High-energy elastic hadron collisions and space structure of hadrons

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Abstract

The elastic and inelastic profiles have been derived from the measured elastic differential cross section data with the help of exact impact parameter representation of elastic scattering amplitude. The results obtained for \( pp \) scattering at energy of 53 GeV and \( 
\bar{p}p \) scattering at energy of 541 GeV have been presented. They indicate that nucleons can be regarded as objects characterized by a small core (diameter \( \sim 0.4\,\text{÷}0.8 \text{ fm} \)) and a half transparent outer shell (responsible mainly for elastic hadron scattering).
1 Introduction

High-energy elastic hadron collisions are realized mainly at small absolute values of four-momentum transfer squared $|t|$. Corresponding differential cross sections exhibit a typical diffractive behavior characterized by approximate exponential decrease for small $|t|$, being followed by diffractive minimum. That has invoked a natural expectation that the hadron elastic processes are peripheral (i.e., correspond to higher impact parameter $b$ values) in contradistinction to inelastic processes exhibiting a central character. However, the majority of models have described the high-energy elastic hadron collisions as central, i.e., a significant part of elastic hadron collisions should occur at $b \sim 0$ (see, e.g., Refs. [1]-[4]); hadrons have been interpreted as transparent objects, which has represented a puzzle [5].

The centrality of elastic processes has been shown [4] to be a direct consequence of one basic assumption being involved in these models: imaginary part $\Im F^N(s,t)$ of elastic hadron scattering amplitude being regarded as dominant in a broad interval of $t$ around zero and vanishing only at the diffractive minimum. However, the dominance of imaginary part can be justified theoretically in a very small region of momentum transfers around forward direction only [6] at very high $\sqrt{s}$ center-of-momentum total energies. That does not eliminate peripheral behavior requiring $\Im F^N(s,t)$ to go to zero approximately at $|t| \sim 0.1 \text{ GeV}^2$ (see Ref. [4]).

The standard description of elastic hadron scattering involves, however, one puzzle more. It concerns the fact that diffractive production processes are being described as peripheral (see Refs. [1, 7, 8]) even if they exhibit the characteristics very similar to elastic processes (regarded at the same time as central) and differing significantly (like elastic hadron collisions) from inelastic non-diffractive ones.

All these problems seem to follow from the fact that only the modulus $|F^N(s,t)|$ can be determined in the whole $t$ region from elastic experiments almost uniquely while amplitude phase remains arbitrary at least to some extent. Its $t$ dependence can be estimated in principle from a small part of elastic scattering data only, i.e., from a narrow interval of small $|t|$ where Coulomb and hadron scatterings interfere. Making use of sufficiently general parameterization of the $t$ dependent elastic hadron amplitude $F^N(s,t)$ the peripheral interpretation of elastic hadron processes have been shown to be preferred [4], even if some characteristics cannot be determined quite uniquely. Nevertheless, some new features of colliding nucleons can be derived.

We will start by introducing corresponding basic formulas enabling to relate the distributions of elastic and inelastic processes in the impact parameter space to the shape of elastic hadron scattering amplitude $F^N(s,t)$ (Sect. 2). Average values of the squares of impact parameter values (mean-squares) for different processes can be then shown to be derived directly from the $t$ dependent elastic hadron scattering amplitude $F^N(s,t)$ in Sect. 3.

Sect. 4 describes the results of the analysis performed on $pp$ elastic scattering data at energy of 53 GeV and on $\bar{p}p$ elastic scattering data at energy of 541 GeV. It will be shown that unique values of the total cross section $\sigma_{tot}(s)$, the diffractive
slope $B(s)$ and the quantity $\rho(s,0)$, i.e., the ratio of the real and imaginary parts of elastic hadron scattering amplitude in forward direction cannot be derived. Only some admissible values can be established. In addition to, the root-mean-squared values of the impact parameters characterizing the range of forces responsible for the total, elastic and inelastic scatterings will be derived, too. Some preliminary results concerning $\bar{p}p$ scattering case have been presented already in Ref. [9]. The actual impact parameter profiles describing the $\bar{p}p$ elastic hadron scattering at energy of 541 GeV as peripheral process in the impact parameter space will be presented then in Sect. 5.

In Sect. 6 the problem of transparent nucleons in 'head-on' collisions is discussed. Transparent nucleons are to be considered as an artifact having nothing to do with the physical reality; such a property follows directly from the a priori assumption of imaginary-part amplitude dominance.

All presented results are based on the rigorous representation of elastic scattering amplitude in the impact parameter space (valid at any $s$ and $t$) proposed by Adachi and Kotani [10] (our approach being based on), being described in Appendix A.

2 Elastic hadron scattering amplitude and impact parameter representation

If the spins of colliding hadrons are not taken into account the elastic hadron scattering can be described fully with the complex elastic hadron scattering amplitude $F_N(s,t)$. This amplitude is bounded together with all production amplitudes $T(s,t,...)$ by unitarity condition

$$\Im <f|F_N|i> = \sum_{|e><e|} <f|F_N|e><e|F_N|i> + \sum_{|n><n|} <f|T|n><n|T|i>, \quad (2.1)$$

where $\sum_{|e><e|}$ stands for an integration over all possible elastic intermediate states $|e>$ and $\sum_{|n><n|}$ includes summation over all possible production (inelastic) states as well as integration over all other kinematical variables; $|i>$ and $|f>$ are initial and final states. Eq. (2.1) can be rewritten according to van Hove (see Refs. [11] or [12]) as

$$\Im F_N(s,t) = \frac{p}{4\pi\sqrt{s}} \int dQ' F_{N*}(s,t')F_N(s,t'') + G_{inel}(s,t), \quad (2.2)$$

where $G_{inel}(s,t)$ is the so called inelastic overlap function and $p$ is the value of momentum of one particle in the center-of-momentum system; $dQ' = \sin \theta'd\theta'd\Phi'$, $t' = 2p\sin \theta'/2$, $t'' = 2p\sin \theta''/2$ and $\cos \theta'' = \cos \theta \cos \theta' + \sin \theta \sin \theta'' \cos \Phi'$. According to the optical theorem the imaginary part of the elastic hadron scattering amplitude in the forward direction is related to the total cross section by

