelerator and reactor data, future experiments at short baselines could easily test or rule out the 1 + 3 scenarios. MiniBooNE [36] will search for $\nu_\mu \to \nu_e$ oscillations and test $A_{LSND}$. Reactor experiments such as at Palo Verde [37], KamLAND [38], and the proposed ORLaND [39], will test $A_{BUGEY}$, but only ORLaND is expected to significantly improve the bound on $\epsilon$. A future precision measurement of $\nu_\mu$ disappearance at short baselines could test $A_{CDHS}$.

4 Discussion and Conclusions

The recent Super-Kamiokande data and global fits present new constraints on the mixing of light sterile neutrinos. It is important to assess the viability of a light sterile neutrino, because its existence is required if the solar, atmospheric, reactor, and accelerator data are all to be understood in terms of neutrino oscillations.

We have examined at a qualitative level two schemes for four-neutrino mass and mixing which nearly accommodate all present data. In the 2 + 2 scheme, two neutrino pairs are separated by the LSND mass scale. One pair maximally mixes $\nu_\mu$ and $\nu_\tau$ to explain the atmospheric data, while the other pair maximally mixes $\nu_e$ and $\nu_s$ to explain the solar deficit with energy-independent oscillations. All data except the low $\nu_e$ capture rate on $^{37}$Cl are explained in this model. A very small day-night effect is expected, at the level of a per cent or less. Suppression of the $^7$Be component of the solar neutrino flux by approximately 50% is expected for the BOREXINO experiment.

In the 1 + 3 scheme, the three active neutrinos are separated by the LSND scale from a sterile neutrino state. The active neutrino $3 \times 3$ submatrix explains the atmospheric and solar data, including the $^{37}$Cl rate. In our example we assigned the bimaximal model [33] to this submatrix, but any $3 \times 3$ active submatrix that describes the atmospheric and solar data may be used. A small mixing of $\nu_e$ and $\nu_\mu$ via the sterile state explains the LSND data. The smaller oscillation amplitude recently reported by the LSND collaboration allows marginal
accommodation with the null $\bar{\nu}_e$ and $\nu_\mu$ disappearance results previously obtained in reactor and accelerator experiments.

The above two classes of models allow a viable sterile neutrino having either large mixing with $\nu_\text{e}$ or small mixing with all active neutrinos. In both classes we made the simplifying assumption that $\nu_\tau$-$\nu_s$ mixing is negligible. Present data may allow considerable $\nu_\tau$-$\nu_s$ mixing, in which case both solar and atmospheric neutrino oscillations could have a sizable sterile component. Quantitative fits to atmospheric and solar data are needed to determine how much $\nu_\tau$-$\nu_s$ mixing is allowed (see, e.g., Ref. [40]).

Neutrinos also provide a hot dark matter component, which is relevant to large-scale structure formation [41]. From the upper limit of $m_\beta = 2.5$ to 3 eV on the effective $\nu_\text{e}$ mass obtained from tritium beta decay endpoint measurements[42], the mass $m_{\text{max}}$ of the heaviest neutrino is bounded by [43]

$$\sqrt{\delta m^2_{\text{LSND}}} \lesssim m_{\text{max}} \lesssim \sqrt{m^2_\beta + \delta m^2_{\text{LSND}}}.$$  \hspace{1cm} (9)

The contribution of the neutrinos to the mass density of the universe is given by $\Omega_\nu = \sum m_\nu/(h^2 93 \text{ eV})$, where $h$ is the present Hubble expansion parameter in units of 100 km/s/Mpc [44]; with $h = 0.65$, both the 2+2 and 1+3 models give $\Omega_\nu \geq 0.02$. For an inverted neutrino mass spectrum (where the $\nu_\text{e}$ is associated predominantly with the heavier neutrinos), the bound is $\sqrt{\delta m^2_{\text{LSND}}} \lesssim m_{\text{max}} \lesssim m_\beta$, which yields $\Omega_\nu \geq 0.02$ and $\Omega_\nu \geq 0.07$ for the 2+2 and 1+3 models, respectively. The MAP[45] and PLANCK[46] satellite measurements of the cosmic microwave background radiation may be sensitive to these neutrino densities.

