TOWARDS A THEORY OF HADRONIC $B$ DECAYS

Matthias Neubert
Newman Laboratory of Nuclear Studies, Cornell University
Ithaca, New York 14853, U.S.A.

We review recent advances in the theory of strong-interaction effects and final-state interactions in hadronic weak decays of heavy mesons. In the heavy-quark limit, the amplitudes for most nonleptonic, two-body $B$ decays can be calculated from first principles and expressed in terms of semileptonic form factors and light-cone distribution amplitudes. We discuss the features of this novel QCD factorization and illustrate its phenomenological implications.

1 Introduction

The theoretical description of hadronic weak decays is difficult due to nonperturbative strong-interaction dynamics. This will affect the interpretation of the data collected at the $B$ factories, including studies of CP violation and searches for New Physics. If these strong-interaction effects could be computed in a model-independent way, this would enhance our ability to uncover the origin of CP violation. The complexity of the problem is illustrated in the cartoon on the left-hand side of Fig. 1. It is well known how to control the effects of hard gluons with virtuality between the electroweak scale $M_W$ and the scale $m_B$ characteristic to the decays of interest. They can be dealt with by constructing a low-energy effective weak Hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_i \lambda^{\text{CKM}}_i (M_W/\mu) C_i(\mu) O_i(\mu),$$

where $\lambda^{\text{CKM}}_i$ are products of CKM matrix elements, $C_i(M_W/\mu)$ are calculable short-distance coefficients, and $O_i(\mu)$ are local operators renormalized at a scale $\mu = \mathcal{O}(m_B)$. The challenge is to calculate the hadronic matrix elements of these operators with controlled theoretical uncertainties, using a systematic approximation scheme.

Previous field-theoretic attempts to evaluate these matrix elements have employed dynamical schemes such as lattice field theory, QCD sum rules, or the hard-scattering approach. The first two have great difficulties in accounting for final-state rescattering, which however is very important for predicting direct CP asymmetries. The hard-scattering approach misses the leading soft contribution to the $B \rightarrow$ meson transition form factors and thus falls short of reproducing the correct magnitude of the decay amplitudes. In view of these difficulties, most previous analyses of hadronic decays have employed phenomenological models such as “naive” or “generalized factorization”, in which the complicated matrix elements of four-quark operators in the effective weak Hamiltonian are replaced (in an ad hoc way) by products of current matrix elements.

\textsuperscript{a}Invited review talk presented at the XXXVth Rencontres de Moriond, QCD and High-Energy Hadronic Interactions, Les Arcs, France, 18–25 March 2000
Corrections to this approximation are accounted for by introducing a set of phenomenological parameters $a_i$. A different strategy is to classify nonleptonic decay amplitudes according to flavor topologies ("trees" and "penguins"), which can be decomposed into SU(3) or isospin amplitudes. This leads to relations between different decay amplitudes in the flavor-symmetry limit. No attempt is made, however, to compute these amplitudes from first principles.

## 2 QCD Factorization Formula

Here we summarize recent progress in the theoretical understanding of nonleptonic decay amplitudes in the heavy-quark limit. The underlying idea is to exploit the presence of a large scale, i.e., the fact that $m_b \gg \Lambda_{\text{QCD}}$. In order to disentangle the physics associated with these two scales, we factorize and compute hard contributions to the decay amplitudes arising from gluons with virtuality of order $m_b$, and parameterize soft and collinear contributions. Considering the cartoon in Fig. 1, we denote by $M_1$ the meson that absorbs the spectator quark of the $B$ meson, and by $M_2$ the meson at the upper vertex, to which we refer as the "emission particle". We find that nonleptonic decay amplitudes simplify in the heavy-quark limit if $M_2$ is a light meson. Then at leading power in $\Lambda_{\text{QCD}}/m_b$ all long-distance contributions to the matrix elements can be factorized into semileptonic form factors and meson light-cone distribution amplitudes, which are much simpler quantities than the nonleptonic amplitudes themselves. (Light-cone distribution amplitudes enter because the partons in the emission particle carry large energy and are almost collinear.) "Nonfactorizable" effects connecting the partons of the emission particle with the rest of the diagram are dominated by hard gluon exchange and can be computed using perturbation theory. A graphical representation of the resulting "factorization formula" is shown on the right-hand side in Fig. 1. The physical picture underlying factorization is color transparency. If the emission particle is a light meson, its constituents carry large energy of order $m_b$ and are nearly collinear. Soft gluons coupling to this system see only its net zero color charge and hence decouple. Interactions with the color dipole of the small $q\bar{q}$-pair are power suppressed in the heavy-quark limit.

