DETERMINATION OF COSMOLOGICAL PARAMETERS FROM LARGE SCALE STRUCTURE OBSERVATIONS

B. Novosyadlyj1, R. Durrer2, S. Gottlöber3, V.N. Lukash4, S. Apunevych1
1Astronomical Observatory of L’viv State University, Kyryla and Mephodia str.8, 290005, L’viv, Ukraine
2Department de Physique Théorique, Université de Genève, Quai Ernest Ansermet 24, CH-1211 Genève 4, Switzerland
3Astrophysikalisches Institut Potsdam, An der Sternwarte 16, D-14482 Potsdam, Germany
4Astro Space Center of Lebedev Physical Institute of RAS, Profsoyuznaya 84/32, 117810 Moscow, Russia

Abstract

The possibility of determining cosmological parameters on the basis of a wide set of observational data including the Abell-ACO cluster power spectrum and mass function, peculiar velocities of galaxies, the distribution of Ly-α clouds and CMB temperature fluctuations is analyzed. Using a χ² minimization method, assuming ΩΛ + Ωm = 1 and no contribution from gravity waves, we found that a tilted ΛCDM model with one sort of massive neutrinos and the parameters n = 1.12 ± 0.10, Ωm = 0.41 ± 0.11 (ΩΛ = 0.59 ± 0.11), Ωcdm = 0.31 ± 0.15, Ων = 0.059 ± 0.028, Ωb = 0.039 ± 0.014 and h = 0.70 ± 0.12 (standard errors) matches observational data best. Ων is higher for more species of massive neutrinos, ~ 0.1 for two and ~ 0.13 for three species. Ωm raises by ~ 0.08 and ~ 0.15 respectively. The 1σ (68.3%) confidence limits on each cosmological parameter, which are obtained by marginalizing over the other parameters, are 0.82 ≤ n ≤ 1.39, 0.19 ≤ Ωm ≤ 1 (0 ≤ ΩΛ ≤ 0.81), 0 ≤ Ων ≤ 0.17, 0.021 ≤ Ωb ≤ 0.13 and 0.38 ≤ h ≤ 0.85. Varying only a subset of parameters and fixing the others shows also that the observational data set used here rules out pure CDM models with h ≥ 0.5, scale invariant primordial power spectrum, zero cosmological constant and spatial curvature at a very high confidence level, > 99.99%. The corresponding class of MDM models are ruled out at ~ 95% C.L. It is notable also that this data set determines the amplitude of scalar fluctuations approximately at the same level as COBE four-year data. It indicates that a possible tensor component in the COBE data cannot be very substantial.

1 Introduction

The last years of the past century are marked by huge eorts of the community of astronomers, physicists and astrophysicists devoted to determine the most fundamental parameters of our Universe, the cosmological parameters. The most important among them are the mass densities of baryons Ωb (in units of the critical density) and of cold dark matter Ωcdm, the neutrino rest masses mν and their total density Ων, the value of cosmological term Λ or the Hubble constant H0, the spatial curvature parameter Ωk and the slopes n and amplitudes A of the primordial power spectra of scalar and tensor fluctuations.

The primordial ratio of the number of deuterium to hydrogen nuclei (D/H) created in Big Bang nucleosynthesis is the most sensitive measure of the cosmological density of baryons Ωb. Quasar absorption systems give definite measurements of the primordial deuterium and the most accurate value of baryon density obtained recently in this way is Ωbh^2 = 0.019 ± 0.0024 (10).

The measurements of the neutrino rest mass is not so certain, unfortunately. Up-to-day we have only some indications for the range where it may be found. The oscillations of solar and atmospheric neutrinos registered
by the SuperKamiokande experiment show that the difference of rest masses between $\tau$- and $\mu$-neutrinos is $0.02 < \Delta m_{\tau\mu} < 0.08eV$ \cite{17,32}. This also provides a lower limit for the neutrino mass, $m_\nu \geq |\Delta m|$ and does not exclude models with cosmologically significant values. The value is close to the flat Universe, $\Omega_m \approx 0.75$ and the mean density of the universe is $\Omega_m = 0.75 \pm 0.1$ \cite{29,34,3}. The upper limit for the neutrino mass comes from the observed large scale structure of our Universe: $\sum_i m_\nu_i / 94eV \leq 0.3h^2$. Since observations give for the Hubble parameter an upper limit of $h = 0.8$ one gets $\sum_i m_\nu_i \leq 18eV$. It is interesting to note that this upper limit coincides roughly with the upper limit for the electron neutrino mass obtained from the supernova explosion SN1987A and tritium $\beta$-decay experiments.

