Density Distribution and Shape of Galactic Dark Halo Can be Determined by Low Frequency Gravitational Waves?

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ABSTRACT

Under the assumption that the Milky Way’s dark halo consists of primordial black hole MACHOs (PBHMACHOs), the mass density of the halo can be measured by the low frequency gravitational waves ($10^{-3}$ Hz $\lesssim \nu_{gw} \lesssim 10^{-1}$ Hz) from PBHMACHO binaries whose fraction is $\sim 10^{-6}$. We find that ten years observation by LISA will detect $\sim 700$ PBHMACHO binaries and enable us to determine the power index of the density profile within 10% (20%) and the core radius within 25% (50%) in about 90% (99%) confidence level, respectively. The axial ratios of the halo may also be determined within $\sim 10\%$. LISA and OMEGA may give us an unique observational method to determine the density profile and the shape of the dark halo to open a new field of observational astronomy.

Subject headings: black hole physics — dark matter — Galaxy: halo — Galaxy: structure — gravitation — gravitational lensing
1. INTRODUCTION

It is important to determine the density profile of the Milky Way’s dark halo observationally in order to gain insights into the galaxy formation and evolution. Unfortunately, little has been known about the halo density profile (HDP), since the dark halo emits little light. A HI rotation curve tells us about the radial distribution of dark matter (e.g. Fich & Tremaine 1991). However it has not been accurately measured for $r > D_0 \sim 8.5$ kpc and recently there is also an argument that the Galactic rotation curve may deviate from that of the standard halo model (Honma & Sofue 1996, 1997, Olling & Merrifield 1998). The dynamics of satellite galaxies and globular clusters can provide the mass inside $r \sim 50$ kpc with some biases (e.g. Lin, Jones, & Klemola 1995, Kochanek 1996, Zaritsky et al. 1989, Einasto & Lynden-Bell 1982, Peebles 1995). The HDP may be recovered by using the tidal streams from Galactic satellites (e.g. Johnston et al. 1999). All methods so far are, however, indirect methods. In this paper we investigate a possibility of direct measurement of the density distribution of Galactic dark halo.

Recently, Ioka, Tanaka, & Nakamura (1998b) (hereafter ITN) proposed a possibility to determine a map of a HDP by low frequency gravitational waves ($10^{-4}$–$10^{-1}$ Hz) from PBHMACHO binaries, which can be detected by the planned Laser Interferometer Space Antenna (LISA) (Bender et al. 1998) and Orbiting Medium Explorer for Gravitational Astrophysics (OMEGA). ITN was motivated by the observations of gravitational microlensing toward the Large Magellanic Cloud (LMC). The analysis of the first 2.1 years of photometry of $8.5 \times 10^6$ stars in the LMC by the MACHO Collaboration (Alcock et al. 1997) suggests that the fraction $0.62^{+0.3}_{-0.2}$ of the halo consists of massive compact halo objects (MACHOs) of mass $0.5^{+0.3}_{-0.2}M_{\odot}$ assuming the standard spherical flat rotation halo model. At present, we do not know what MACHOs are. There are several candidates proposed to explain MACHOs, such as brown dwarfs, red dwarfs, white dwarfs and so on.
(Chabrier 1999, Gould, Flynn, & Bahcall 1998, see also references in ITN). Any objects clustered somewhere between the LMC and the sun with the column density larger than \(25M_\odot pc^{-2}\) can also explain the data (Nakamura, Kan-ya, & Nishi 1996). They include the possibilities: LMC-LMC self-lensing, the thick disk, warps, tidal debris and so on (Sahu 1994, Zhao 1998a,b, Evans et al. 1998, Gates et al. 1998, Bennett 1998, see also discussions on SMC events, Afonso et al. 1998, Albrow et al. 1998, Alcock et al. 1998, Honma 1999). However, none of them do not convincingly explain the microlensing events toward the LMC and SMC.