For $B$ decays into final states containing a heavy charm meson and a light meson $L$, the factorization formula takes the form

$$
\langle D^{(*)}L^- | O_i(\mu) | \bar{B}_d \rangle = \sum_j F_j^{\bar{B} \to D^{(*)}} \int_0^1 du T_{ij}^L(u, \mu) \Phi_L(u, \mu) + O\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right), \tag{2}
$$

where $O_i$ is an operator in the effective weak Hamiltonian (1), $F_j^{\bar{B} \to D^{(*)}}$ are transition form factors, $\Phi_L$ is the leading-twist light-cone distribution amplitude of the light meson, and $T_{ij}^L$ are process-dependent hard-scattering kernels. For decays into final states containing two light mesons in the final state, such as $\bar{B} \to \pi\pi$. Contributions not shown (such as weak annihilation graphs) are power suppressed.
mesons there is a second type of contribution to the factorization formula, which involves a
hard interaction with the spectator quark in the $B$ meson. It is contained in the second graph
on the right-hand side in Fig. 1. Below we focus on $\bar{B} \rightarrow D L$ decays, where this second term
is power suppressed and can be neglected. Decays into two light final-state mesons are more
complicated and have been discussed elsewhere.

In order to prove factorization one must first separate hard from infrared (soft and collinear)
contributions to the decay amplitudes. This is done at the level of Feynman diagrams. One must
then show that “nonfactorizable contributions”, i.e., contributions not associated with $\bar{B} \rightarrow M_1$
form factors or meson wave functions, are dominated by hard gluon exchange. This amounts to
showing that the soft and collinear singularities, which are present in individual Feynman dia-
grams, cancel in the sum of all contributions. For the case of decays into a heavy–light final state
this cancellation has been demonstrated by an explicit two-loop analysis. General arguments
support factorization to all orders of perturbation theory. The fact that these cancellations
occur is far from trivial, given that by power counting the $\bar{B} \rightarrow M_1$ form factors (both for a
heavy and a light final-state meson) are dominated by soft gluon exchange. To complete the
proof of factorization one must also show that contributions from transverse momenta of the
partons in the final-state mesons, from asymmetric parton configurations where one (or several)
partons are soft, and from non-valence Fock states (containing additional hard or soft partons)
are power suppressed, and that competing flavor topologies such as weak annihilation are power
suppressed, too. All this is discussed in detail in our recent work.

The factorization formula for nonleptonic decays provides a model-independent basis for
the analysis of these processes in an expansion in powers and logarithms of $\Lambda_{\text{QCD}}/m_b$. At
leading power, but to all orders in $\alpha_s$, the decay amplitudes assume the factorized form shown
in (2). Having such a formalism based on power counting in $\Lambda_{\text{QCD}}/m_b$ is of great importance
to the theoretical description of hadronic weak decays, since it provides a well-defined limit
of QCD in which these processes admit a rigorous, theoretical description. (For instance, the
possibility to compute systematically $O(\alpha_s)$ corrections to “naive factorization”, which emerges
as the leading term in the heavy-quark limit, solves the old problem of renormalization-scale
and scheme dependences of nonleptonic amplitudes.) The usefulness of this new scheme may be
compared with the usefulness of the heavy-quark effective theory for the analysis of exclusive
semileptonic decays of heavy mesons, or of the heavy-quark expansion for the analysis of inclusive
decay rates. In all three cases, it is the fact that hadronic uncertainties can be eliminated up to
power corrections in $\Lambda_{\text{QCD}}/m_b$ that has advanced our ability to control theoretical errors.

