This talk is based on my work in collaboration with Thibault Damour. We compare the probing power of different classes of gravity experiments: solar-system tests (weak-field regime), binary-pulsar tests (strong-field regime), and future gravitational-wave observations of inspiralling binaries (strong-field effects detected in our weak-gravitational-field conditions). This is done within the most natural class of alternative theories to general relativity, namely tensor-scalar theories, in which the gravitational interaction is mediated by one tensor field ($g_{\mu\nu}$) together with one or several scalar fields ($\phi$). Our main conclusion is that strong-field tests are qualitatively different from weak-field experiments: They constrain theories which are strictly indistinguishable from general relativity in the solar system. We also show that binary-pulsar data are so precise that they already rule out the theories for which scalar effects could have been detected with LIGO or VIRGO. This proves that it is therefore sufficient to compute the ‘chirp’ templates within general relativity.

1 Introduction

Solar-system experiments probe the weak-field regime of gravity. Indeed, the largest deviations from the flat metric are at the surface of the Sun, and are of order $Gm_\odot/R_\odot c^2 \approx 2 \times 10^{-6}$ (where $m_\odot$ and $R_\odot$ denote the mass and the radius of the Sun). On the other hand, binary-pulsar data allow us to test the strong-field regime, since the compactness $Gm/Rc^2$ of a neutron star is of order 0.2 (not far from the theoretical maximum of 0.5, corresponding to black holes). The forthcoming gravitational-wave observations of inspiralling compact binaries, by interferometers like LIGO or VIRGO, will also allow us to test gravity in strong-field conditions. It is therefore interesting to compare and contrast the probing power of these different classes of experiments.

A convenient quantitative way of doing this comparison is to embed general relativity into a class of alternative theories. For instance, the Parametrized Post-Newtonian (PPN) formalism is very useful to study weak-field gravity, at order $1/c^2$ with respect to the Newtonian interaction.

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The original idea was formulated by Eddington, who wrote the usual Schwarzschild metric in isotropic coordinates, but introduced some phenomenological parameters $\beta_{\text{PPN}}, \gamma_{\text{PPN}}$, in front of the different powers of the dimensionless ratio $Gm/rc^2$:

$$-g_{00} = 1 - 2 \frac{Gm}{rc^2} + 2 \beta_{\text{PPN}} \left( \frac{Gm}{rc^2} \right)^2 + O \left( \frac{1}{c^4} \right), \quad g_{ij} = \delta_{ij} \left[ 1 + 2 \gamma_{\text{PPN}} \frac{Gm}{rc^2} + O \left( \frac{1}{c^4} \right) \right].$$  (1)

General relativity, which corresponds to $\beta_{\text{PPN}} = \gamma_{\text{PPN}} = 1$, is thus embedded into a two-dimensional space of theories. [The third parameter that one may introduce in front of $Gm/rc^2$ in $g_{00}$ can be reabsorbed in the definition of the mass $m$.] The constraints imposed in this space by solar-system experiments are displayed in Fig. 1. We have also indicated the tight bounds obtained in 1997 with Very Long Baseline Interferometry (VLBI), although they are not yet published. See K. Nordtvedt’s contribution to the present Proceedings for even more precise limits recently extracted from Lunar Laser Ranging (LLR) data. For the most general formulation of the PPN formalism, see the works of Nordtvedt & Will.

Figure 1 shows clearly that the different solar-system experiments are complementary. We wish to compare the strong-field tests in a similar way, but an extension of the PPN formalism to all orders in $1/c^n$ would need the introduction of an infinite number of phenomenological parameters. It will be more convenient to focus instead on the most natural class of alternative theories to general relativity.

## 2 Tensor-scalar theories of gravity

The existence of scalar partners to the graviton is predicted by all unified and extra-dimensional theories, notably superstrings. Moreover, tensor-scalar theories are the only consistent massless field theories able to satisfy exactly the weak equivalence principle (universality of free fall of laboratory-size objects). They are also the only known theories satisfying “extended Lorentz invariance”, i.e., such that the physics of subsystems, influenced by external masses, exhibit Lorentz invariance. Finally, they explain the key role played by $\beta_{\text{PPN}}$ and $\gamma_{\text{PPN}}$ in the PPN formalism (all the 8 extra parameters introduced by Will and Nordtvedt vanish identically), and they are general enough to describe many possible deviations from general relativity. These reasons show that tensor-scalar theories are privileged alternatives to general relativity.