Freese, Fields and Graff (1999) claimed that on theoretical grounds one is pushed to either exotic explanations or a non-MACHO halo. We here simply adopt the suggestion by the MACHO Collaboration (Alcock et al. 1997) and consider an example of exotic explanations: primordial black hole MACHO (Nakamura et al. 1997). This possibility is free from observational constraints at present (Fujita et al. 1998) and PBHMACHOs may be identified by LIGO, VIRGO, TAMA and GEO within next 5 years (Nakamura et al. 1997, Ioka et al. 1998a), if they exist as dark matter.

If primordial black holes (PBHs) were formed in the early universe at \(t \sim 10^{-5}\) s (Yokoyama 1997, Kawasaki & Yanagida 1999, Jedamzik 1997), a part of them evolved into binaries through the three body interactions (Nakamura et al. 1997, Ioka et al. 1998a). Some of these binaries emit gravitational waves (GWs) at low frequencies at present. ITN found that one year observation by LISA will be able to identify at least several hundreds of PBHMACHO binaries. Since LISA can measure distances and positions of PBHMACHO binaries (Bender et al. 1998, Cutler 1998), it may be possible to obtain HDP from the distribution map of PBHMACHO binaries.

In this paper, we will quantitatively investigate how well HDP can be determined by the observation of the low frequency GWs from PBHMACHO binaries and show that LISA
and OMEGA will serve as excellent instruments for determination of the shape of our dark halo.

2. PBHMACHO MODEL

For simplicity, we assume that PBHs dominate the dark matter, i.e., $\Omega = \Omega_{BH}$, where $\Omega_{BH}$ is the density parameter of PBHs at present, and that all PBHs have the same mass $M_{BH}$. Throughout this paper, we will set $M_{BH} = 0.5M_\odot$ and $\Omega h^2 = 0.1$, where $h$ is the present Hubble parameter $H_0$ in unit of 100 km s$^{-1}$ Mpc$^{-1}$.

Assuming that PBHs are distributed randomly at their formation, we can obtain the probability distribution function (PDF) for orbital frequency $\nu_p$ and eccentricity $e$ of the binary, $f_{\nu,e}(\nu_p, e)d\nu_p de$ (Nakamura et al. 1997, Ioka et al. 1998a). For $e \ll 1$, an approximate PDF is given by

$$f_{\nu,t}(\nu_p; t_0) d\nu_p \sim \frac{425}{3552} \left( \frac{t_0}{t} \right)^{3/37} \left( \frac{a}{a_0} \right)^4 \Gamma \left( \frac{58}{37} \right) \frac{d\nu_p}{\nu_p},$$

where $a = (GM_{BH}/2\pi^2\nu_p^2)^{1/3}$ is the semimajor axis, $t_0 = 10^{10}$ years is the age of the universe, $a_0 = 2.0 \times 10^{11}(M_{BH}/M_\odot)^{3/4}$ cm is the semimajor axis of a binary in a circular orbit which coalesces in $t_0$, $\bar{x} = 1.2 \times 10^{16}(M_{BH}/M_\odot)^{1/3}(\Omega h^2)^{-4/3}$ cm is the mean separation of black holes at the time of matter-radiation equality and $\bar{t} = \beta^7(\alpha \bar{x}/a_0)^4 t_0$ (ITN). $\alpha$ and $\beta$ are constants of order unity. In this paper we adopt $\alpha = 0.5$ and $\beta = 0.7$ (ITN). Note that a circular binary with orbital frequency $\nu_p$ emits GWs at GW frequency $\nu_{gw} = p\nu_p$ with the second harmonic $p = 2$ (Peter & Mathew 1963, Hils 1991).
3. INDIVIDUALLY OBSERVABLE SOURCES