However, it must be stressed that we are just beginning to explore the theory of nonleptonic
$B$ decays. Some important conceptual problems remain to be better understood. In the next
few years it will be important to further develop this novel approach. This should include
proving factorization at leading power to all orders in $\alpha_s$, developing a formalism for dealing
with power corrections to factorization, understanding the light-cone structure of heavy mesons,
and understanding the relevance (or irrelevance) of Sudakov form factors. Also, we must gauge
the accuracy of the approach by learning about the magnitude of corrections to the heavy-
quark limit from extensive comparisons of theoretical predictions with data. As experience with
previous heavy-quark expansions has shown, this is going to be a long route. Yet, already we
have obtained important insights. Let us mention three points here:

1. Corrections to “naive factorization” (usually called “nonfactorizable effects”) are process
dependent, in contrast with a basic assumption underlying models of “generalized factorization”.

2. The physics of nonleptonic decays is both rich and complicated. In general, it is character-
ized by an interplay of several small parameters (Wilson coefficients, CKM factors, $1/N_c$, etc.) in
addition to the small parameter $\Lambda_{\text{QCD}}/m_b$ relevant to QCD factorization. In some cases, terms
that are formally power suppressed may be enhanced by factors such as $2m_u^2/(m_u+m_d) \approx 3 \text{GeV}$,
which are larger than naive dimensional analysis would suggest. Finally, several not-so-well-


known input parameters (e.g., heavy-to-light form factors and light-cone distribution ampli-
tudes) introduce sizable numerical uncertainties in the predictions.

3. Strong-interaction phases arising from final-state interactions are suppressed in the heavy-
quark limit. More precisely, the imaginary parts of nonleptonic decay amplitudes are suppressed
by at least one power of $\alpha_s(m_b)$ or $\Lambda_{QCD}/m_b$. At leading power, the phases are calculable
from the imaginary parts of the hard-scattering kernels in the factorization formula. Since this
observation is of paramount importance to the phenomenology of direct CP violation, we will
discuss it in some more detail.

3 Final-State Interactions and Rescattering Phases

Final-state interactions are usually discussed in terms of intermediate hadronic states. This is
suggested by the unitarity relation (taking $\bar{B} \to \pi\pi$ for definiteness)

$$\text{Im} A_{\bar{B} \to \pi\pi} \sim \sum_n A_{\bar{B} \to n} A^*_{n \to \pi\pi}. \tag{3}$$

However, because of the dominance of hard rescattering in the heavy-quark limit we can also
interpret the sum as extending over intermediate states of partons. In the limit $m_b \to \infty$ the
number of physical intermediate states is arbitrarily large. We may then argue on the grounds of
parton–hadron duality that their average is described well enough (up to $\Lambda_{QCD}/m_b$ corrections,
say) by a partonic calculation. This is the picture implied by the factorization formula. The
hadronic language is in principle exact. However, the large number of intermediate states makes
it intractable to observe systematic cancellations, which usually occur in an inclusive sum over
hadronic states. An example familiar from previous applications of the heavy-quark expansion is
the calculation of the inclusive semileptonic decay width of a heavy hadron. Here the leading
term is given by the free quark decay, but attempts to reproduce this obvious result by summing
over exclusive modes has been successful only in two-dimensional toy models,\textsuperscript{7,8} not in QCD.

In many phenomenological discussions of final-state interactions it has been assumed that
systematic cancellations are absent. It is then reasonable to consider the size of rescattering
effects for a subset of intermediate states (such as the two-body states), assuming that this will
provide a correct order-of-magnitude estimate for the total rescattering effect. This strategy un-
derlies all estimates of final-state phases using dispersion relations and Regge phenomenology.\textsuperscript{8,10}
These approaches suggest that soft rescattering phases do not vanish in the heavy-quark limit.
However, they also leave open the possibility of systematic cancellations. The QCD factorization
formula implies that systematic cancellations do indeed occur in the sum over all intermediate
states. The underlying physical reason is that the sum over all states is accurately represented
by a $q\bar{q}$ fluctuation in the emitted light meson of small transverse size of order $1/m_b$. Because the
$q\bar{q}$ pair is small, the physics of rescattering is very different from elastic $\pi\pi$ scattering, and hence
the Regge phenomenology applied to $B$ decays is difficult to justify in the heavy-quark limit.
Consequently, the numerical estimates for rescattering effects and final-state phases obtained
using Regge models are likely to overestimate the correct size of the effects.