The existence of the LSND mass gap will be tested by the MiniBooNE experiment [36]. A definitive test of the presence of the sterile state in the 2+2 model will be the measurement of the NC/CC ratio for solar neutrinos by the SNO experiment. With the solar solution given by maximal $\nu_\text{e} \rightarrow \nu_s$ mixing in this model, the NC should show the same suppression as the CC. The existence of the sterile neutrino in the 1+3 model will not be tested by SNO NC/CC data since the ratio is approximately that of $\nu_\text{e}$ oscillations to active neutrinos. The sterile state in the 1+3 model can be tested by searches for small amplitude oscillations at short baselines.

Observation of the flavor ratio of extragalactic neutrinos may serve as a test of the models. Oscillations of neutrinos from distant sources will have averaged, leaving definite predictions for flavor ratios. If cosmic neutrinos are mainly produced in pion/muon decay, their initial flavor ratio is $\nu_\tau : \nu_\mu : \nu_\text{e} : \nu_s \approx 0 : 2 : 1 : 0$. A simple calculation [47] then gives the asymptotic ratios 1 : 1 : 0.5 : 0.5 for the 2+2 model with two maximally-mixed pairs, and 1 : 1 : 1 : 0 for the 1+3 model with bimaximal mixing of the active neutrinos.

A virtue of active-sterile neutrino oscillations is that they may aid r-process nucleosynthesis of heavy elements in neutrino-driven supernovae ejecta. The basic requirement is that the $\nu_\text{e}$ flux be diminished in the region where inverse $\beta$-decay would otherwise transform neutrons into protons. The 2+2 and 1+3 mass spectra and mixing matrices presented here are of the forms previously discussed to enhance r-process nucleosynthesis. In the 2+2 model, this is accomplished with a two-step process: first, a matter-enhanced $\nu_\mu/\nu_\tau \rightarrow \nu_s$ transition beyond the neutrino sphere removes the energetic $\nu_\mu/\nu_\tau$ before they can convert to energetic $\nu_\text{e}$, and then a large $\nu_\text{e} \rightarrow \nu_\mu/\nu_\tau$ transition reduces the $\nu_\text{e}$ abundance [48]; the required mass-squared parameter is $\delta m^2_{\text{LSND}} \gtrsim 1 \text{ eV}^2$. In the 1+3 scheme, the relevant oscillation
is a matter-enhanced $\nu_e \rightarrow \nu_s$, obtained with $\delta m^2_{\text{LSND}} \gtrsim 2 \text{ eV}^2$ and $\sin^2 2\theta_{\text{es}} \gtrsim 10^{-4}$ [49]. It appears that the r-process enhancements discussed in Refs. [48, 49] may be accomplished by tuning the small parameters in our mixing schemes.

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[46] See http://astro.estec.esa.nl/Planck/ for information on PLANCK.


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Table 1: Oscillation probabilities in the $2 + 2$ model defined by Eq. (2), to leading order for each oscillation scale. The oscillation arguments are defined by $\Delta_j \equiv 1.27 \delta m_j^2 L/E$, with $\delta m_j^2$ in eV$^2$, $L$ in km, and $E$ in GeV.

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<tr>
<td>$e$</td>
<td>$1 - 8\epsilon^2 \sin^2 \Delta_{\text{LSND}}$</td>
<td>$- \sin^2 \Delta_{\text{SOLAR}}$</td>
<td>$8\epsilon^2 \sin^2 \Delta_{\text{LSND}}$</td>
<td>$2\epsilon^2 \sin^2 \Delta_{\text{ATM}}$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$1 - 8\epsilon^2 \sin^2 \Delta_{\text{LSND}}$</td>
<td>$- \sin^2 \Delta_{\text{ATM}}$</td>
<td>$\sin^2 \Delta_{\text{ATM}}$</td>
<td>$1 - \sin^2 \Delta_{\text{ATM}}$</td>
</tr>
</tbody>
</table>

Table 2: Oscillation probabilities in the $1 + 3$ model defined by Eq. (4), to leading order for each oscillation scale. The oscillation arguments are defined by $\Delta_j \equiv 1.27 \delta m_j^2 L/E$, with $\delta m_j^2$ in eV$^2$, $L$ in km, and $E$ in GeV.