To be observed as individual sources, the amplitudes of the GWs from the binaries have to exceed the threshold amplitude $h^{th} = 5h_\nu(\Delta \nu)^{1/2}$ which is determined by the GW background $h_\nu$ and the frequency resolution $\Delta \nu = 1/T$ (Schutz 1997, Thorne 1987). Here $T$ is the observation time, and we set the signal-to-noise ratio ($SNR$) as 5.\(^1\) In our model, the GW background $h_\nu$ is determined by PBHMACHO binaries themselves (ITN, Hiscock 1998). We use Figure 6 in ITN to estimate the GW background $h_\nu$. The amplitude of the GW at $\nu_{gw}$ from a binary with eccentricity $e$, the harmonic $p$ and the distance from the earth $d$ is given by

$$h_i = 2\sqrt{GF_i/c^3\pi\nu_{gw}^2},$$

where $F_i = L_i^0(\nu_{gw}, e)/4\pi d^2 := L_0 \nu_{gw}^{10/3} p^{-10/3} g(p, e)/4\pi d^2$ is the GW flux and $L_0 = (32c^5/5G)^2(2\pi G M/c^3)^{10/3}$. The function $g(p, e)$ is given by Peter & Mathew (1963), and $M = M_{BH}/2^{1/5}$ is the charp mass. Then, the requirement that the signal exceeds the threshold, $h_i > h^{th}$, determines the maximum distance to individually observable sources with $p = 2$ and $e = 0$ as

$$D_{\text{max}}^{(p)}(\nu_{gw}, e) = 87.2 \left(\frac{h^{th}}{10^{-23}}\right)^{-1} \left(\frac{\nu_{gw}}{10^{-2}\text{Hz}}\right)^{2/3} \text{kpc.} \quad (2)$$

At frequencies above $\sim \nu_{dis} = 10^{-2.23} (T/1\text{year})^{-6/11} (M_{BH}/0.5M_\odot)^{-5/11} \text{Hz}$, LISA can measure distances to PBHMACHO binaries since binaries change their GW frequencies $\nu_{gw}$ by more than $\Delta \nu$ through GW emission within the observation time $T$ (Bender et al. 1998, Schutz 1986). Since binaries with high orbital frequencies ($\nu_p \gtrsim 10^{-3} \text{Hz}$) are almost in circular orbits at present (ITN), we can assume $e = 0$ and $p = 2$. If a source with a circular orbit changes its GW frequency by $\xi \Delta \nu = \xi/T$

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\(^1\)For simplicity we do not consider effects of the inclination of binaries and a reduction factor due to the antenna pattern of the detector in details. These effects can be absorbed in the observation time $T$, and the conclusion of this paper will hold if the effective $T$ is increased correspondingly.
during the observation, the change in the binding energy of the binary is given by
\[ \Delta E = \left( \frac{c^5 \xi}{3 \pi \nu_{gw}^2 G T} \right) \left( \frac{\pi \nu_{gw} G M}{c^3} \right)^{5/3}. \]
From the relation \[ L^{(2)}(\nu_{gw}, 0) = \Delta E/T, \]
we can obtain the charp mass as \[ M = \left( \frac{5 \pi \xi}{96} \right)^{3/5} \left( \frac{\pi \nu_{gw} G}{c^3} \right)^{5/3}. \]
Substituting the charp mass to the amplitude \[ h_i = \left( \frac{32}{5} \right)^{1/2} \left( \frac{c}{\pi \nu_{gw} d} \right) \left( \frac{\pi \nu_{gw} G M}{c^3} \right)^{5/3}, \]
we can obtain the distance to the source, \[ d = \left( \frac{5}{288} \right)^{1/2} \left( \frac{c \xi}{\pi^2 \nu_{gw}^2 h_i T^2} \right). \]
The observable parameters, \( h_i, \nu_{gw}, \) and \( \xi, \) contain observational errors as, \( h_i(1 \pm \epsilon_1), \nu_{gw}(1 \pm \epsilon_2) \) and \( \xi(1 \pm \epsilon_3), \) which can be estimated as \( \epsilon_1 = 1/(SNR), \epsilon_2 = 1/\nu_{gw} T \) and \( \epsilon_3 = 1/\xi. \) Therefore the observational error in the distance can be estimated as \[ d[1 \pm (\epsilon_1 + 3 \epsilon_2 + \epsilon_3)]. \] The angular resolution of LISA is estimated to be a few degree (Cutler 1998)\(^2\).