4 Applications to $\bar{B}_d \to D^{(*)}+L^-$ Decays

Our results for the nonleptonic $\bar{B}_d \to D^{(*)}+L^-$ decay amplitudes (with $L$ a light meson) can be
compactly expressed in terms of the matrix elements of a "transition operator"

$$T = \frac{G_F}{\sqrt{2}} V^*_{ud} V_{cb} [a_1(DL) Q_V - a_1(D^*L) Q_A]. \tag{4}$$
where the hadronic matrix elements of the operators \( Q_V = \tilde{c} \gamma^\mu b \otimes \tilde{d} \gamma_\mu (1 - \gamma_5) u \) and \( Q_A = \tilde{c} \gamma^\mu \gamma_5 b \otimes \tilde{d} \gamma_\mu (1 - \gamma_5) u \) are understood to be evaluated in factorized form. Eq. (4) defines the quantities \( a_1(D^{(*)} L) \), which include the leading “nonfactorizable” corrections, in a renormalization-group invariant way. To leading power in \( \Lambda_{QCD}/m_b \) these quantities should not be interpreted as phenomenological parameters (as is usually done), because they are dominated by hard gluon exchange and thus calculable in QCD. At next-to-leading order in \( \alpha_s \), we obtain

\[
a_1(D^{(*)} L) = \tilde{C}_1(m_b) + \frac{\tilde{C}_2(m_b)}{N_c} \left[ 1 + \frac{C_F\alpha_s(m_b)}{4\pi} \int_0^1 du F(u, \pm z) \Phi_L(u) \right],
\]

where \( \tilde{C}_i(m_b) \) are the so-called “renormalization-scheme independent” Wilson coefficients, \( z = m_c/m_b \), and the upper (lower) sign in the second argument of the function \( F(u, \pm z) \) refers to a \( D (D^*) \) meson in the final state. An exact analytic expression for this function is known but not relevant to our discussion here. Note that the coefficients \( a_1(DL) \) and \( a_1(D^*L) \) are nonuniversal, i.e., they are explicitly dependent on the nature of the final-state mesons. Politzer and Wise have computed the “nonfactorizable” vertex corrections to the ratio of the \( \bar{B}_d \to D^\mp \pi^- \) and \( \bar{B}_d \to D^+ \pi^- \) decay rates. \(^{11}\) This requires the symmetric part (with respect to \( u \leftrightarrow 1 - u \)) of the difference \( F(u, z) - F(u, -z) \). We agree with their result.

The expressions for the decay amplitudes obtained by evaluating the hadronic matrix elements of the transition operator \( T \) involve products of CKM matrix elements, light-meson decay constants, \( \bar{B} \to D^{(*)} \) transition form factors, and the QCD parameters \( a_1(D^{(*)} L) \). A numerical analysis shows that \( |a_1| = 1.055 \pm 0.025 \) for the decays considered below. Below we will use this as our central value.

### 4.1 Test of factorization

A particularly clean test of our predictions is obtained by relating the \( \bar{B}_d \to D^{+}L^- \) decay rates to the differential semileptonic \( \bar{B}_d \to D^{+} l^- \nu \) decay rate evaluated at \( q^2 = m_L^2 \). In this way the parameters \( |a_1(D^*L)| \) can be measured directly. \(^3\) One obtains

\[
\Gamma(\bar{B}_d \to D^{+}L^-) / \Gamma(\bar{B}_d \to D^{+} l^- \nu)/|q^2 = m_L^2| = 6\pi^2 |V_{ud}|^2 f_L^2 |a_1(D^*L)|^2.
\]

With our result for \( a_1 \) this relation becomes a prediction based on first principles of QCD. This is to be contrasted with the usual interpretation of this formula, where \( a_1 \) plays the role of a phenomenological parameter that is fitted from data.

Using data reported by the CLEO Collaboration, \(^{12}\) we find

\[
|a_1(D^*\pi)| = 1.08 \pm 0.07, \quad |a_1(D^*\rho)| = 1.09 \pm 0.10, \quad |a_1(D^*a_1)| = 1.08 \pm 0.11,
\]

in good agreement with our prediction. It is reassuring that the data show no evidence for large power corrections to our results. However, a further improvement in the experimental accuracy would be desirable in order to become sensitive to process-dependent, nonfactorizable effects.