$$\sigma_{tot}(s) = \frac{4\pi}{p\sqrt{s}} \Im F_N(s,0). \quad (2.3)$$
The elastic scattering amplitude $h_{el}(s, b)$ in the impact parameter space can be obtained with the help of Fourier-Bessel (FB) transform of the elastic hadron scattering amplitude $F^N(s, t)$

$$h_{el}(s, b) = \frac{1}{4p\sqrt{s}} \int_{t_{min}}^{0} dt \ J_0(b\sqrt{-t}) \ F^N(s, t), \quad (2.4)$$

where $J_0(x)$ is the Bessel function of the zeroth order. At finite energies the region of kinematically allowed $t$ values is limited: $t \in (t_{min}, 0)$; e.g., in the case of nucleon-nucleon scattering $t_{min} = -s + 4m^2$, where $m$ is the nucleon mass. In order to define the FB transform exactly (existence of inverse transform) at finite energies, the elastic hadron scattering amplitude is to be defined also in the unphysical region of $t$, i.e., in the interval $(-\infty, t_{min})$. Consequently, the actual shape of elastic hadron scattering amplitude $h_{el}(s, b)$ determined with the help of Eq. (2.4) might be deformed in an uncontrolled way. These problems were solved by Adachi and Kotani [10] and by Islam [12]; the corresponding mathematically exact theory of elastic scattering amplitude is briefly described in the Appendix A.

The amplitude $h_{el}(s, b)$ has been divided into two terms. The first contribution $h_1(s, b)$ is determined by the FB transform of elastic hadron amplitude defined in the physical region of $t$ while the second one comes from the unphysical region. The FB transform of the inelastic overlap function $G_{inel}(s, t)$ has been divided in a similar way. The unitarity condition (A.9) in the impact parameter space contains only the physically relevant terms based on the FB transforms of the functions $F^N(s, t)$ and $G_{inel}(s, t)$ in the physical region of $t$; and also the correction term is defined with the help of amplitude values from the physical region.

The elastic profile $|h_1(s, b)|^2$ is always non-negative and generates the total elastic cross section; it can be, therefore, considered as the distribution of elastic collisions at the impact parameter $b$. The other profiles, i.e., $\Re h_{el}(s, b)$ and the FB transform of $G_{inel}(s, t)$ oscillate around zero for higher impact parameter values. They cannot be, therefore, considered as the corresponding distributions even if their integrals performed over all impact parameter values give the corresponding cross sections.

However, as shown in the Appendix A, physically relevant profiles can be derived. Using the ambiguity of the FB transform of inelastic overlap function $G_{inel}(s, t)$ in the unphysical region of $t$, the unitarity condition (A.9) can be modified in such a way that the new total and inelastic profiles can be made to be non-negative, too. The new unitarity condition (A.15) is obtained where all individual profiles are represented by the non-negative distribution functions in the impact parameter space.

As already mentioned in Sec. 1 the currently used models of elastic scattering have led to the central impact parameter distribution of Gaussian type with the center at $b = 0$ (see, e.g., Refs. [1]-[4]); the distribution being narrower than that exhibited by inelastic processes (see, e.g., Ref. [1]). Such a picture has followed directly from the a priori assumption requiring the real part of amplitude to be
neglected in a broad interval of \( t \) around zero. If such a limitation is not applied to better fits of experimental data can be obtained. In such a case the elastic hadron scattering should be interpreted as a peripheral process. However, even if the peripherality is to be preferred the experimental data may hardly lead to unique picture, as equivalent \( \chi^2 \) values can be obtained with different degrees of peripherality; the peripherality degree being characterized, e.g., by the ratio of root-mean-squares of impact parameters for elastic and inelastic processes. Even if the available experimental data do not allow to establish these values quite uniquely, they represent surely a more qualified physical characterization of hadron collision processes than the values of \( B \) and \( \rho \) (being added to \( \sigma_{\text{tot}} \)).

3 Mean-squares of impact parameter distributions

The mentioned mean-square of elastic impact parameter distribution can be defined as

\[
< b^2(s) >_{el} = \frac{\int_0^{\infty} b db \ b^2 \ \left| h_1(s, b) \right|^2}{\int_0^{\infty} b db \ \left| h_1(s, b) \right|^2}, \quad (3.1)
\]

where \( h_1(s, b) \) is the FB transform of elastic hadron scattering amplitude \( F^N(s, t) \) in the physical region of \( t \) (see Appendix A, Eqs. (A.2) - (A.3)). This quantity can be expressed in a form containing the \( t \) dependent elastic hadron scattering amplitude \([4, 13]\):

\[
< b^2(s) >_{el} = \frac{\int_0^{t_{\text{min}}} dt \ |t| \ \left| \frac{d}{dt} F^N(s, t) \right|^2}{\int_{t_{\text{min}}}^{0} dt \ \left| F^N(s, t) \right|^2}, \quad (3.2)
\]

which can be rewritten further as a sum of the two terms

\[
< b^2(s) >_{el} = < b^2(s) >_{mod} + < b^2(s) >_{ph} = \frac{\int_0^{t_{\text{min}}} dt \ |t| \ \left( \frac{d}{dt} |F^N(s, t)| \right)^2}{\int_{t_{\text{min}}}^{0} dt \ \left| F^N(s, t) \right|^2} + \frac{\int_0^{t_{\text{min}}} dt \ |t| \ \left| F^N(s, t) \right|^2 \left( \frac{d}{dt} \zeta^N(s, t) \right)^2}{\int_{t_{\text{min}}}^{0} dt \ \left| F^N(s, t) \right|^2} \quad (3.3)
\]

where the contributions of the modulus \(|F^N(s, t)|\) and the phase \(\zeta^N(s, t)\) of the elastic hadron amplitude \(F^N(s, t)\) defined by

\[
F^N(s, t) = i |F^N(s, t)| e^{-i\zeta^N(s, t)}, \quad (3.4)
\]

are separated and both are non-negative. The derivation of Eq. (3.2) given in Ref. [4] enabled its generalization to the case when the spins of all particles involved are taken into account [14].
The mean-square of the total impact parameter (i.e., for all collision processes) can be defined in analogy to Eq. (3.1) as

$$< b^2(s) >_{tot} = \frac{\int_0^\infty b db \ b^2 \ h_{tot}(s, b)}{\int_0^\infty b db \ h_{tot}(s, b)} ,$$  \hspace{1cm} (3.5)$$

where $h_{tot}(s, b)$ is the distribution of all collisions in the impact parameter space - see Appendix, Eq. (A.15).