4. DENSITY PROFILE RECONSTRUCTION

In this section we show one of simulations of real observations. For simplicity, we assume that the distribution of the number density of PBHMACHOs obeys the law as
\[ n(r) = \frac{n_s}{\left[ 1 + (r^2/D_a^2) \right]^\lambda}, \tag{3} \]
where \( r, n_s, D_a \) and \( \lambda \) are the galactcentric radius, the number density of PBHMACHOs at the galactic center, the core radius and the power index, respectively. As the “real” parameters for a simulation we set \( n_s = 2.60 \times 10^{-2} \text{ pc}^{-3}, D_a = 5 \text{ kpc} \) and \( \lambda = 1. \) The

\(^2\)The results of Cutler (1998) are only valid for large \( SNR \) and the angular resolution may be worse in a realistic detection with \( SNR = 5 \) (Balasubramanian, Sathyaprakash, & Dhurandhar 1996), although we here simply adopt the Cutler’s results. Note also that the relative velocities of the sources to the solar system are not taken into account in Cutler (1998).
total number of PBHMACHOs within \( r < D_{\text{halo}} \) is given by
\[
N_{\text{total}} = \int_{r < D_{\text{halo}}} n(r) d^3x
\]
where \( D_{\text{halo}} \) is the size of the halo. Since the fraction of PBHMACHO binaries with
\( (10^{-3} \text{ Hz} \lesssim \nu_{\min} < \nu_{gw} < \nu_{\max}) \) is given by
\[
F_b(\nu_{\min}, \nu_{\max}) := \int_{\nu_{\min}}^{\nu_{\max}} \int_{\nu_p(t_0)}^{\nu_p(t_0) + d\nu_p/d\nu_{gw}} d\nu_p d\nu_{gw}
\]
with \( p = 2 \) from equation (1), the total number of PBHMACHO binaries with \( (10^{-3} \text{ Hz} \lesssim \nu_{\min} < \nu_{gw} < \nu_{\max}) \) is given by
\[
N_b = N_{\text{total}} F_b(\nu_{\min}, \nu_{\max}).
\]
Note that, as long as \( \nu_{gw} \gtrsim 10^{-3} \text{ Hz} \), it is sufficient to consider the case \( \epsilon \ll 1 \) and the second harmonic \( p = 2 \). For example,
\[
N_{\text{total}} = 4.03 \times 10^{12}, \quad F_b(\nu_{\min}, \nu_{\max}) = 2.14 \times 10^{-8}
\]
and hence \( N_b = 8.62 \times 10^4 \), for \( D_{\text{halo}} = 500 \text{ kpc} \), \( \nu_{\min} = 4 \times 10^{-3} \text{ Hz} \), and \( \nu_{\max} = 1 \times 10^{-1} \text{ Hz} \).

The following algorithm explains a method of our simulations to see how well HDP can be determined by the observation of the low frequency GWs.

1. We distribute \( N_b \) PBHMACHO binaries randomly following the adopted HDP in \( r < D_{\text{halo}} \) and assigning frequencies according to the PDF in equation (1) for \( \nu_{\min} < \nu_{gw} < \nu_{\max} \).

2. We make an observation in this numerically generated galactic halo. Note that we cannot use all individual sources to reconstruct the density profile, since for low frequency a maximum distance to be observed as individual sources in equation (2) is short. If we want to determine the HDP within \( r < D_{\text{obs}} \), we have to use binaries with frequencies \( \nu_{gw} > \nu_{obs} \) where \( \nu_{obs} \) is determined by \( D_{\text{max}}^2(\nu_{obs}, 0) = D_{\text{obs}} + D_0 \). Here \( D_0 = 8.5 \text{ kpc} \) is the distance from the galactic center to the earth. For simplicity, we use a uniform probability distribution to assign the observational error of \( \epsilon_1 + 3\epsilon_2 + \epsilon_3 \) to the distance and the error of \( 3^\circ \) to the angular resolution.