Like in Einstein’s theory, the action of matter is given by a functional $S_m[\psi_m, \tilde{g}_{\mu\nu}]$ of some matter fields $\psi_m$ (including gauge bosons) and one second-rank symmetric tensor $b_{\mu\nu}$. The

\[b_{\mu\nu}\]
difference with general relativity lies in the kinetic term of $\tilde{g}_{\mu\nu}$. Instead of being a pure spin-2 field, it is here a mixing of spin-2 and spin-0 excitations. More precisely, it can be written as $\tilde{g}_{\mu\nu} = \exp^{2a(\varphi)}g_{\mu\nu}$, where $a(\varphi)$ is a function of a scalar field $\varphi$, and $g_{\mu\nu}$ is the Einstein (spin-2) metric. The action of the theory reads thus

$$S = \frac{c^3}{16\pi G} \int d^4x \sqrt{-g} \left( R - 2g^{\mu\nu}\partial_\mu \varphi \partial_\nu \varphi \right) + S_m \left[ \psi_m, e^{2a(\varphi)}g_{\mu\nu} \right].$$  \hspace{1cm} (2)

[Our signature is $-+++$, $R$ is the scalar curvature of $g_{\mu\nu}$, and $g$ its determinant.]

Our discussion will now be focused on the function $a(\varphi)$, which characterizes the coupling of matter to the scalar field. It will be convenient to expand it around the background value $\varphi_0$ of the scalar field (i.e., its value far from any massive body):

$$a(\varphi) = \alpha_0(\varphi - \varphi_0) + \frac{1}{2!} \beta_0(\varphi - \varphi_0)^2 + \frac{1}{3!} \beta_0'(\varphi - \varphi_0)^3 + \cdots ,$$  \hspace{1cm} (3)

where $\alpha_0, \beta_0, \beta_0', \ldots$ are constants defining the theory. In particular, the slope $\alpha_0$ measures the coupling strength of the linear interaction between matter and the scalar field. This slope and the curvature $\beta_0$ of $a(\varphi)$ are the only parameters which appear at the post-Newtonian order, i.e., when measuring effects of order $1/c^2$ in weak-field conditions. The same solar-system experiments as those of Fig. 1 give now the constraints displayed in Fig. 2. [The bold line will be discussed in the next section.] We see that the main information provided by weak-field tests is that the slope $\alpha_0$ must be small, i.e., that the background value $\varphi_0$ must correspond almost to an extremum of $a(\varphi)$. On the other hand, the curvature $\beta_0$ is not constrained at all if $\alpha_0$ is small enough. In particular, its sign is not known, and $\varphi_0$ may thus be close either to a maximum ($\beta_0 < 0$) or to a minimum ($\beta_0 > 0$) of the function $a(\varphi)$. [Note that action (2) contains only positive-energy excitations, independently of the shape of the function $a(\varphi)$, and therefore independently of the sign of $\beta_0$.]

Let us underline that in Fig. 2, the vertical axis ($\beta_0 = 0$) corresponds to the Jordan-Fierz-Brans-Dicke theory, where the constant slope $\alpha_0$ is related to the usual Brans-Dicke parameter by $\alpha_0^2 = 1/(2\omega_{BD} + 3)$. On the other hand, the horizontal axis ($\alpha_0 = 0$) corresponds to theories

\hspace{1cm} to simplify again, we will restrict our discussion to a single scalar field, although the case of tensor-multi-scalar theories can also be treated in a similar way.\hspace{1cm}
which are perturbatively equivalent to general relativity. Indeed, it can be shown\(^6\) that any deviation from Einstein’s theory (at any order \(1/c^2\)) involves a factor \(\alpha_0^2\).