Important conclusions about measurements of matter density $\Omega_m (\equiv \Omega_k + \Omega_{cdm} + \Omega_\nu)$ come from the Supernova Cosmology Project and the High-z Supernova Search. In particular, the relation of observed brightness vs. redshift for SNeIa shows that distant supernovae are fainter than expected for a decelerating Universe, and, thus, more distant. This can be interpreted as an accelerated expansion rate, or $\Omega_\Lambda > 0$. The best-fit value is $\Omega_\Lambda = \frac{4}{3} \Omega_m + \frac{4}{3} \pm 0.1$ (1$\sigma$ error) and $\Omega_m = 1$ models are ruled out at the 8$\sigma$ level \cite{20}. For a flat Universe $\Omega_m + \Omega_\Lambda = 1$ ($\Omega_k = 0$) the best-fit values are $\Omega_m = 0.25 \pm 0.1$ and $\Omega_\Lambda = 0.75 \pm 0.1$ \cite{29,34,3}.

An upper limit of $\Omega_\Lambda < 0.7$ (95% C.L.) follows from gravitational lensing statistics\cite{4,16}, just consistent with distant supernovas results.

Strong evidence against an open Universe can be derived from recent measurements of the position of the first acoustic peak in the cosmic microwave background (CMB) power spectrum by the Boomerang experiment\cite{25}. The 1$\sigma$ range for the curvature parameter derived from this experiment is $-0.25 \leq \Omega_k \leq 0.15$ \cite{26} and the mean value is close to the flat Universe, $\Omega_k \approx 0$.

Currently there are a few completely independent and broad routes to the determination of the Hubble constant $H_0$. The direct experiments can be divided into three groups: the gravitational lens time delay methods, the Sunyaev-Zel’dovich method for clusters and extra-galactic distance measurements. Almost all observations yield values of $H_0$ in the range 50-80 km/sec/Mpc.

Other independent methods for the determination of cosmological parameters are based on large scale structure (LSS) observations. Their advantage is that all parameters mentioned above can be determined together because the form and amplitude of the power spectrum of density fluctuations are rather sensitive to all of them. Their disadvantage is that they are model dependent. This approach has been carried out in several papers \cite{1,23,37,8,27,30} and references therein) and it is also the goal of this paper. The papers on this subject differ by the number of parameters and the set of observational data included into the analysis. In this paper a total of 23 measurements from sub-galaxy scales (Ly-\alpha clouds) over cluster scales up to horizon scale (CMB quadrupole) is used to determine eight cosmological parameters, namely the tilt of the primordial spectrum $n$, the densities of cold dark matter $\Omega_{cdm}$, hot dark matter $\Omega_\nu$, baryons $\Omega_b$ and cosmological constant $\Omega_\Lambda$, the number of massive neutrino species $N_\nu$, the Hubble parameter $H_0$ and, in addition, the bias parameter $b_{cl}$ for rich clusters of galaxies. We restrict ourselves to the analysis of spatially flat cosmological models with $\Omega_\Lambda + \Omega_m = 1$ ($\Omega_k = 0$) and to an inflationary scenario without tensor mode. We also neglect the effect of a possible early reionization which could reduce the amplitude of the first acoustic peak in the CMB anisotropy spectrum.

In comparison to the companion paper\cite{28} the influence of the uncertainties in the normalization of the scalar mode amplitude caused by experimental errors on determination of cosmological parameters is also taken into account here.
The experimental data set and our methods

We use the power spectrum of Abell-ACO clusters \(^{14,33}\), measured in the range \(0.03 \leq k \leq 0.2h/{\text{Mpc}}\), as observational input. Its amplitude and slope at lower and larger scales are quite sensitive to baryon content \(\Omega_b\), Hubble constant \(h\), neutrino mass \(m_\nu\) and number of species of massive neutrinos \(N_\nu\). \(^{27}\). The total number of Abell-ACO data points with their errors used for minimization is 13, but not all of these points can be considered as independent measurements. Since we can accurately fit the power spectrum by an analytic expression depending on three parameters only (the amplitude at large scales, the slope at small scales and the scale of the bend); we assign to the power spectrum 3 effective degrees of freedom.