Using the impact parameter representation of the elastic hadron scattering amplitude introduced by Adachi and Kotani one can write

$$\frac{d}{dt} \ln \Im F^N(s, t) = \frac{1}{2\sqrt{-t}} \frac{\int_0^\infty b db \ b^2 \ h_{tot}(s, b) \ J_1(b\sqrt{-t})}{\int_0^\infty b db \ h_{tot}(s, b) \ J_0(b\sqrt{-t})} ,$$  \hspace{1cm} (3.6)$$

where $J_1(x)$ is the Bessel function of the first order. Going to limit $\sqrt{-t} \to 0$ in Eq. (3.6) one obtains

$$\frac{d}{dt} \ln \Im F^N(s, t) \bigg|_{t=0} = \frac{1}{4} < b^2(s) >_{tot} .$$  \hspace{1cm} (3.7)$$

On the other hand the logarithmic derivative of $\Im F^N(s, t)$ can be also expressed with the help of its modulus and phase (see Eq. (3.4)) as

$$\frac{d}{dt} \ln \Im F^N(s, t) = \frac{d}{dt} \frac{|F^N(s, t)|}{|F^N(s, t)|} - \cot \zeta^N(s, t) \frac{d}{dt} \zeta^N(s, t) .$$  \hspace{1cm} (3.8)$$

Assuming the first derivative of the phase $\zeta^N(s, t)$ to vanish at $t = 0$ (which is quite plausible) we obtain from Eqs. (3.6) - (3.8), that (as generally $\zeta^N(s, 0) \neq 0$)

$$< b^2(s) >_{tot} = 2B(s, 0) ,$$  \hspace{1cm} (3.9)$$

where $B(s, 0)$ is the diffractive slope at $t = 0$, defined generally as

$$B(s, t) = \frac{2}{|F^N(s, t)|} \frac{d}{dt} |F^N(s, t)| .$$  \hspace{1cm} (3.10)$$

Eq. (3.9) was derived already in Ref. [15] under two assumptions: differential cross section having a purely exponential $t$ dependence for small $|t|$ values, and the real part of $F^N(s, t)$ being neglected. Our derivation shows that it holds under more general conditions.

The mean-squares of impact parameter for elastic and total processes have been determined by Eqs. (3.2) and (3.9). The mean-square of inelastic impact parameter
can be then established with the help of modified unitarity equation (A.15). Multi-
plying this equation by \( b^3 \) and integrating over all possible impact parameter values \( b \) one obtains

\[
\int_0^{\infty} b db \ b^2 \ h_{\text{tot}}(s, b) = \int_0^{\infty} b db \ b^2 \ |h_1(s, b)|^2 + \int_0^{\infty} b db \ b^2 \ g_{\text{inel}}(s, b). \tag{3.11}
\]

Defining the mean-square of the inelastic impact parameter in analogy with Eqs. (3.1) and (3.5) as

\[
< b^2(s) >_{\text{inel}} = \frac{\int_0^{\infty} b db \ b^2 g_{\text{inel}}(s, b)}{\int_0^{\infty} b db \ g_{\text{inel}}(s, b)}. \tag{3.12}
\]

Eq. (3.11) can be rewritten as

\[
< b^2(s) >_{\text{tot}} = \frac{\sigma_{\text{el}}(s)}{\sigma_{\text{tot}}(s)} < b^2(s) >_{\text{el}} + \frac{\sigma_{\text{inel}}(s)}{\sigma_{\text{tot}}(s)} < b^2(s) >_{\text{inel}}. \tag{3.13}
\]

As both the elastic and total mean-squares can be calculated directly from elastic amplitude \( F^N(s, t) \) in \( t \) variable with the help of Eqs. (3.2), (3.9) and (3.10), Eq. (3.13) can be used for establishing their inelastic analogue. Eq. (3.13) represents a rigorous result: the mean-square of total impact parameter can be expressed as the weighted sum of mean-squares of elastic and inelastic impact parameters with the weights representing the corresponding branching ratios.

It is useful to mention that Eqs. (3.11) and (3.13) are valid not only for the second power of \( b \) but for any function of \( b \) provided the corresponding integrals exist.

4 Elastic nucleon - nucleon scattering and analysis of experimental data

In the experiments with charged nucleons the hadron scattering is always accompa-
 nied by Coulomb scattering playing a dominant role at very small \( |t| \). The analysis of corresponding experimental data (i.e., in the interference region) was being performed in the past currently with the help of a simplified West and Yennie (WY) formula (see, e.g., Refs. [16] - [18]) for the total amplitude \( F^{C+N}(s, t) \)

\[
F^{C+N}(s, t) = e^{\imath \phi} F^C(s, t) + F^N(s, t) = \pm \frac{\alpha_s}{t} f_1(t) f_2(t) e^{\imath \phi} + \frac{\sigma_{\text{tot}}}{4\pi} p \sqrt{s} (p + i) e^{Bt/2}, \tag{4.1}
\]

derived within the framework of QED with the help of Feynman diagrams representing one photon exchange. There are two form factors \( f_1(t) \) and \( f_2(t) \) corresponding to
individual colliding hadrons in the first term representing Coulomb elastic scattering amplitude $F^C(s,t)$. The upper (lower) sign corresponds to the $pp$ ($\bar{p}p$) scatterings. It holds \cite{16} for the relative phase in the standard WY expression (4.1):

$$\alpha \Psi = \mp \alpha(\ln(-Bt/2) + \gamma) \quad (4.2)$$

where $\gamma = 0.577215$ is the Euler constant and $\alpha = 1/137.036$ is the fine structure constant. The elastic hadron scattering is fully characterized in this scheme by the total cross section $\sigma_{tot}$, the quantity $\rho$ (the ratio of the real to imaginary parts of elastic hadron scattering amplitude in forward direction) and the diffractive slope $B$ (all these quantities being constant).

There are, however, two main deficiencies in using Eq. (4.1) for the analysis of experimental data. First, different formulas must be made use of for the lower and higher values of $|t|$ ($F^{C+N}(s,t)$ and/or $F^N(s,t)$), while the boundary is not clearly defined. Second, the formula is based on some simplifying assumptions that are not fully justified \cite{6}.