### 4.2 Predictions for class-I decay amplitudes

We now consider a larger set of so-called class-I decays of the form \( \bar{B}_d \to D^{(*)+}L^- \), all of which are governed by the transition operator (4). In Table 1 we compare the QCD factorization predictions with experimental data. As previously we work in the heavy-quark limit, i.e., our predictions are model independent up to corrections suppressed by at least one power of \( \Lambda_{QCD}/m_b \). There is good agreement between the predictions and the data within experimental errors, which however are still large. It would be desirable to reduce these errors to the
percent level. Note that we have not attempted to adjust the semileptonic form factors $F_+^{B \to D}$ and $A_0^{B \to D^*}$ entering our results so as to obtain a best fit to the data. (The fact that with $F_+(0) = A_0(0) = 0.6$ our predictions for the $\bar{B}_d \to D^{(*)} \pi^-$ branching ratios come out higher than the central experimental results reported by the CLEO Collaboration must not be taken as evidence against QCD factorization. As we have seen above, the value of $|a_1(D^* \pi)|$ extracted in a form-factor independent way is in good agreement with our theoretical result.)

The observation that the experimental data on class-I decays into heavy-light final states show good agreement with our predictions may be taken as (circumstantial) evidence that in these decays there are no unexpectedly large power corrections. In our recent work we have addressed the important question of power corrections theoretically by providing estimates for two sources of power-suppressed effects: weak annihilation and nonfactorizable spectator interactions. We stress that a complete account of power corrections to the heavy-quark limit cannot be performed in a systematic way, since these effects are no longer dominated by hard gluon exchange. However, we believe that our estimates are nevertheless instructive.

We parameterize the annihilation contribution to the $\bar{B}_d \to D^+ \pi^-$ decay amplitude in terms of an amplitude $A$ such that $A(B_d \to D^+ \pi^-) = T + A$, where $T$ is the “tree topology”, which contains the dominant factorizable contribution. We find that $A/T \sim 0.04$, indicating that the annihilation contribution is a correction of a few percent. We have also obtained an estimate of nonfactorizable spectator interactions, which are part of the $T$, finding that $T_{\text{spec}}/T_{\text{had}} \sim -0.03$. In both cases, the results exhibit the expected linear power suppression $\sim \Lambda_{\text{QCD}}/m_b$ in the heavy-quark limit. We conclude that the typical size of power corrections to the heavy-quark limit in class-I $B$ decays into heavy-light final states is at the level of 10$\%$ or less, and thus our predictions for the values and the near universality of the parameters $a_1$ governing these decay modes appear robust.

### 4.3 Remarks on class-II and class-III decay amplitudes

In the class-I decays $\bar{B}_d \to D^{(*)} L^-$ considered above, the flavor quantum numbers of the final-state mesons ensure that only the light meson $L$ can be produced by the $(d\bar{u})$ current contained in the operators of the effective weak Hamiltonian (1). The factorization formula then predicts that the corresponding decay amplitudes are factorizable in the heavy-quark limit. It also predicts that other topologies, in which the heavy charm meson is created by a $(\bar{c}u)$ current, are power

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>Theory (HQL)</th>
<th>CLEO data</th>
<th>PDG98</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{B}_d \to D^+ \pi^-$</td>
<td>$3.27 \times [F_+(0)/0.6]^2$</td>
<td>2.50 ± 0.40</td>
<td>3.0 ± 0.4</td>
</tr>
<tr>
<td>$\bar{B}_d \to D^+ K^-$</td>
<td>$0.25 \times [F_+(0)/0.6]^2$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\bar{B}_d \to D^+ \rho^-$</td>
<td>$7.64 \times [F_+(0)/0.6]^2$</td>
<td>7.89 ± 1.39</td>
<td>7.9 ± 1.4</td>
</tr>
<tr>
<td>$\bar{B}_d \to D^+ K^{*-}$</td>
<td>$0.39 \times [F_+(0)/0.6]^2$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\bar{B}_d \to D^+ a_1^-$</td>
<td>$7.76 \times [F_+(0)/0.6]^2$</td>
<td>8.34 ± 1.66</td>
<td>6.0 ± 3.3</td>
</tr>
<tr>
<td>$\bar{B}_d \to D^{*+} \pi^-$</td>
<td>$3.05 \times [A_0(0)/0.6]^2$</td>
<td>2.34 ± 0.32</td>
<td>2.8 ± 0.2</td>
</tr>
<tr>
<td>$\bar{B}_d \to D^{*+} K^-$</td>
<td>$0.22 \times [A_0(0)/0.6]^2$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\bar{B}_d \to D^{*+} \rho^-$</td>
<td>$7.59 \times [A_0(0)/0.6]^2$</td>
<td>7.34 ± 1.00</td>
<td>6.7 ± 3.3</td>
</tr>
<tr>
<td>$\bar{B}_d \to D^{<em>+} K^{</em>-}$</td>
<td>$0.40 \times [A_0(0)/0.6]^2$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\bar{B}_d \to D^{*+} a_1^-$</td>
<td>$8.53 \times [A_0(0)/0.6]^2$</td>
<td>11.57 ± 2.02</td>
<td>13.0 ± 2.7</td>
</tr>
</tbody>
</table>