The second observational data set which we use are the position and amplitude of the first acoustic peak derived from the data on the angular power spectrum of CMB temperature fluctuations. To determine the position and amplitude of the first acoustic peak we use a 6-th order polynomial fit to the data set on CMB temperature anisotropy, accumulated in Table 2 of our accompanied paper \(^{28}\), 51 data points in total. The amplitude \(A_p\) and position \(\ell_p\) of first acoustic peak determined from this fit are \(79.6 \pm 16.5\mu\text{K}\) and \(253 \pm 70\) correspondingly. The statistical errors are estimated by edges of the \(\chi^2\)-hyper-surface in the space of polynomial coefficients which corresponds to 68.3\% (1\sigma) probability level under the assumption of Gaussian statistics. Also the mean weighted bandwidth of each experiment around \(\ell_p\) is added to obtain total \(\Delta \ell_p\).

A constraint on the amplitude of the matter density fluctuation power spectrum at cluster scale can be derived from the cluster mass and X-ray temperature functions. It is usually formulated in terms of the density fluctuation in a top-hat sphere of \(8h^{-1}\text{Mpc}\) radius, \(\sigma_8\), which can be easily calculated for the given initial power spectrum. According to the recent optical determination of the mass function of nearby galaxy clusters \(^{18}\) and taking into account the results from other authors (for references see \(^{7}\)) we use the value \(\tilde{\sigma}_8\Omega_m^{0.46-0.09\Omega_m} = 0.60 \pm 0.08\). From the existence of three most massive clusters of galaxies observed at \(z > 0.5\) a further constraint has been established by Bahcall & Fan \(^{21}\): \(\tilde{\sigma}_8\Omega_m^0 = 0.8 \pm 0.1\) , where \(\alpha = 0.24\) if \(\Omega_\Lambda = 0\) and \(\alpha = 0.29\) if \(\Omega_\Lambda > 0\) with \(\Omega_\Lambda + \Omega_m = 1\).

A constraint on the amplitude of the linear power spectrum of density fluctuations in our vicinity comes from the study of galaxy bulk flow, the mean peculiar velocity of galaxies in sphere of radius 50h\(^{-1}\text{Mpc}\) around our position. We use the data given by Kollat & Dekel \(^{21}\), \(\tilde{V}_{50} = (375 \pm 85)\text{km/s}\).

A further essential constraint on the linear power spectrum of matter clustering at galactic and sub-galactic scales \(k \sim (2 - 40)h/{\text{Mpc}}\) can be obtained from the Ly-\(\alpha\) forest of absorption lines seen in quasar spectra \(^{19,13}\). Assuming that the Ly-\(\alpha\) forest is formed by discrete clouds of a physical extent near Jeans scale in the reionized inter-galactic medium at \(z \sim 2 - 4\), Gnedin \(^{19}\) has obtained a constraint on the value of the r.m.s. linear density fluctuations \(1.6 < \tilde{\sigma}_P(z = 3) < 2.6\) (95\% C.L.) at Jeans scale for \(z = 3\) equal to \(k_F \approx 38\Omega_m^{1/2}/h/{\text{Mpc}}\). \(^{20}\)

The procedure to recover the linear power spectrum from the Ly-\(\alpha\) forest has been elaborated by Croft et al.\(^{13}\). Analyzing the absorption lines in a sample of 19 QSO spectra, they have obtained the following 95\% C.L. constraint on the amplitude and slope of the linear power spectrum at \(z = 2.5\) and \(k_p = 1.5\Omega_m^{1/2}/h/{\text{Mpc}}\)

\[
\hat{\Delta}_p^2(k_p) \equiv k_p^3P(k_p)/2\pi^2 = 0.57 \pm 0.26, \\
\hat{n}_p \equiv \frac{\Delta \log P(k)}{\Delta \log k} \bigg|_{k_p} = -2.25 \pm 0.1.
\]

In addition to the power spectrum measurements we use the constraints on the value of Hubble constant \(\hat{h} = 0.65 \pm 0.15\) which is a compromise between measurements made by two groups; \(^{36}\) and \(^{24}\). We also employ the nucleosynthesis constraints on the baryon density of \(\Omega_b h^2 = 0.019 \pm 0.0024\) (95\% C.L.) \(^{10}\).

In order to find the best fit model we must evaluate the above mentioned quantities for a given cosmological model.

To this end we use the accurate analytic approximations of the MDM transfer function \(T(k; z)\) depending on the parameters \(\Omega_m, \Omega_b, \Omega_\nu, N_\nu, h\) by Eisenstein & Hu \(^{15}\).
The linear power spectrum of matter density fluctuations is

\[ P(k; z) = A k^n T^2(k; z)D_1^2(z)/D_1^2(0), \]

where \( A \) is the normalization constant and \( D_1(z) \) is the growth factor, useful analytical approximation for which has been given by Carrol et al.\(^{12}\).