To obtain more precise and sufficiently consistent results it is necessary to use a more general formula for the total elastic scattering amplitude \cite{6, 19}:

$$F^{C+N}(s,t) = \pm \frac{\alpha s}{t} f_1(t)f_2(t) + F^N(s,t) \left\{ 1 \mp i\alpha \int_{t_{min}}^{0} dt' \left[ \ln \left( \frac{t'}{t} \right) f_1(t')f_2(t') \right]' - \frac{1}{2\pi} \left[ \frac{F^N(s,t')}{F^N(s,t)} - 1 \right] I(t,t') \right\}, \quad (4.3)$$

where

$$I(t,t') = \int_0^{2\pi} d\Phi f_1(t'')f_2(t'') \frac{t''}{t''} \quad (4.4)$$

and $t'' = t + t' + 2\sqrt{tt'} \cos \Phi$. This formula is valid at any $s$ and $t$ up to the terms linear in $\alpha$. At difference to the standard analysis based on the simplified WY formula \cite{17, 18} the new approach enables to perform the analysis in the whole region of experimental differential cross section data, i.e., in the Coulomb, interference and hadronic domains simultaneously with the help of one common formula. It enables to separate the Coulomb and hadron elastic scatterings practically in a model-independent way. It turns out that contrary to general belief the influence of Coulomb scattering even at higher $|t|$ values cannot be fully neglected \cite{19}.

The corresponding differential cross section is given by

$$\frac{d\sigma(s,t)}{dt} = \frac{\pi}{sp^2} |F^{C+N}(s,t)|^2. \quad (4.5)$$

The total cross section $\sigma_{tot}$ equals now \cite{19}

$$\sigma_{tot} = \frac{4\pi}{p\sqrt{s}} |F^N(s,0)| \frac{1}{\sqrt{1+p^2(s,0)}}, \quad (4.6)$$
where

\[ \rho(s, t) = \tan \zeta^N(s, t) \]  

(4.7)

is the ratio of real to imaginary parts at individual values of \( t \); the value of \( \rho(s, 0) \) being compared to \( \rho \) in Eq. (4.1); and similarly \( B(s, 0) \) (see Eq. (3.10)) to \( B \).

To derive the \( t \) dependence of the elastic hadron scattering amplitude from experimental data (in the absence of any reliable theory of diffraction scattering) some convenient parametrization of its has to be used. The following parameterizations of modulus and phase (proposed in Ref. [19]) has been made use of:

\[
|F^N(s, t)| = (a_1 + a_2 t) \exp(b_1 t + b_2 t^2 + b_3 t^3) + (c_1 + c_2 t) \exp(d_1 t + d_2 t^2 + d_3 t^3),
\]

(4.8)

\[
\zeta^N(s, t) = \zeta_0 + \zeta_1 \left| \frac{t}{t_0} \right|^k e^{\nu t} + \zeta_2 \left| \frac{t}{t_0} \right|^\lambda,
\]  

(4.9)

Any behavior including central as well as peripheral pictures of elastic hadron scattering may be then described with the help of these formulas; the quantities \( a_k, c_k, b_j, d_j, \zeta_i, \kappa, \nu \) and \( \lambda \) being free (energy dependent) parameters.

Some analysis has been performed also with the help of the phase parameterized as

\[
\zeta^N(s, t) = \arctan \frac{\rho_0}{1 - \left| \frac{t}{t_{\text{diff}}} \right|^\kappa},
\]

(4.10)

where \( t_{\text{diff}} \) corresponds to diffractive minimum (\( \rho_0 \) and \( t_{\text{diff}} \) being energy dependent free parameters). This formula always leads to a central distribution of elastic hadron scattering.

It was already mentioned that the \( t \) dependence of the phase cannot be uniquely derived from experimental data with the help of general parameterization (4.9). Consequently we have added the constraint by requiring for the ratio

\[
\eta = \frac{|h_{el}(s, b = 0)|^2}{|h_{el}(s, b_{\text{max}})|^2}
\]

(4.11)

to have in the optimization procedure a fixed value lying between 0 and 1. Here, \( b_{\text{max}} \) is the value of impact parameter for which \( |h_{el}(s, b)|^2 \) has its maximum. Evidently, the case when \( b_{\text{max}} = 0 \) (i.e., \( \eta = 1 \)) corresponds to a central picture, while \( b_{\text{max}} > 0 \) leads to a peripheral picture. The degree of peripherality will be maximal when \( \eta \) approaches 0.

The given formulas have been applied to experimental data on \( pp \) elastic scattering at energy of 53 GeV and \( \bar{p}p \) elastic scattering at energy of 541 GeV. Fits for different values of \( \eta \) have been found.

### 4.1 \( pp \) elastic scattering at energy of 53 GeV

The standardly normalized data for \( pp \) elastic scattering at energy of 53 GeV were taken from Ref. [20]. To combine the data from different experiments the normalization coefficients in individually measured kinematical regions were considered as
free parameters in the fits. Their values were admitted to change within the corresponding statistical errors; once established they were kept fixed in all remaining constraint fits.

Table I contains the results of the analysis performed with the help of formula (4.3) for both the cases of central and peripheral pictures of elastic hadron scattering in the impact parameter space. The modulus of elastic hadron amplitude was parameterized with the help of formula (4.8) in both the cases; for its phase formula (4.9) was used in the peripheral case while in the central case formula (4.10) was applied to.

Using formula (4.10) the phase $\zeta^N(s,t)$ can be uniquely established from experimental data, and consequently, the values of $\sigma_{tot}$, $B$ and $\rho$ can be uniquely determined from the available experimental data. With the help of Eq. (4.9) only some admissible regions of these values can be derived.

Fixing the quantity $\rho = \rho(s,0)$ to different values we looked then for the best fits in both the peripheral and central cases of elastic hadron scattering. The corresponding values of $\chi^2$ (Table I) indicate that the admissible values of $\rho$ in the peripheral case of $pp$ elastic collisions can lie in the interval (-0.12, 0.08) while in the central case the interval of admissible $\rho$ values is much narrower, i.e., (0.07, 0.08) - see Fig. 1.