<table>
<thead>
<tr>
<th></th>
<th>$e$</th>
<th>$\mu$</th>
<th>$\tau$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>$1 - 4\epsilon^2 \sin^2 \Delta_{\text{LSND}}$</td>
<td>$- \sin^2 \Delta_{\text{SOLAR}}$</td>
<td>$4\epsilon^2 \delta^2 \sin^2 \Delta_{\text{LSND}}$</td>
<td>$4\epsilon^2 \sin^2 \Delta_{\text{LSND}}$</td>
</tr>
<tr>
<td></td>
<td>$+ \frac{1}{4} \sin^2 \Delta_{\text{SOLAR}}$</td>
<td>$+ \frac{1}{4} \sin^2 \Delta_{\text{SOLAR}}$</td>
<td>$- \frac{1}{4} \sin^2 \Delta_{\text{SOLAR}}$</td>
<td>$+ \frac{1}{4} (\delta^2 - 2\epsilon^2) \sin^2 \Delta_{\text{SOLAR}}$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$1 - 4\delta^2 \sin^2 \Delta_{\text{LSND}}$</td>
<td>$- \sin^2 \Delta_{\text{ATM}}$</td>
<td>$\sin^2 \Delta_{\text{ATM}}$</td>
<td>$4\delta^2 \sin^2 \Delta_{\text{LSND}}$</td>
</tr>
<tr>
<td></td>
<td>$- \frac{1}{4} \sin^2 \Delta_{\text{SOLAR}}$</td>
<td>$- \frac{1}{4} \sin^2 \Delta_{\text{SOLAR}}$</td>
<td>$- \frac{1}{4} \sin^2 \Delta_{\text{SOLAR}}$</td>
<td>$+ \frac{1}{4} (2\epsilon^2 - \delta^2) \sin^2 \Delta_{\text{SOLAR}}$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$1 - \sin^2 \Delta_{\text{ATM}}$</td>
<td>$- \frac{1}{4} \sin^2 \Delta_{\text{SOLAR}}$</td>
<td>$\delta^2 \sin^2 \Delta_{\text{ATM}}$</td>
<td>$1 - 4(\epsilon^2 + \delta^2) \sin^2 \Delta_{\text{LSND}}$</td>
</tr>
<tr>
<td></td>
<td>$- \frac{1}{4} \sin^2 \Delta_{\text{SOLAR}}$</td>
<td>$- \frac{1}{4} \sin^2 \Delta_{\text{SOLAR}}$</td>
<td>$+ \frac{1}{4} (2\epsilon^2 - \delta^2) \sin^2 \Delta_{\text{SOLAR}}$</td>
<td>$- \delta^2 (2\epsilon^2 + \delta^2) \sin^2 \Delta_{\text{ATM}}$</td>
</tr>
</tbody>
</table>
Table 3: Summary of constraints on four-neutrino oscillation parameters in the 1+3 scheme. The upper limit on $A_{\text{LSND}}$ at $\delta m^2_{\text{LSND}} \simeq 6 \text{ eV}^2$ is from the E776 experiment at BNL [35]. All experimental limits are at 90% C.L., except for LSND, which is at 99% C.L.

<table>
<thead>
<tr>
<th>$\delta m^2_{\text{LSND}}$ (eV$^2$)</th>
<th>$\epsilon_{\text{max}}$ (BUGEY)</th>
<th>$\delta_{\text{max}}$ (CDHS)</th>
<th>$(4\epsilon^2\delta^2)_{\text{max}}$</th>
<th>$(A_{\text{LSND}})_{\text{min}}$ (LSND)</th>
<th>$(A_{\text{LSND}})_{\text{max}}$ (KARMEN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.0</td>
<td>0.19</td>
<td>0.14</td>
<td>$2.8 \times 10^{-3}$</td>
<td>$1.5 \times 10^{-3}$</td>
<td>$2.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>1.7</td>
<td>0.16</td>
<td>0.12</td>
<td>$1.5 \times 10^{-3}$</td>
<td>$0.8 \times 10^{-3}$</td>
<td>$1.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.9</td>
<td>0.12</td>
<td>0.16</td>
<td>$1.5 \times 10^{-3}$</td>
<td>$1.4 \times 10^{-3}$</td>
<td>$3.0 \times 10^{-3}$</td>
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<tr>
<td>0.3</td>
<td>0.10</td>
<td>0.45</td>
<td>$8 \times 10^{-3}$</td>
<td>$10 \times 10^{-3}$</td>
<td>$30 \times 10^{-3}$</td>
</tr>
</tbody>
</table>