Table 1: Model-independent predictions for the branching ratios (in units of 10$^{-3}$) of $\bar{B}_d \to D^{(*)} L^-$ decays in the heavy-quark limit. Predictions are in units of ($|V_{cb}|/0.04)^2 \times (|a_1|/1.05)^2 \times (\tau_{\bar{B}_d}/1.56 \text{ps})$. We show experimental results reported by the CLEO Collaboration$^{13}$ and the Particle Data Group.$^{14}$
suppressed. To study these topologies one may consider decays with a neutral charm meson in the final state. In the class-II decays $\bar{B}_d \to D^{(*)0}L^0$ the only possibility is to have the charm meson as the emission particle, whereas for the class-III decays $B^- \to D^{(*)0}L^-$ both final-state mesons can be the emission particle. The factorization formula predicts that in the heavy-quark limit class-II decay amplitudes are power suppressed with respect to the corresponding class-I amplitudes, whereas class-III amplitudes should be equal to the corresponding class-I amplitudes up to power corrections.

Experimental data indicate sizable corrections to these predictions, which are mainly due to significant heavy-quark scaling violations in the values of the semileptonic form factors and meson decay constants. For a detailed analysis of this problem the reader is referred to our recent work. Note that, whereas the QCD factorization formula (2) allows us to compute the coefficients $a_1$ in the heavy-quark limit, it does not allow us to compute the corresponding parameters $a_2$ in class-II decays. Because in these decays the emission particle is a heavy charm meson, the mechanism of color transparency is not operative. For a rough estimate of $a_2$ in $B \to \pi D$ decays we have considered the limit in which the charm meson is treated as a light meson, however with a highly asymmetric distribution amplitude. In this limit we found that $a_2 \approx 0.25 e^{-\pi i \phi}$ with large theoretical uncertainties. Remarkably, this crude estimate indicates a significant correction to naive factorization (which gives $a_2 \approx 0.12$). It yields the right order of magnitude for $|a_2|$ and, at the same time, a large strong-interaction phase.

5 Summary and Outlook

With the recent commissioning of the $B$ factories and the planned emphasis on heavy-flavor physics in future collider experiments, the role of $B$ decays in providing fundamental tests of the Standard Model and potential signatures of New Physics will continue to grow. In many cases the principal source of systematic uncertainty is a theoretical one, namely our inability to quantify the nonperturbative QCD effects present in these decays. This is true, in particular, for almost all measurements of direct CP violation. Our work provides a rigorous framework for the evaluation of strong-interaction effects for a large class of exclusive, two-body nonleptonic decays of $B$ mesons. It gives a well-founded field-theoretic basis for phenomenological studies of exclusive hadronic $B$ decays and a formal justification for the ideas of factorization.

We hope that the factorization formula (2) and its generalization to decays into two light mesons will form the basis for future phenomenological studies of nonleptonic $B$ decays. We stress, however, that a considerable amount of conceptual work remains to be completed. Theoretical investigations along the lines discussed here should be pursued with vigor. We are confident that, ultimately, this research will result in a theory of nonleptonic $B$ decays, which should be as useful for this area of heavy-flavor physics as the large-$m_b$ limit and the heavy-quark effective theory were for the phenomenology of semileptonic weak decays.

Acknowledgments

It is a pleasure to thank M. Beneke, G. Buchalla and C. Sachrajda for an ongoing collaboration on the subject of this talk. This work was supported in part by the National Science Foundation.

References