Different values of $\sigma_{tot}$ and $B$ correspond, of course, to changing values of $\rho$. The values of $\sigma_{tot}$ in the peripheral case lie within the interval (42.65, 42.97) mb (see Table I and Fig. 2), while the values of the diffractive slope $B$ can lie in the interval (13.45, 13.68) GeV$^2$ (see Table I and Fig. 3). In the central case $\sigma_{tot}$ lies within the interval (42.65, 42.75) mb (see Fig. 2) and $B$ in the interval (13.25, 13.35) GeV$^2$ (see Fig. 3). The typical errors corresponding to individual basic quantities, i.e., to the total cross section, the diffractive slope and the $\rho$ quantity are given in Table I, too (quantities in brackets).

All the optimal peripheral fits (corresponding to different values of $\rho$) exhibit roughly the same peripheral profiles characterized by the value of $\eta \sim 0.38$ (see Eq. (4.11)) and also approximately by the same value of elastic root-mean-square $\sim 1.80$ fm as shown in Table II and Fig. 4 while in the central case the elastic root-mean-square is practically constant and equals approximately only to 0.68 fm (see Fig. 5).

The value of the root-mean-square of elastic impact parameter (1.78 $\div$ 1.80) fm calculated with the help of Eq. (3.3) is composed of two terms. The first term representing the contribution of the modulus of the elastic hadron scattering amplitude contributes to the elastic root-mean-square by the value of 0.68 fm. The second term, containing a contribution of the phase of elastic hadron amplitude, represents then the main contribution to the root-mean-square of elastic impact parameter in a peripheral case, reaching the value of about 1.65 fm. It equals practically zero for the central behavior.

Table II also contains the root-mean-square values of the total and inelastic impact parameters. The root-mean-squares of the total impact parameter determined
with the help of Eq. (3.9) are slightly higher than 1 fm. They do not depend practically on actual $t$ dependence of the phase $\zeta^N(s, t)$ used in the analysis.

The values of the root-mean-square of inelastic impact parameter calculated with the help of Eq. (3.13) are significantly smaller in the peripheral case of elastic hadron scattering than in the central case. In the case of central behavior of elastic hadron scattering the root-mean-square ($\sim 0.68$ fm) of the elastic impact parameter is even lesser than the root-mean-square of the inelastic one ($\sim 1.1$ fm); the elastic hadron scattering being more central (see also Ref. [1]) than inelastic scattering processes. The situation is reversed in the case of peripheral elastic hadron profiles. The more pronounced centrality of the deep inelastic scattering processes, the higher peripherality of elastic hadron scattering. Figs. 4, 5 exhibit the graphs of the root-mean-squares for the total, elastic and inelastic impact parameters calculated with the help of Eqs. (3.9), (3.3) and (3.13) for corresponding $\rho$ values.

Further analysis of the $pp$ elastic hadron scattering at this energy shows that also in the case of higher peripherality, characterized by the higher values of the elastic root-mean-squares, e.g., $\sqrt{<b^2>_{el}} \sim 1.95 \div 1.98$ fm, the region of admissible $\rho$ values is not changed substantially (and of $\sigma_{tot}$ and $B$, too).

The typical $t$ dependence of elastic hadron phase $\zeta^N(s, t)$ is shown in Fig. 6. It may be seen that there is a fundamental difference in the $t$ dependence for peripheral (full line) and the central (dashed line) behaviors. Corresponding impact parameter profiles are shown in Fig. 7.

### 4.2 $\bar{p}p$ elastic scattering at energy of 541 GeV

The data for the analysis of $\bar{p}p$ elastic hadron scattering at energy of 541 (546) GeV were taken from Refs. [21, 22] (data at energy of 630 GeV [23] being also included). The data in individual measured kinematical regions were normalized in a similar way as in the previous $pp$ case. Moreover, the normalization condition, used in UA4/2 experiment [21], i.e., $\sigma_{tot}(1 + \rho^2) = 63.3 \pm 1.5$ mb, was also taken into account.

The numerical values of basic physical characteristics of the elastic hadron scattering amplitude, i.e., the total cross section, the diffractive slope and the quantity $\rho$ are shown in lower part of Table I. The results of standard analysis obtained with the simplified form of WY total elastic scattering amplitude (see Ref. [21]) are also included.

Fig. 8 shows the $\chi^2$ distributions for different $\rho$ values in both the peripheral and central cases where the data from the total measured interval of momentum transfers were taken into account. Fig. 9 shows the values of the total cross section (obtained with the help of Eq. (4.6)) corresponding to different $\rho$ values. The condition $\sigma_{tot}(1 + \rho^2) = 63.3 \pm 1.5$ mb used in the normalization of experimental data at small $|t|$ [21] limits significantly the region of admissible total cross section values; it admits for the $\rho$ to be in the interval (0.11, 0.18) in the peripheral case; and within the interval (0.08, 0.14) in the central case.
However, the interval of admissible $\rho$ values in the peripheral case would be broader, i.e., $(0.11, 0.23)$, if instead of the previous normalization condition the value of $\sigma_{\text{tot}} = 63.0 \pm 2.1$ mb were used; the value of total cross section being estimated with the help of another (luminosity dependent) method [24]. In the central case the value of $\rho$ would lie within the interval $(0.08, 0.16)$.

Fig. 10 shows the values of the diffractive slope $B$ (determined with the help of Eq. (3.10) in forward direction) corresponding to different $\rho$ values. The used normalization condition limits its values to the interval $(16.20, 16.55)$ GeV$^2$ in the peripheral case. In the central case its value would lie within the interval $(15.80, 16.00)$. And finally the dependence of the root-mean-squares of the total, elastic and inelastic impact parameters for different $\rho$ values are shown in Figs. 11 and 12. The root-mean-square values of the total impact parameter determined with the help of Eq. (3.9) equal approximately 1.1 fm (they are a little bit higher than in the $pp$ case).

As it was mentioned, the value of the root-mean-square of elastic impact parameter calculated with the help of Eq. (3.3) is composed of two terms. The first term representing the contribution of the modulus of the elastic hadron amplitude gives the value of 0.76 fm; a little bit higher than in the previous $pp$ case. The second term, containing the contribution of the phase of elastic hadron amplitude, reaches in a peripheral case the value about 2.1 fm and is again practically zero for the central behavior. The values of the root-mean-square of inelastic impact parameter calculated with the help of Eq. (3.13) are in the peripheral case of elastic hadron scattering significantly smaller: $\sim 0.4 \div 0.5$ fm; see Table II. The $t$ dependence of the phase in peripheral and central cases is represented in Fig. 13; and corresponding elastic profiles in the impact parameter space in Fig. 14.