We normalize the spectra using the 4-year COBE data which can be expressed by the value of the density perturbation at the horizon crossing scale, \( \delta_h \). The normalization constant is related to \( \delta_h \) by

\[ A = 2\pi^2 \delta_h^2 (3000/h)^{3+n} \text{ Mpc}^{3+n}. \]

The Abell-ACO power spectrum is given by the matter power spectrum at \( z = 0 \) multiplied by a linear and scale independent cluster biasing parameter \( b_{cl} \), which we include as a free parameter

\[ P_{A+ACO}(k) = b_{cl}^2 P(k; 0). \]

For a given set of parameters \( n, \Omega_m, \Omega_b, h, \Omega_\nu, N_\nu \) and \( b_{cl} \) the theoretical value of \( P_{A+ACO}(k_j) \) can now be calculated for each observed scale \( k_j \). Let's denote these values by \( y_{j} \) (\( j = 1, ..., 13 \)).

The dependence of position and amplitude of the first acoustic peak in the CMB power spectrum on cosmological parameters has been investigated using the public code CMBfast by Seljak & Zaldarriaga\(^{15}\). As expected, these characteristics are independent on the hot dark matter content. We determine the values \( \ell_p \) and \( A_p \) for given parameters \( (n, h, \Omega_b \) and \( \Omega_\Lambda) \) on a 4-dimensional grid for parameter values in between the grid points we determine \( \ell_p \) and \( A_p \) by linear interpolation. We denote \( \ell_p \) and \( A_p \) by \( y_{14} \) and \( y_{15} \) respectively.

The theoretical values of the other experimental constraints are obtained as follows: The density fluctuation \( \sigma_8 \) is calculated according to

\[ \sigma_8^2 = \frac{1}{2\pi^2} \int_0^\infty k^2 P(k; 0)W^2(8\text{Mpc} \; k/h)dk; \]

with \( P(k; z) \) from Eq. (3). We set \( y_{16} = \sigma_8 \Omega_m^{0.46 - 0.09\Omega_m} \) and \( y_{17} = \sigma_8 \Omega_\alpha \), where \( \alpha = 0.24 \) for \( \Omega_\Lambda = 0 \) and \( \alpha = 0.29 \) for \( \Omega_\Lambda > 0 \), respectively.

The r.m.s. peculiar velocity of galaxies in a sphere of radius \( R = 50h^{-1}\text{Mpc} \)

\[ V_{50}^2 = \frac{1}{2\pi^2} \int_0^\infty k^2 P^{(v)}(k) e^{-k^2 R_j^2} W^2(50\text{Mpc} \; k/h)dk, \]

where \( P^{(v)}(k) \) is the density-weighted power spectrum for the velocity field\(^{15}\), \( W(50\text{Mpc} \; k/h) \) is a top-hat window function, and \( R_j = 12h^{-1}\text{Mpc} \) is the radius of a Gaussian filter used for smoothing of the raw data. For the scales considered \( P^{(v)}(k) \approx (\Omega_0^{0.6} H_0^2 P(k; 0)/k^2 \). We denote the r.m.s. peculiar velocity by \( y_{18} \).

The value of the r.m.s. linear density perturbation from the formation of Ly-\( \alpha \) clouds at redshift \( z \) and scale \( k_F \) is given by

\[ \sigma_F^2(z) = \frac{1}{2\pi^2} \int_0^\infty k^2 P(k; z)e^{(-k/k_F)^2} dk. \]

We set \( \sigma_F^2(z = 3)y_{19} \).

The value of \( \Delta_F^2(k_p, z) \) and the slope \( n(z) \) are obtained from the linear power spectrum \( P(k; z) \) by Eq. (1) and Eq. (2) at \( z = 2.5 \) and \( k_p = 0.008H(z)/(1+z)(\text{km/s})^{-1} \), and are denoted by \( y_{20} \) and \( y_{21} \) accordingly.

For all tests except Gnedin’s Ly-\( \alpha \) test we use the density weighted transfer function \( T_{\text{cl0}}(k; z) \) from\(^{15}\). For \( \sigma_F \) the function \( T_{\text{cl0}}(k; z) \) is used according to the prescription given by Gnedin\(^{19}\). Note, however, that even in the model with maximal \( \Omega_\nu \) (\( \sim 0.2 \)) the difference between \( T_{\text{cl0}}(k, z) \) and \( T_{\text{cl0}}(k, z) \) is less than 12\% for \( k \leq k_p \).