3. By fitting the distribution map of the HDP, we can compare the reconstructed HDP with the “real” HDP. Note that observationally the normalization \( n_s \) in equation (3) has to be replaced by the density of PBHMACHO binaries at the galactic center \( n_{sb} \). \( n_s \) is obtained from
\[
n_s = n_{sb}/F_b(\nu_{obs}, \nu_{\max}).
\]
An example of simulated observations is shown in Figure 1. This histogram shows the number \( N_i \) of the observed PBHMACHO binaries whose distances from the galactic center are in \((i-1)\delta r \leq r < i\delta r \) \((i = 1, 2, \cdots)\). We adopt \( \delta r = 1 \) kpc to determine the structure within a few kpc from the galactic center. Here we set \( T = 10 \) years and \( D_{\text{obs}} = 50 \) kpc, which corresponds to \( \nu_{\text{obs}} = 9.63 \times 10^{-3} \) Hz. In this realization, \( N_{\text{map}} \), the total number of PBHMACHO binaries which can be used to determine the HDP, is 719. In order to estimate the fitted parameters \( (n_{sb}, D_a, \lambda) \), we apply the least squares method\(^3\) minimizing \( \chi^2 = \sum_i [N_i - n_i(n_{sb}, D_a, \lambda)]^2/\sigma_i^2 \), where \( n_i(n_{sb}, D_a, \lambda) = \int_{(i-1)\delta r}^{i\delta r} 4\pi n(r)dr \). The variance \( \sigma_i \) of \( N_i \) can be estimated by \( \sigma_i = \sqrt{n_i(n_{sb}, D_a, \lambda)} \), since the distribution of \( N_i \) will follow the Poisson distribution with mean \( n_i(n_{sb}, D_a, \lambda) \) assuming that the statistical uncertainty dominates the instrumental uncertainty due to the observational errors. For this realization, the fitted parameters turn out to be \( n_s/n_{s\text{real}} = 0.779 \), \( D_a = 6.20 \) kpc and \( \lambda = 1.04 \), where \( n_{s\text{real}} = 2.60 \times 10^{-2} \) pc\(^{-3} \) is the “real” value. The reduced \( \chi^2 \) is 0.913 with \( 47(=D_{\text{obs}}/\delta r - 3) \) degrees of freedom. In Figure 2, the “real” parameter and the fitted parameter are marked with a filled square and a cross respectively in the \( D_a-\lambda \) plane. The contours of constant \( \Delta \chi^2 \) are also plotted with \( \Delta \chi^2 = 1.00, 2.30, 4.00 \) and 6.17. In Figure 3, the “real” HDP and the reconstructed HDP normalized by \( n_{s\text{real}} \) are shown. It seems that in our method HDP is reconstructed quite well except for the central region.

We performed \( 10^4 \) simulations of observations with (or without) the instrumental error to obtain the probability distributions of the core radius \( D_a \) and the power index \( \lambda \). 

\(^3\)Strictly speaking, we may have to maximize the probability for observing \( N_i \) PBHMACHO binaries in \( i \)-th bin from the Poisson distribution, \( P(n_{sb}, D_a, \lambda) = \prod_i \left\{ n_i(n_{sb}, D_a, \lambda) \right\}^{N_i} e^{-n_i(n_{sb}, D_a, \lambda)}/N_i! \}, \) instead of minimizing \( \chi^2 \). However, since almost all \( N_i \) is larger than 10, it will be a reasonable assumption that the shape of the Poisson distributions governing the fluctuations is nearly Gaussian.
mean values $\langle w \rangle$ and the dispersions $\Delta w = (\langle w^2 \rangle - \langle w \rangle^2)^{1/2}$ of these parameters $w$ with (or without) the instrumental error are shown in Table 1. From Table 1, we find that the instrumental error does not affect the results so much. The probabilities that these parameters $w$ are within $|w - \langle w \rangle| < \Delta w$ and $2\Delta w$ turn out to be about 70% and 95% respectively from these realizations. Although the power index $\lambda$ is determined within 10% (20%) error in 89% (99.7%) confidence level (CL), respectively, by ten years observation, the dispersion of the core radius $D_a$ is somewhat large, 25% (50%) error in 63% (93%) CL.