5 Actual inelastic and total profiles

The function $c(s, b)$ enabling to derive the densities of total and inelastic scatterings in the impact parameter space has been introduced in Sect. 2. It can be hardly derived analytically; however, its numerical values can be derived with the help of numerical procedure. Its $b$ dependence found in such a way for the peripheral case of $\bar{p}p$ elastic scattering at energy of 541 GeV is presented in Fig. 15. The corresponding total and inelastic profiles are shown, too. They are both of central character, while the original elastic profile remains unchanged and is peripheral. Also values of the total and inelastic root-mean-squares derived from the $t$ dependent elastic hadron scattering amplitude are reproduced by these newly established total and inelastic profiles.

Fig. 16 shows the shape of correction function $K(s, b)$ calculated from the $t$ dependent elastic hadron scattering amplitude $F_N^N(s, t)$ with the help of formula (A.10). It corresponds to the peripheral picture of elastic $\bar{p}p$ hadron scattering. The absolute values of $K(s, b)$ at the given value of $b$ are about 19 orders of magnitude
lesser compared to the remaining quantities in unitarity equation (A.9) at the same 
b and can be, therefore, practically neglected.

6 Transparent or hard nucleon?

The $pp$ elastic scattering at energy of 53 GeV was analyzed by Miettinen [1] provided 
that the imaginary part of elastic hadron scattering amplitude is dominant in a broad 
interval of $t$. Having used the FB transform and unitarity condition he established 
elastic, inelastic and total profiles in the impact parameter space; all these profiles 
being central. Comparing the inelastic profile to that corresponding to the black disc 
model he concluded that for $b = 0$ there is cca 6 % of events in which no inelastic 
(absorption) process occurs; the value being regarded very high from the point of 
realistic conditions. Consequently, the nucleon was claimed by Miettinen to be a 
transparent object enabling to exhibit elastic scattering even in central collisions.

The approach of Miettinen was repeated by Henzi and Valin [25] and applied to 
elastic hadron $\bar{p}p$ scattering at energy of 546 GeV. They obtained that nucleons at 
this energy should be more opaque with elastic impact parameter profile being more 
edgier and having a greater range than in the case of ISR energies.

It is evident that the idea of transparent nucleon (having influenced significantly 
all discussions concerning quark structure of hadrons) has followed from the assu-
ption of dominant imaginary part of elastic amplitude, representing an $a$ priori 
strongly limited condition. There is not any need of transparency if elastic colli-
sions are described with the help of a more exact formula and are allowed to be 
peripheral. In such a case the root-means-square of elastic impact parameter (for 
$pp$ elastic scattering at energy of 53 GeV, e.g., $1.78 \div 1.80$ fm, and for $\bar{p}p$ scattering 
at energy of 541 GeV, e.g., $2.24 \div 2.35$ fm) are significantly higher than these for 
inelastic processes (i.e., 0.68 or 0.76 fm in these two cases - see Table II).

Peripheral behavior is preferred by the total $\chi^2$ values being significantly lower 
than for any alternatives requiring the central behavior. There is not, therefore, any 
reason to regard nucleons in elastic high-energy collisions as transparent objects.

However, the nucleons cannot be denoted as standard (classical) matter objects 
having a fixed dimension in the transverse direction at any time. On the basis of 
our analysis they can be regarded as objects with a rather hard core of diameter cca 
$0.4 \div 0.8$ fm and with a practically transparent outer shell of diameter $1.8 \div 2.4$ fm; 
the given core being responsible for inelastic processes and the outer shell for elastic 
scattering (or perhaps for some diffractive production processes, too). The value 
of core diameter is a little bit smaller than the proton charge diameter determined 
with the help of different charge distribution methods inside proton (see, e.g., Ref. 
[26]).

Such a collision structure may result from two different reasons. First, all expen-
imental data must be regarded as an average over divers spin orientations. And a 
transverse momentum may correspond to different nucleon polarizations in individ-
ual events. The other reason might be related to the possibility that the dimensions of nucleons consisting of internal moving partons need not to be quite constant in all time. Experiments with polarized nucleons might contribute to the solution of this problem.

7 Conclusion

The new formulas describing elastic hadron scattering and presented in this paper have enabled to establish the impact parameter profiles for elastic and inelastic processes. The values of corresponding root-mean-squares may be taken as characteristics of the ranges of hadron forces responsible for elastic and inelastic collision processes.

The corresponding numerical values were derived from experimental data of elastic $pp$ scattering at energy of 53 GeV and $\bar{p}p$ scattering at energy of 541 GeV. They depend to some extent (if not substantially) on the peripherality degree imposed on the elastic profiles in establishing the $t$ dependence of elastic hadron scattering amplitude (especially, of its phase) from the experimental data. Higher peripherality degrees are preferred by total $\chi^2$ values, which indicates that there is not any reason to regard protons as transparent objects in elastic high-energy collisions.

The presented analysis also leads to the conclusion that it is not sufficient to characterize high-energy elastic collisions only by usual three quantities: total cross section $\sigma_{tot}$, diffractive slope $B$ and the ratio $\rho$ of real to imaginary parts of elastic hadron amplitude in the forward direction. All these quantities depend on peripherality degree imposed, significant dependence being exhibited especially by the quantity $\rho$. The presented results have opened a series of new questions concerning the actual structure of hadrons playing a role in collisions of different types.

Appendix A: Exact impact parameter representation of scattering amplitude

The approach used in Refs. [10] and [12] starts from the possibility of defining the amplitude $A(s, t)$ being identical with the elastic hadron amplitude $F^N(s, t)$ in the physical $t$ region, while in the unphysical region of $t$ the amplitude $A(s, t)$ equals unknown complex function $\lambda(s, t)$ which is assumed to be of a bounded variation for all $t$ values from $-\infty < t < t_{min}$ and which further fulfills condition that the integral

$$\int_{-\infty}^{t_{min}} \lambda(s, t)(-t)^{-1/4} dt$$

(A.1)

is absolutely convergent. Both the conditions guarantee (according to Hankel’s theorem [27]) that $A(s, t)$ has a Fourier-Bessel representation for $-\infty < t < 0$. 