Finally, the values of \( \Omega_b h^2 \) and \( h \) are denoted by \( y_{22} \) and \( y_{23} \) respectively.

Under the assumption that the errors on the data points are Gaussian, the deviations of the theoretical values from their observational counterparts can be characterized by \( \chi^2 \):

\[ \chi^2 = \sum_{j=1}^{23} \left( \frac{y_j - y_j}{\Delta y_j} \right)^2, \]
where \( \tilde{y}_j \) and \( \Delta \tilde{y}_j \) are the experimental data and their dispersions, respectively. The set of parameters \( n, \Omega_m, \Omega_b, h, \Omega_\nu, N_\nu \) and \( b_{cl} \) are then determined by minimizing \( \chi^2 \) using the Levenberg-Marquardt method \(^\text{31}\). The derivatives of the predicted values w.r.t the search parameters required by this method are calculated numerically using a relative step size of \( 10^{-5} \).

This method has been tested and has proven to be reliable, independent on the initial values of parameters and it has good convergence.

### 3 Results

The determination of the parameters \( n, \Omega_m, \Omega_b, h, \Omega_\nu, N_\nu \) and \( b_{cl} \) by the Levenberg-Marquardt \( \chi^2 \) minimization method is realized in the following way: we vary the set of parameters \( n, \Omega_m, \Omega_b, h, \Omega_\nu \) and \( b_{cl} \) and find the minimum of \( \chi^2 \), using all observational data described in previous section. Since the \( N_\nu \) is discrete, we repeat the procedure three times for \( N_\nu = 1, 2, \) and \( 3 \). The lowest of the three minima is the minimum of \( \chi^2 \) for the complete set of free parameters.

Table 1: Cosmological parameters determined for the tilted \( \Lambda \)CDM model with one, two and three species of massive neutrinos.

<table>
<thead>
<tr>
<th>( N_\nu )</th>
<th>( \chi^2_{\text{min}} )</th>
<th>( n )</th>
<th>( \Omega_m )</th>
<th>( \Omega_\nu )</th>
<th>( \Omega_b )</th>
<th>( h )</th>
<th>( b_{cl} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.64</td>
<td>1.12±0.09</td>
<td>0.41±0.11</td>
<td>0.059±0.028</td>
<td>0.039±0.014</td>
<td>0.70±0.12</td>
<td>2.23±0.33</td>
</tr>
<tr>
<td>2</td>
<td>4.82</td>
<td>1.13±0.10</td>
<td>0.49±0.13</td>
<td>0.103±0.042</td>
<td>0.039±0.014</td>
<td>0.70±0.13</td>
<td>2.33±0.36</td>
</tr>
<tr>
<td>3</td>
<td>5.09</td>
<td>1.13±0.10</td>
<td>0.56±0.14</td>
<td>0.132±0.053</td>
<td>0.040±0.015</td>
<td>0.69±0.13</td>
<td>2.45±0.37</td>
</tr>
</tbody>
</table>

The results are presented in the Table 1. The errors in the determined parameters are the square roots of diagonal elements of the covariance matrix of standard errors. More information about the accuracy of the determination of parameters and their sensitivity to the data used can be obtained from the contours of confidence levels presented in Fig. 1 for the tilted \( \Lambda \)CDM model with parameters from Table 1 (case \( N_\nu = 1 \)). These contours show the confidence regions which contain 68.3% (solid line), 95.4% (dashed line) and 99.73% (dotted line) of the total probability distribution in the two dimensional sections of the six-dimensional parameter space, if the probability distribution is Gaussian. Since the number of degrees of freedom is 7 they correspond to \( \Delta \chi^2 = 8.2, 14.3 \) and 21.8 respectively. The parameters not shown in a given diagram are set to their best-fit value.

As one can see in Fig.1a the iso-\( \chi^2 \) surface is rather prolate from the low-\( \Omega_m \) - high-\( n \) corner to high-\( \Omega_m \) - low-\( n \). This indicates a degeneracy in \( n - \Omega_m \) parameter plane. Within the 1\( \sigma \) the 'maximum likelihood ridge' in this plane can be approximated by the equation \( n\sqrt{\Omega_m} = 0.73 \). A similar degeneracy is observed in the \( \Omega_\nu - \Omega_m \) plane in the range \( 0 \leq \Omega_\nu \leq 0.17, 0.25 \leq \Omega_m \leq 0.6 \) (Fig.1c). The equation for the 'maximum likelihood ridge' or 'degeneracy equation' has here the form: \( \Omega_\nu = 0.023 - 0.44\Omega_m + 1.3\Omega_m^2 \).