After we know the power index $\lambda$ accurately by this global observation, we can analyze the HDP for shorter distances $r < \hat{D}_{\text{obs}}$ using the PBHMACHO binaries with lower frequencies $\nu_{gw} > \hat{\nu}_{\text{obs}}$, where $\hat{\nu}_{\text{obs}}(< \nu_{\text{obs}})$ is determined by $D_{\text{max}}^{(2)}(\hat{\nu}_{\text{obs}}, 0) = \hat{D}_{\text{obs}} + D_0$ from equation (2). For example, for $T = 10$ years and $\hat{D}_{\text{obs}} = 10$ kpc, which corresponds to $\hat{\nu}_{\text{obs}} = 5.15 \times 10^{-3}$ Hz, the mean value and the dispersion of $D_a$ are found to be $\langle D_a \rangle = 4.81$ kpc and $\Delta D_a = 0.710$ kpc from $10^4$ realizations with $\delta r = 0.5$ kpc and $\lambda = 1$. The dispersion $\Delta D_a$ is reduced by a factor 0.5. Then, the core radius $D_a$ is determined within 25% (50%) error in 91% (99.8%) CL.

5. DISCUSSIONS

In this paper we have quantitatively investigated how well the HDP consisting of PBHMACHOs can be determined by the observation of the low frequency GWs, assuming the spherical HDP in equation (3). We have found that ten years observation by LISA will

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4This also justifies the assumption that the statistical error dominates the instrumental error. Note also that the dispersions $\Delta w$ are consistent with the extent of the contour $\Delta \chi^2 = 1.00$ in Figure 2. This justifies the assumption that the distribution of the number $N_i$ of the observed PBHMACHO binaries in $i$-th bin is nearly Gaussian.
be able to determine $\lambda$, the power index of the HDP, within 10% (20%) error and $D_a$, the core radius, within 25% (50%) error in about 90% (99%) CL, respectively.

The halo of our galaxy may be non-spherical (e.g., Olling and Merrifield 1997). For a non-spherical halo if we calculate quadrupole moments of positions of PBHMACHO binaries we can determine axial ratios $\langle c/a \rangle$ and $\langle b/a \rangle$ of the dark halo (for details see Dubinski & Carlberg 1991). Since about 700 PBHMACHO binaries can be used by ten years observation, errors in the axial ratios are estimated as less than 10% if the axial ratios are less than 0.8.

We have assumed that MACHOs are PBHs. However they may be white dwarfs, or some other compact objects (e.g., Freese et al. 1999). For such cases also it may not be so strange to expect that some of them are binaries. If a fraction $\sim 10^{-6}$ of them is in binary systems emitting GWs in the frequency range of $10^{-3}$ Hz $\lesssim \nu_{gw} \lesssim 10^{-1}$ Hz, similar arguments to this paper will hold even for non-black hole MACHOs (see also Bond & Carr 1984).