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The elastic scattering amplitude $h_{el}(s, b)$ can be then expressed as a sum of two terms
\begin{equation}
 h_{el}(s, b) = h_1(s, b) + h_2(s, b), \tag{A.2}
\end{equation}
where
\begin{equation}
 h_1(s, b) = \frac{1}{4p\sqrt{s}} \int_{t_{\text{min}}}^{0} dt \, F^N(s, t) \, J_0(b\sqrt{-t}), \tag{A.3}
\end{equation}
and
\begin{equation}
 h_2(s, b) = \frac{1}{4p\sqrt{s}} \int_{-\infty}^{t_{\text{min}}} dt \, \lambda(s, t) \, J_0(b\sqrt{-t}). \tag{A.4}
\end{equation}

One can divide the inelastic overlap function into two parts in the same way \[12\]
\begin{equation}
 g_{\text{inel}}(s, b) = g_1(s, b) + g_2(s, b), \tag{A.5}
\end{equation}
where
\begin{equation}
 g_1(s, b) = \frac{1}{4p\sqrt{s}} \int_{t_{\text{min}}}^{0} dt \, G_{\text{inel}}(s, t) \, J_0(b\sqrt{-t}), \tag{A.6}
\end{equation}
and
\begin{equation}
 g_2(s, b) = \frac{1}{4p\sqrt{s}} \int_{-\infty}^{t_{\text{min}}} dt \, \mu(s, t) \, J_0(b\sqrt{-t}). \tag{A.7}
\end{equation}

The real function $\mu(s, t)$ introduced in Eq. (A.7) is restricted by the same two conditions as function $\lambda(s, t)$.

It also holds \[10\]
\begin{equation}
 \int_{0}^{\infty} b db \, \Im h_2(s, b) = \int_{0}^{\infty} b db \, g_2(s, b) = 0. \tag{A.8}
\end{equation}

Consequently, the following unitarity equation in the impact parameter space can be written which binds together only the physically relevant amplitudes $h_1(s, b)$ and $g_1(s, b) \[10, 12\]:
\begin{equation}
 \Im h_1(s, b) = |h_1(s, b)|^2 + g_1(s, b) + K(s, b). \tag{A.9}
\end{equation}

The function $K(s, b)$ can be determined with the help of elastic hadron amplitude $F^N(s, t)$ by
\begin{equation}
 K(s, b) = \frac{1}{16\pi^2 s} \int_{t_{\text{min}}}^{0} dt_1 \int_{t_{\text{min}}}^{0} dt_2 \, F^N(s, t_2) \, F^N(s, t_1) \left[ J_0 \left( \frac{b}{2p} \sqrt{-t_1(4p^2 + t_2)} \right) - J_0 \left( \frac{b}{2p} \sqrt{-t_2(4p^2 + t_1)} \right) - J_0(b\sqrt{-t_1}) \, J_0(b\sqrt{-t_2}) \right]. \tag{A.10}
\end{equation}
The function $K(s, b)$ vanishes at $b = 0$ and $b \to \infty$.

Adachi and Kotani [10] showed then that contrary to general belief the functions $\Im h_1(s, b)$ and $g_1(s, b)$ cannot be non-negative at all finite positive $b$ at finite $s$. It must hold only $\Im h_1(s, 0) \geq 0$ and both the functions $\Im h_1(s, b)$ and $g_1(s, b)$ must vanish when $b$ tends to $\infty$. At higher finite values of $b$ these functions should oscillate. The oscillations vanish at infinite energies only.

Multiplying the both sides of Eq. (A.9) by $8\pi b$ and integrating over the all possible impact parameter values $b$ one obtains in our normalization

$$\sigma_{\text{tot}}(s) = \sigma_{\text{el}}(s) + \sigma_{\text{inel}}(s) + 8\pi \int_0^\infty bdb \, K(s, b),$$

which shows that the function $K(s, b)$ fulfills the condition:

$$\int_0^\infty bdb \, K(s, b) = 0.$$  

Here $\sigma_{\text{tot}}(s)$, $\sigma_{\text{el}}(s)$ and $\sigma_{\text{inel}}(s)$ are the total, elastic and inelastic cross sections defined as integrals

$$\sigma(s) = 8\pi \int_0^\infty bdb \, Z(s, b),$$

where $Z(s, b)$ stands for one of the amplitudes $\Im h_{\text{el}}(s, b)$, $|h_{\text{el}}(s, b)|^2$ or $g_{\text{inel}}(s, b)$.

All the integrals $\sigma(s)$ in Eq. (A.13) have the physical meaning as introduced earlier but only $|h_1(s, b)|^2$ is non-negative at any $b$. Thus, only $|h_1(s, b)|^2$ can be interpreted as the density distribution (i.e., of elastic hadron scattering) in the impact parameter space. The other functions $\Im h_1(s, b)$ and $g_1(s, b)$ can turn to oscillate at some higher values of $b$; therefore, they cannot be interpreted directly as corresponding density distributions.

However, they can be modified to fulfil such a goal. Adding a suitable real function $c(s, b)$, fulfilling the condition

$$\int_0^\infty bdb \, c(s, b) = 0,$$

to the both sides of unitarity equation (A.9), both the functions $h_{\text{tot}}(s, b) = \Im h_1(s, b) + c(s, b)$ and $g_{\text{inel}}(s, b) = g_1(s, b) + K(s, b) + c(s, b)$ can be brought to be non-negative for all the values of $b$. Such a function $c(s, b)$ exists owing to the properties of function $g_2(s, b)$, or equivalently owing to properties of the function $\mu(s, b)$ defined in the unphysical region of $t$. Then the functions $h_{\text{tot}}(s, b)$ and $g_{\text{inel}}(s, b)$ can be regarded
as new density distributions of the total and inelastic scatterings in the impact parameter space and one has instead of unitarity equation (A.9) the modified unitarity condition

\[ h_{tot}(s,b) = |h_1(s,b)|^2 + g_{inel}(s,b). \]  

(A.15)

Thus, we have in principle a fully consistent description of elastic scattering in the impact parameter space.