The important question is: which is the confidence limit of each parameter marginalized over the other ones. The straightforward answer is the integral of the likelihood function over the allowed range of all the other parameters. But for a 6-dimensional parameter space this is computationally time consuming. Therefore, we have estimated the 1\( \sigma \) confidence limits for all parameters in the following way. By variation of all parameter we determine the 6-dimensional \( \chi^2 \) surface which contains 68.3% of the total probability distribution. We then project the surface onto each axis of parameter space. Its shadow on the parameter axes gives us the 1\( \sigma \) confidence limits on cosmological parameters. For the best \( \Lambda \)CDM model with one sort of massive neutrinos the 1\( \sigma \) confidence limits on parameters obtained in this way are presented in Table 2.
Figure 1: Likelihood contours (solid line - 68.3%, dashed - 95.4%, dotted - 99.73%) of the tilted ΛMDM model with \( N_e = 1 \) and parameters from Table 1 (\( N_e = 1 \)) in the different planes of \( n - \Omega_m - \Omega_\nu - \Omega_b - h \) space. The parameters not shown in a given diagram are set to their best fit value.
**Table 2:** The best fit values of all the parameters with errors obtain by maximizing the (Gaussian) 68% confidence contours over all other parameters.

<table>
<thead>
<tr>
<th>parameter</th>
<th>central value and errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_m$</td>
<td>0.41$^{+0.59}_{-0.22}$</td>
</tr>
<tr>
<td>$\Omega_\nu$</td>
<td>0.06$^{+0.11}_{-0.06}$</td>
</tr>
<tr>
<td>$\Omega_b$</td>
<td>0.039$^{+0.09}_{-0.018}$</td>
</tr>
<tr>
<td>$h^*$</td>
<td>0.70$^{+0.15(+0.31)}_{-0.32}$</td>
</tr>
<tr>
<td>$n$</td>
<td>1.12$^{+0.27}_{-0.30}$</td>
</tr>
<tr>
<td>$b_{cl}$</td>
<td>2.29$^{+1.3}_{-0.7}$</td>
</tr>
</tbody>
</table>

*) - the upper limit is obtained by including the lower limit on the age of the Universe due to the age of oldest stars, $t_0 \geq 13.2 \pm 3.0^{11)}$. The value obtained without this constraint is given in parenthesis.

It must be noted that, using the observational data described above, the upper 1σ edge for $h$ is equal 1.08 when we marginalized over all other parameters. But this contradicts the age of the oldest globular clusters $t_0 = 13.2 \pm 3.0^{11)}$. Thus we have included this value into the marginalization procedure for the upper limit of $h$. We then have 8 degrees of freedom (24 data points) and the 6-dimensional $\chi^2$ surface which contains 68.3% of the probability is confined by the value 13.95. We did not use the age of oldest globular cluster for searching of best fit parameters in general case because it is only a lower limit for age of the Universe.

The errors given in Table 2 represent 68% likelihood, of course, only when the probability distribution is Gaussian. As one can see from Fig.1 (all panels without degeneracy) the ellipticity of the likelihood contours in most of planes is close to what is expected from a Gaussian distribution. This indicates that around their maxima the likelihood functions are close to Gaussian. However, the asymmetry of the error bars obtained, shows that away from the maxima this is no longer the case. Therefore, our estimates of the confidence limits have to be taken with a grain of salt. The errors define the range of each parameter within which the best-fit values obtained for the remaining parameters lead to $\chi^2_{min} \leq 12.84$. Of course, the best-fit values of the remaining parameters lay within the range given in Table 2. However clearly not every set of parameters from these ranges satisfies the condition, $\chi^2_{min} \leq 12.84$. For example, standard CDM model ($\Omega_m = 1$, $h = 0.5$, $\Omega_b = 0.05$, $n = 1$ and best-fit value of cluster biasing parameter $b_{cl} = 2.17$ ($\sigma_S = 1.2$)) has $\chi^2_{min} = 142$ (!), which excludes it at very high confidence level, > 99,999%. When we use the baryon density inferred from nucleosynthesis ($h^2\Omega_b = 0.019$ ($b_{cl} = 2.25$, $\sigma_S = 1.14$)) the situation does not improve much, $\chi^2_{min} = 112$. Furthermore, even if we leave $h$ as free parameter we still find $\chi^2_{min} = 16$ (> 1σ) with the best-fit values $h = 0.37$ and $b_{cl} = 3.28$ ($\sigma_S = 0.74$); this variant of CDM is ruled out by direct measurements of the Hubble constant.

