The Halo Formation Rate and its link to the Global Star Formation Rate

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Abstract. The star formation history of the universe shows strong evolution with cosmological epoch. Although we know mergers between galaxies can cause luminous bursts of star formation, the relative importance of such mergers to the global star formation rate (SFR) is unknown. We present a simple analytic formula for the rate at which halos merge to form higher-mass systems, derived from Press-Schechter theory and confirmed by numerical simulations (for high halo masses). A comparison of the evolution in halo formation rate with the observed evolution in the global SFR indicates that the latter is largely driven by halo mergers at $z > 1$. Recent numerical simulations by Kolatt et al. (1999) and Knebe & Müller (1999) show how merging systems are strongly biased tracers of mass fluctuations, thereby explaining the strong clustering observed for Lyman-break galaxies without any need to assume that Lyman-break galaxies are associated only with the most massive systems at $z \sim 3$.

1. Calculating the Halo Formation Rate

In our analysis, a halo formation event is considered to have occurred when all the mass in the halo is assembled. Note that this is different to that previously used by some authors (Lacey & Cole 1993). We wish to answer the question: ‘Given a halo of mass $M$ forms at some time, what is the probability $P(t|M)dt$ that it forms in the time interval $(t,t+dt)$?’

Standard PS theory calculates $P(M|t)dt$, the distribution of halo mass at fixed epoch, and we have shown that it is possible to calculate $P(t|M)dt$ from this using Bayes’ theorem. We can also calculate the same formula using intrinsic properties of Brownian random walks invoked in PS theory. The mass of halo a small volume element resides in at time $t$, is given by the first upcrossing of the line $\delta = \delta_c$ by a Brownian random walk in $(\delta, \sigma^2_M)$ space, where $\delta$ is a function of time and $\sigma^2_M$ is a function of mass. Using the theory of random walks we can calculate the distribution of first upcrossing times at $\sigma^2_M$, $P(\delta_c|\sigma^2_M)$, from which a simple change of variables can be used to obtain $P(t|M)dt$:

$$P(t|M)dt = \frac{\delta_c}{\sigma^2_M} \exp \left( -\frac{\delta^2_c}{2\sigma^2_M} \right) \left| \frac{d\delta_c}{dt} \right| dt.$$  \hspace{1cm} (1)

This equation is in good agreement with Monte-Carlo realisations of Brownian random walks (Fig. 1). We have also run a large N-body simulation, using
Figure 1. Comparison of N-body results (solid circles) with Press-Schechter predictions of the halo formation rate for halos of mass \( \sim 1.3 \times 10^{13} M_\odot \). The curve shows the prediction of equation 1 at this mass with parameters as given in the text and normalised to the N-body values. The distribution of \( 10^4 \) ‘formation events’ at the required mass from Monte-Carlo realisations of random walks is also shown (open circles).

the Hydra N-body hydrodynamics code (Couchman, Thomas & Pearce 1995). Groups of between 45 and 47 particles (\( 1.3 \times 10^{13} M_\odot \)) were identified using a standard friends-of-friends algorithm at 362 output times. The number which could have formed in each time interval is compared to the expected distribution in Fig. 1.

Now suppose we are only interested in a subset of formation events - e.g. those which involve similar mass objects merging together. The formation rate from such mergers is the same as that derived above, because all walks which pass through a given point can be thought of as new walks starting from that point. Consequently the mass distribution of progenitors immediately prior to the formation event is independent of the formation epoch. Turning this argument around, placing constraints on the progenitors of halos immediately prior to their formation doesn’t affect the distribution of formation times, although this might affect the bias (see later).

2. The Star Formation Rate

It is interesting to compare the halo formation rate with the observed global star formation rate. At \( z < 1 \) the rate of halo formation falls off very rapidly with cosmic time, independently of halo mass, and it is likely that this effect is primarily responsible for the observed rapid evolution in the CFRS. Such
Figure 2. The observed mean comoving volume-averaged SFR as determined from the CFRS (Lilly et al. 1996) (solid circles), optical HDF data (Madau et al. 1996; Pettini et al. 1997) (crosses) and Connolly et al. (1997) (solid stars), extinction-corrected Lyman break galaxies (Steidel et al. 1998) (solid squares), sub-mm data (Hughes et al. 1998) (open circle), and Hα surveys (Gallego et al. 1995) (solid triangle) and (Glazebrook et al. 1998) (open triangle). We assume a Salpeter IMF and flat \( \Omega_M = 1 \) cosmology. Dotted lines are the predicted halo formation rates normalised to the local SFR (convolved with a nominal starburst lifetime of 0.6 Gyr, Bruzual et al. 1993) for masses of \( 10^{10.0}, 10^{11.0}, 10^{11.5} \) M\(_\odot\) and for a mass of \( 10^{10.6} \) M\(_\odot\) (dashed line). As an illustration, we combine the formation rate of different mass halos weighted by a Gaussian in dn/dlogM centred at a mass of \( 10^{10.6} \) M\(_\odot\) (solid line). The low-z evolution is unaffected, but an increasing contribution from lower-mass halos flattens the curve at high z.

strong evolution is not seen in semi-analytic models that only include a quiescent component of star formation (Guiderdoni et al. 1998).

3. The Clustering of Lyman-break Galaxies

The strong clustering of Lyman-break galaxies has been explained as being due to the high bias of the most massive overdensities, and consideration of the abundance of Lyman-break galaxies and their clustering leads to an interpretation of them as being associated with massive halos, \( M \sim 8 \times 10^{11} h^{-1} \) M\(_\odot\) (Adelberger et al. 1998) for an \( \Omega = 0.3 \) flat universe. If star formation is initiated after halo formation then these most massive halos form too late in the universe to reproduce the observed evolution in SFR in a simple way: we would require the efficiency of star-formation to evolve with redshift. Conversely, if the Lyman-break galaxies have lower mass but are associated with newly-formed,
merging halos, then we might suppose that such mergers are also highly biased, and recent simulations indicate this to be the case (Kolatt et al. 1999, Knebe & Müller 1999).

4. Conclusions

We have presented the key points involved in deriving a simple formula for the rate of formation of new halos using Press-Schechter theory. It agrees with Monte-Carlo and N-body simulation results. We have argued that the strong cosmological evolution observed in the SFR is primarily driven by the cosmic variation in the rate of halo formation. Given that quiescent star formation does not provide enough evolution (Guiderdoni et al. 1998) we suggest that merger-induced starbursts are extremely important for star formation at \( z \sim 1 \) and are perhaps the principal sites of the observed star formation at high redshifts. At high \( z \), a more physically-motivated model is needed to deduce the relative contributions of a range of halo masses, but we have shown that a simple combination of such a range can produce evolution consistent with present data. Recent results indicate that such merging-halo systems are also sufficiently highly biased to explain the strong clustering of Lyman-break galaxies at \( z \sim 3 \).

The work highlighted here is more comprehensively covered in our recent paper available as astro-ph/9906204. We are also continuing to work on the bias of merging halos from numerical simulations.

References

Hughes D. et al., 1998, Nat, 394, 241