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Fig. 1.— The number $N_i$ of the observed PBHMACHO binaries whose distances from the galactic center are within $(i - 1)\delta r \leq r < i\delta r$ ($i = 1, 2, \cdots$) in one experimental realization is shown. We adopt $\delta r = 1$ kpc. Here we set $T = 10$ years and $D_{\text{obs}} = 50$ kpc, which corresponds to $\nu_{\text{obs}} = 9.63 \times 10^{-3}$ Hz. The fitted curve (solid line) and the “real” curve (dashed line) are also shown. The fitted parameters are $n_s/n_s^{\text{real}} = 0.779$, $D_a = 6.20$ kpc and $\lambda = 1.04$, where $n_s^{\text{real}} = 2.60 \times 10^{-2}$ pc$^{-3}$ is the “real” value. The reduced $\chi^2$ is 0.913 with 47($= D_{\text{obs}}/\delta r - 3$) degrees of freedom.
Fig. 2.— The “real” parameter and the fitted parameter obtained from one experimental realization in Figure 1 are marked with a filled square and a cross respectively in the $D_a$-$\lambda$ plane. The contours of constant $\Delta \chi^2$ are also plotted with $\Delta \chi^2 = 1.00, 2.30, 4.00$ and 6.17. Note that for the Gaussian fluctuations the projections of the contours $\Delta \chi^2 = 1.00$ and 4.00 onto one axis contain 68.3% and 95.4% of data projected onto the axis respectively, and the contours $\Delta \chi^2 = 2.30$ and 6.17 contain 68.3% and 95.4% of data respectively.
Fig. 3.— The “real” density profile with $D_a = 5$ kpc and $\lambda = 1$ (dashed line) and the density profile fitted from Figure 1 with $n_s/n_s^{\text{real}} = 0.779$, $D_a = 6.20$ kpc and $\lambda = 1.04$ (solid line) are shown. These density profiles are normalized by $n_s^{\text{real}} = 2.60 \times 10^{-2}$ pc$^{-3}$. 
Table 1. The mean values $\langle w \rangle$ and the dispersions $\Delta w = (\langle w^2 \rangle - \langle w \rangle^2)^{1/2}$ of the core radius $D_a$ and the power index $\lambda$ obtained from $10^4$ experimental realizations with (out of parentheses) and without (in parentheses) the instrumental error are shown for several observational time $T$. $\nu_{dis}$ is the minimum frequency of the binaries to which LISA can measure distances. $\nu_{obs}$ is the minimum frequency of the binaries which we can use to determine the density profile within $r < D_{obs} = 50$ kpc. $\langle N_{map} \rangle$ is the mean number of the PBHMACOH binaries that can be used to determine the density profile.

<table>
<thead>
<tr>
<th>$T$ [year]</th>
<th>$\nu_{dis}$ [mHz]</th>
<th>$\nu_{obs}$ [mHz]</th>
<th>$\langle N_{map} \rangle$</th>
<th>$\langle D_a \rangle \pm \Delta D_a$ [kpc]</th>
<th>$\langle \lambda \rangle \pm \Delta \lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4.08</td>
<td>15.1</td>
<td>217 (214)</td>
<td>4.60 ± 2.44 (4.68 ± 2.43)</td>
<td>1.00 ± 0.114 (1.01 ± 0.116)</td>
</tr>
<tr>
<td>4</td>
<td>2.80</td>
<td>12.4</td>
<td>367 (363)</td>
<td>4.57 ± 1.90 (4.67 ± 1.90)</td>
<td>0.992 ± 0.0876 (1.00 ± 0.0894)</td>
</tr>
<tr>
<td>6</td>
<td>2.24</td>
<td>11.1</td>
<td>496 (492)</td>
<td>4.62 ± 1.64 (4.71 ± 1.64)</td>
<td>0.992 ± 0.0763 (1.00 ± 0.0771)</td>
</tr>
<tr>
<td>8</td>
<td>1.92</td>
<td>10.2</td>
<td>614 (608)</td>
<td>4.65 ± 1.47 (4.74 ± 1.46)</td>
<td>0.991 ± 0.0684 (0.999 ± 0.0689)</td>
</tr>
<tr>
<td>10</td>
<td>1.70</td>
<td>9.63</td>
<td>723 (716)</td>
<td>4.67 ± 1.35 (4.76 ± 1.34)</td>
<td>0.990 ± 0.0626 (0.999 ± 0.0632)</td>
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