The standard MDM model ($\Omega_m = 1$, $h = 0.5$, $\Omega_b = 0.5$, $n = 1$, $\Omega_\nu = 0.2$, $N_\nu = 1$ with a best value of the cluster biasing parameter $b_{cl} = 2.74$ ($\sigma_S = 0.83$)) does significantly better: it has $\chi^2_{min} = 23.1$ (99% C.L.) which is out of the 2σ confidence contour but inside 3σ. With the nucleosynthesis constrain the situation does not change: $\chi^2_{min} = 22$; also if we leave $h$ as free parameter: $\chi^2_{min} = 21$, $h = 0.48$. But if, in addition, we let vary $\Omega_\nu$, we obtain $\chi^2_{min} = 13$ with best-fit values of $\Omega_\nu = 0.09$, $h = 0.43$, $b_{cl} = 3.2$ ($\sigma_S = 0.73$). This means that the model is ruled out by the data set considered in this work at $\sim 70\%$ confidence level only. But also here the best-fit value for $h$ is very low. If we fix it at lower observational limit $h = 0.5$ then $\chi^2_{min} = 18.9$ (the best fit values are: $\Omega_\nu = 0.15$, $b_{cl} = 2.8$ ($\sigma_S = 0.83$)), which corresponds to a confidence level of 95% .
Therefore, we conclude that the observational data set used here rules out CDM models with \( h \geq 0.5 \), a scale invariant primordial power spectrum \( n = 1 \) and \( \Omega_b = \Omega_A = 0 \) at very high confidence level, > 99.99%. MDM models with \( h \geq 0.5 \), \( n = 1 \) and \( \Omega_b = \Omega_A = 0 \) are ruled out at \( \sim 95\% \) C.L.

One can see the model with one sort of massive neutrinos provides the best fit to the data, \( \chi^2_{min} \approx 4.6 \). Note, however, that there are only marginal differences in \( \chi^2_{min} \) for \( N_\nu = 1, 2, 3 \). Therefore, with the given accuracy of the data we cannot conclude whether – if massive neutrinos are present at all – their number of species is one, two, or three.

The number of degrees of freedom is \( N_F = N_{\exp} - N_{par} = 7 \). The \( \chi^2_{min} \) for all cases is within the expected range, \( N_F - \sqrt{2N_F} \leq \chi^2_{min} \leq N_F + \sqrt{2N_F} \) for the given number of degrees of freedom. This means that the cosmological paradigm which has been assumed is consistent with the data.

One important question is how each point of the data influences our result. To estimate this we have excluded some data points from the searching procedure. Excluding any part of observable data results only in a change of the best-fit values of \( n, \Omega_m \) and \( h \) within the range of their corresponding standard errors. This indicates that the data are mutually in agreement, implying consistent cosmological parameters (within the still considerable error bars). The small scale constraints, the Ly-\( \alpha \) tests reduce the hot dark matter content from \( \Omega_\nu \sim 0.22 \) to \( \sim 0.075 \). The \( \sigma_8 \)-tests further reduce \( \Omega_\nu \) to \( \sim 0.06 \). Including of the Abell-ACO power spectrum in the search procedure, tends to enhance \( \Omega_\nu \) slightly. The most crucial test for the baryon content is of course the nucleosynthesis constraint. Its \( \sim 6\% - 1\sigma \)-accuracy safely keeps \( h^2\Omega_b \) near its median value 0.019. The parameter \( \Omega_\Omega \), in turn is only known to \( \sim 36\% \) accuracy due to the large errors of other experimental data used here, especially of the Hubble constant. The accuracy of \( h (\sim 17\%) \) is better than the one assumed from direct measurements, \( \sim 23\% \). Summarizing, we conclude that all data points used here are important for searching the best-fit cosmological parameters and do not contradict each other.

Up to this point we ignored the uncertainties in the COBE normalization. Indeed, the statistical uncertainty of the fit to the four-year COBE data, \( \delta_h \), is \( 7\% (1\sigma) \) and we want to study how this uncertainty influences the accuracy of cosmological parameters which we determine?