References


Figure 1: Dependence of the total $\chi^2$ values upon the chosen value of $\rho$ for $pp$ elastic scattering at energy of 53 GeV

Figure 2: Dependence of the total cross section in $pp$ elastic scattering on the chosen value of $\rho$ at energy of 53 GeV

Figure 3: Dependence of the diffractive slope $B$ values upon $\rho$ for $pp$ elastic scattering at energy of 53 GeV

Figure 4: Dependence of the root-mean-squares for the total (dashed line), elastic (full line) and inelastic (dotted line) impact parameters upon the chosen value of $\rho$; $pp$ elastic scattering at energy of 53 GeV - peripheral case

Figure 5: Dependence of the root-mean-squares for the total (dashed line), elastic (full line) and inelastic (dotted line) impact parameters on the chosen value of $\rho$; $pp$ elastic scattering at energy of 53 GeV - central case

Figure 6: The $t$ dependence of the phase corresponding to the peripheral and the central cases of $pp$ elastic hadron scattering at energy of 53 GeV

Figure 7: Elastic impact parameter profiles in the peripheral and central cases of $pp$ elastic scattering at energy of 53 GeV

Figure 8: Dependence of the total $\chi^2$ values upon $\rho$ for $\bar{p}p$ elastic scattering at energy of 541 GeV

Figure 9: Dependence of the total cross section $\sigma_{tot}$ values upon $\rho$ for $\bar{p}p$ elastic scattering at energy of 541 GeV

Figure 10: Dependence of the diffractive slope $B$ values upon $\rho$ for $\bar{p}p$ elastic scattering at energy of 541 GeV

Figure 11: Dependence of the root-mean-squares for the total (dashed line), elastic (full line) and inelastic (dotted line) impact parameters upon $\rho$ for $\bar{p}p$ elastic scattering at energy of 541 GeV - peripheral case

Figure 12: Dependence of the root-mean-squares for the total (dashed line), elastic (full line) and inelastic (dotted line) impact parameters upon $\rho$ for $\bar{p}p$ elastic scattering at energy of 541 GeV - central case
Figure 13: $t$ dependence of the phase in the peripheral and central cases; $\bar{p}p$ elastic scattering at energy of 541 GeV

Figure 14: Impact parameter distribution of elastic $\bar{p}p$ elastic scattering at energy of 541 GeV

Figure 15: New profiles constructed with the help of function $c(s, b)$ for the peripheral case of $\bar{p}p$ elastic scattering at energy of 541 GeV

Figure 16: Correction function $K(s, b)$ calculated with the help of Eq. (A.10) for the peripheral case of $\bar{p}p$ elastic scattering at energy of 541 GeV
Table 1: Region of admissible values with typical statistical errors obtained by fitting the data of $pp$ elastic scattering at energy of 53 GeV and $\bar{p}p$ elastic scattering at energy of 541 GeV with the help of different formulas for total elastic scattering amplitude

<table>
<thead>
<tr>
<th>Data</th>
<th>Amplitude profile</th>
<th>$\sigma_{tot}$ [mb]</th>
<th>$B$ [GeV$^{-2}$]</th>
<th>$\rho$</th>
<th>$\chi^2/df$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$pp$</td>
<td>(4.3) peripheral</td>
<td>42.65 $\div$ 42.97</td>
<td>13.45 $\div$ 13.68</td>
<td>$-0.12 \div 0.08$</td>
<td>252/201 $\div$ 254/201</td>
</tr>
<tr>
<td></td>
<td>(4.3) central</td>
<td>42.65 $\div$ 42.75</td>
<td>13.25 $\div$ 13.35</td>
<td>$0.07 \div 0.08$</td>
<td>329/204 $\div$ 339/204</td>
</tr>
<tr>
<td>53 GeV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W-Y</td>
<td>central</td>
<td>42.38 $\pm$ 0.15</td>
<td>12.87 $\pm$ 0.14</td>
<td>0.077 $\pm$ 0.009</td>
<td>1.43</td>
</tr>
<tr>
<td>$\bar{p}p$</td>
<td>(4.3) peripheral</td>
<td>60.70 $\div$ 63.30</td>
<td>16.20 $\div$ 16.55</td>
<td>0.11 $\div$ 0.18</td>
<td>233/213 $\div$ 238/213</td>
</tr>
<tr>
<td></td>
<td>(4.3) central</td>
<td>62.20 $\div$ 63.00</td>
<td>15.80 $\div$ 16.00</td>
<td>0.08 $\div$ 0.14</td>
<td>354/217 $\div$ 364/217</td>
</tr>
<tr>
<td>541 GeV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W-Y</td>
<td>central</td>
<td>62.17 $\pm$ 1.50</td>
<td>15.50 $\pm$ 0.10</td>
<td>0.135 $\pm$ 0.015</td>
<td>1.1</td>
</tr>
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Table 2: Values of the total, elastic and inelastic root-mean-squares calculated with the help of formulas (3.9), (3.5) and (3.13) for $pp$ elastic scattering at energy of 53 GeV $\bar{p}p$ elastic scattering at energy of 541 GeV

<table>
<thead>
<tr>
<th>data</th>
<th>profile</th>
<th>$\sqrt{&lt;b^2&gt;_{tot}}$ [fm]</th>
<th>modulus [fm]</th>
<th>$\sqrt{&lt;b^2&gt;_{el}}$ [fm]</th>
<th>phase [fm]</th>
<th>total [fm]</th>
<th>$\sqrt{&lt;b^2&gt;_{inel}}$ [fm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$pp$</td>
<td>peripheral</td>
<td>1.026÷1.033</td>
<td>0.68</td>
<td>1.64÷1.67</td>
<td>~0.</td>
<td>1.78÷1.80</td>
<td>0.76÷0.78</td>
</tr>
<tr>
<td>53 GeV</td>
<td>central</td>
<td>1.03</td>
<td>0.68</td>
<td>~0.</td>
<td>0.68</td>
<td>1.09</td>
<td></td>
</tr>
<tr>
<td>$\bar{p}p$</td>
<td>peripheral</td>
<td>1.123÷1.135</td>
<td>0.76</td>
<td>2.11÷2.22</td>
<td>~0.</td>
<td>2.24÷2.35</td>
<td>0.39÷0.49</td>
</tr>
<tr>
<td>541 GeV</td>
<td>central</td>
<td>1.13</td>
<td>0.76</td>
<td>~0.</td>
<td>0.76</td>
<td>1.21</td>
<td></td>
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</tbody>
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