However, nonperturbative effects can occur in strong-field conditions,\(^7\) even if \(\alpha_0\) vanishes. Indeed, the presence of matter induces an effective potential for the scalar field. If the curvature parameter \(\beta_0\) is negative and the compactness \(Gm/Rc^2\) of a body is large enough, one finds that this effective potential has the shape of a Mexican hat, with a local maximum at \(\varphi = \varphi_0\) and two symmetric minima at nonvanishing values of \(\varphi - \varphi_0 = \pm \varphi_{\text{min}}\). It is therefore energetically favorable for the body to create a scalar field which differs from the background \(\varphi_0\). This phenomenon is similar to the spontaneous magnetization of ferromagnets, and we have called it “spontaneous scalarization”\(^8\) The theory now behaves like in weak-field conditions, but with a matter-scalar coupling strength proportional to \(a'(\varphi_0 \pm \varphi_{\text{min}}) \approx \pm \beta_0 \varphi_{\text{min}} \neq 0\) instead of \(\alpha_0 \approx 0\). Large deviations from general relativity can thus occur in systems involving compact bodies, like neutron stars, even if the theory is indistinguishable from general relativity in the solar system. These conclusions have been confirmed by numerical integrations of the coupled differential equations of the metric \(g_{\mu\nu}\) and the scalar field \(\varphi\), while using either polytropes\(^7,8\) or realistic equations of state\(^9\) to describe nuclear matter inside a neutron star.

3 Binary-pulsar tests

A pulsar is a rapidly rotating neutron star emitting radio waves in a particular direction, like a lighthouse. Experiment tells us that isolated pulsars are very stable clocks. A binary pulsar (a pulsar and a companion orbiting around each other) is thus a moving clock, the best tool that one could dream of to test a relativistic theory. Indeed, the Doppler effect modifies the frequency of the pulses, and the time between two maxima of this frequency is thus a measure of the orbital period \(P_b\). A careful analysis\(^10\) of the Times Of Arrivals can give in fact many other orbital data, like the eccentricity \(e\), the angular position of the periastron \(\omega\), as well as the measure of several relativistic effects.

In the case of PSR 1913+16, three post-Keplerian parameters are determined with great accuracy:\(^11\) (i) an observable denoted \(\gamma_{\text{Timing}}\), which combines the second-order Doppler effect and the redshift due to the companion; (ii) the periastron advance \(\dot{\omega}\); (iii) the variation of the orbital period \(\dot{P}_b\), due to gravitational radiation reaction. For a given theory of gravity, these 3 quantities can be predicted in terms of the 2 unknown masses \(m_A, m_B\), of the pulsar and its companion. This gives \((3-2) = 1\) test of the theory: There must exist a pair of masses \((m_A, m_B)\) such that the three equations \(\gamma_{\text{Timing}}(m_A, m_B) = \gamma_{\text{obs}}\), \(\dot{\omega}_{\text{obs}}(m_A, m_B) = \dot{\omega}_{\text{obs}}\), \(\dot{P}_b^{\text{th}}(m_A, m_B) = \dot{P}_b^{\text{obs}}\), are simultaneously satisfied. It is shown in J. Taylor’s contribution to the present Proceedings that general relativity passes this test with flying colors: In the plane of the masses \((m_A, m_B)\), the three curves defined by the above equations meet in one point.

The predictions of tensor-scalar theories for these three observables can be computed for any shape of the function \(a(\varphi)\). However, to make a quantitative comparison between this binary-pulsar test and solar-system experiments, let us consider a generic class of theories defined by parabolic functions \(a(\varphi)\), i.e., such that the slope \(\alpha_0\) and the curvature \(\beta_0\) are the only nonvanishing coefficients in expansion \((3)\). We find that all the theories lying on the left of the bold line, in Fig. 2, are ruled out by PSR 1913+16 data. Note in particular that the theories corresponding to \(\alpha_0 = 0, \beta_0 < -4.5\), are excluded by this pulsar test\(^4\), although they are strictly indistinguishable from general relativity in the solar system. This is due to the “spontaneous scalarization” of neutron stars which occurs when \(\beta_0\) is negative (see Sec. 2).

The constraints imposed by two other binary-pulsar tests are displayed in Fig. 3. See I. Stairs’ contribution to the present Proceedings for a discussion of PSR 1534+12. The region which lies

\(^{11}\)It is interesting to note that binary-pulsar data are nicely consistent with the results of tensor-scalar cosmological models,\(^{12}\) which privilege the positive values of \(\beta_0\).
on the right of the dashed line is consistent with its four observables $\gamma_{\text{Timing}}, \dot{\omega}, r,$ and $s$ (the last two denoting the range and the shape of the Shapiro time delay). PSR 0655+64 is a dissymmetrical system composed of a neutron star and a white dwarf companion, which generically emits strong scalar dipolar waves in tensor-scalar theories. The region consistent with the small observed value of its $\dot{P}_b$ lies between the two solid lines of Fig. 3.