Varying \( \delta_h \) in the \( 1\sigma \) range we found that the best-fit values of all parameters except \( \Omega_\nu \) do not vary by more than \( 2\% \) from the values presented in Table 1. Only \( \Omega_\nu \) varies in a range of \( 12\% \). These uncertainties are significantly smaller than the standard errors given in Table 1 and neglecting them is thus justified. The normalization constant \( A \), which for best model with one species of massive neutrinos, \( A = 4.68 \times 10^7 (h^{-1}\text{Mpc})^3 + n \), varies in a range of \( 8\% \), and not \( 14\% \) as one might expect from (4) at the first sight. The reason is that variation of \( \delta_h \) is somewhat compensated by correlated variation of \( n \) and \( h \). Moreover, if we disregard the COBE normalization and treat the normalization constant \( A \) as a free parameter to be determined like the others, its best-fit value becomes \( 4.82 \times 10^7 (h^{-1}\text{Mpc})^3 + n \) (for \( N_\nu = 1 \)), consistent with COBE normalization. The best-fit values of the other parameters correspondingly do not vary substantially: \( n = 1.09, \Omega_m = 0.40, \Omega_\nu = 0.052, \Omega_b = 0.041, h = 0.68 \) and \( b_{cl} = 2.19 \) (this is less than \( 5\% \) except for \( \Omega_\nu \), which is reduced by \( \sim 12\% \)). This implies that determinations of the amplitude of scalar fluctuations by the COBE measurement of the large scale CMB anisotropies and by large scale structure data at much smaller scales are in good agreement. It also indicates that a possible tensor component in the COBE data cannot be very substantial.

4 \ Conclusions

We summarize, that the observational data of the LSS of the Universe considered here can be explained by a tilted ΛCDM inflationary model without tensor mode. The best fit parameters are: \( n = 1.12 \pm 0.09, \Omega_m = 0.41 \pm 0.11, \Omega_\nu = 0.06 \pm 0.028, \Omega_b = 0.039 \pm 0.014 \) and \( h = 0.70 \pm 0.12 \). All predictions of measurements are close to the experimental values given above and within the error bars of the data. The CDM density parameter is \( \Omega_{cdm} = 0.31 \pm 0.12 \) and \( \Omega_\Lambda \) is moderate, \( \Omega_\Lambda = 0.59 \pm 0.11 \). The neutrino matter density corresponds to a neutrino mass \( m_\nu = 94\Omega_\nu h^2 \approx 2.7 \pm 1.2 \text{eV} \). The value of the Hubble constant is close to the measurements
by Madore et al.\textsuperscript{24}) The age of the Universe for this model equals 12.3 Gyrs which is in good agreement with the age of the oldest objects in our galaxy \textsuperscript{11}). The spectral index coincides with the COBE prediction. The relation between the matter density $\Omega_m$ and the cosmological constant $\Omega_{\Lambda}$ agrees well with the independent measurements of cosmic deceleration and global curvature based on the SNIa observation.

The $1\sigma$ (68.3\%) confidence limits on each cosmological parameter, obtained by marginalizing over the other parameters, are $0.82 \leq n \leq 1.39$, $0.19 \leq \Omega_m \leq 1$, $0 \leq \Omega_{\Lambda} \leq 0.81$, $0 \leq \Omega_{\nu} \leq 0.17$, $0.021 \leq \Omega_b \leq 0.13$ and $0.38 \leq h \leq 0.85$.

The observational data set used here rules out the CDM models with $h \geq 0.5$, scale invariant primordial power spectrum $n = 1$ and $\Omega_{\Lambda} = \Omega_b = 0$ at very high confidence level, > 99.99\%. Also pure MDM models are ruled out at $\approx 95\%$ C.L.

It is remarkable also that this data set determines the value of normalization constant for scalar fluctuations which approximately equals the value deduced from COBE four-year data. It indicates that a possible tensor component in the COBE data cannot be very substantial.

The coincidence of the values of cosmological parameters obtained by different methods indicates that a wide set of cosmological measurements are correct and that their theoretical interpretation is consistent. However, we must also note that the accuracy of present observational data on the large scale structure of the Universe is still insufficient to determine a set of cosmological parameters with high accuracy.

References

1 Atrio-Barandela, F. et al., 1997, Pis’ma Zh. Eksp. Teor. Fiz. 66, 373
14 Einasto, J. et al., 1997, Nature 385, 139
17 Fukuda, Y. et al. 1998, Phys. Rev. Lett. 81, 1562
20 Gnedin N.Y., 1999, private communication
33 Retzlaff, J. et al., 1998, New Astronomy v.3, 631
34 Riess, A. et al., astro-ph/9805201