4 Detection of gravitational waves

One of the main differences between general relativity and tensor-scalar theories occurs in the expression of the energy flux due to the emission of gravitational waves. It has the form

$$\left\{ \frac{\text{Quadrupole}}{c^5} \right\}_{\text{spin 2}} + \left\{ \frac{\text{Monopole}}{c} \left( \sigma + \frac{1}{c^2} \right)^2 + \frac{\text{Dipole}}{c^3} + \frac{\text{Quadrupole}}{c^5} \right\}_{\text{spin 0}} + O \left( \frac{1}{c^7} \right),$$

where the first curly brackets contain the predictions of Einstein’s theory, and the second ones the extra contributions due to the emission of scalar waves. The symbol $\sigma$ means the “scalar charge” of the system, which is constant if the bodies are at equilibrium, like in the above binary pulsars. The predictions for their $\dot{P}_b$ are thus dominated by the dipolar term $\propto \frac{1}{c^3}$.

On the other hand, in the case of a collapsing star, the time derivative of this scalar charge may be of order unity, and one expects thus a huge emission of monopolar waves $\propto \frac{1}{c}$, much larger than the general relativistic prediction $\propto \frac{1}{c^3}$. However, Fig. 2 shows that matter is very weakly coupled to the scalar field in the solar system ($\alpha_0 \approx 0$), and interferometers like LIGO or VIRGO might be insensitive even to large scalar waves. Actually, numerical calculations suggest that the effects of such helicity-0 waves are too small to be of observational interest.

There remains nevertheless an indirect way to test for the presence of a scalar partner to the graviton, in gravitational-wave observations of inspiralling binaries. Indeed, even if no helicity-0 wave is detected, the time-evolution of the detectable helicity-2 chirp depends on the energy flux, Eq. (4), which can differ significantly from the general relativistic prediction because of nonperturbative strong-field effects. If one analyses the data with filters constructed from the standard general relativistic orbital phase evolution, the signal-to-noise ratio will thus drop. On the contrary, if one detects a coalescence using such standard filters, this will constrain the coupling strength of the bodies to a possible scalar field. Using such a matched-filter analysis,
Will\textsuperscript{15} has computed the bounds that a LIGO advanced detector could bring on Brans-Dicke theory (the vertical axis of Figs.2-3). We have generalized his results to the plane \((\alpha_0, \beta_0)\) of generic tensor-scalar theories.\textsuperscript{9} We found that the detection of a neutron star-black hole coalescence would exclude all the theories lying above the hatched straight line of Fig. 3. Similarly, the detection of a double neutron star coalescence would exclude the bubble of theories labeled NS-NS in this figure. Like binary-pulsar tests, gravity-wave observations have thus the capability of probing theories which are strictly indistinguishable from general relativity in the solar system (namely, those with \(\alpha_0 = 0\) and \(\beta_0\) large and negative). However, we see in Fig. 3 that these theories are already excluded by binary-pulsar data. Paradoxically, this is a good news for the LIGO or VIRGO projects: This proves that it is sufficient to compute the filters within the simpler framework of general relativity. Of course, this does not reduce the great interest of such projects. They will provide the first direct observation of gravitational waves in the wave zone, will (hopefully) lead to additional confirmations of general relativity through the wave forms, and will provide important astrophysical information, like masses and radii of neutron stars, or distance measurements up to hundreds of Mpc.

5 Conclusion

The main conclusion of our study is that strong-field tests of gravity (binary pulsars and gravitational-wave observations) are qualitatively different from solar-system experiments. They constrain theories which are perturbatively equivalent to general relativity. It is interesting to rewrite in terms of the Eddington parameters the bound \(\beta_0 > -4.5\) that we obtained in Sec. 3. We find: \((\beta_{\text{PPN}} - 1)/\gamma_{\text{PPN}} - 1\) < 1.1. The singular (0/0) nature of this ratio vividly expresses why such a conclusion could not be obtained in weak-field experiments.

We also underlined that binary pulsars are the best tools to test alternative theories of gravity. In fact, we have shown\textsuperscript{9} that the best “scalar probe” would be a pulsar orbiting around a black hole companion. One could constrain the deviations from general relativity at the level \(\alpha_0^2 < 10^{-6}\), i.e., three orders of magnitude tighter than the present limits.

Our third conclusion is that general relativistic filters can be used confidently to analyze the data of LIGO or VIRGO interferometers. Indeed, even if there exists a scalar partner to the graviton, binary pulsars already tell us that it is too weakly coupled to matter to change significantly the chirp templates.

References