SIMULATING WEAK LENSING BY CLUSTERS AND
LARGE-SCALE STRUCTURE

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ABSTRACT. Selected results on estimating cosmological parameters from
simulated weak lensing data with noise are presented. Numerical simulations
of ray tracing through N-body simulations have been used to generate shear
and convergence maps due to lensing by large-scale structure. Noise due to
the intrinsic ellipticities of a finite number of galaxies is added. In this contribution
we present our main results on estimation of the power spectrum and density
parameter $\Omega$ from weak lensing data on several degree sized fields. We also
show that there are striking morphological differences in the weak lensing
maps of clusters of galaxies formed in models with different values of $\Omega$.

1 Introduction

Weak lensing by large-scale structure (LSS) shears the images of
distant galaxies. The first calculations of weak lensing by LSS (Blandford
et al. 1991; Miralda-Escude 1991; Kaiser 1992), based on the pioneering
work of Gunn (1967), showed that lensing would induce coherent
 ellipticities of order 1% over regions of order one degree on the sky. Re-
cently several authors have extended this work to probe semi-analytically
the possibility of measuring the mass power spectrum and cosmological
parameters from the second and third moments of the induced ellipti-
city or convergence (Bernardeau et al. 1997; Kaiser 1998; Stebbins 1996;
Jain and Seljak 1997; Schneider et al. 1997).

Analytical calculations suggest that nonlinear evolution of the den-
sity perturbations that provide the lensing effect can significantly al-
ter the predicted signal. It is expected to enhance the power spec-
trum on small scales and makes the probability distribution function
(pdf) of the ellipticity and convergence non-Gaussian. We have car-
ried out numerical simulations of ray tracing through N-body simu-
lation data to compute the fully nonlinear moments and pdf. Details
of the method and results are presented in a separate (Jain, Seljak &
White 1999); here we summarize the method and present some highlights
of the results. Other recent numeri-
cal work includes Wambsganss, Cen
& Ostriker (1998), Premadi, Martel
& Matzner (1998), van Waerbeke,
Bernardeau & Mellier (1998), Bartel-
mann et al (1998) and Couchman,
Barber & Thomas (1998).

The dark matter distribution ob-
tained from N-body simulations of
different models of structure forma-
tion is projected on to 2-dimensional
planes lying between the observer
and source galaxies. Typically we use
galaxies at $z \sim 1$ with $\sim 20 - 30$
planes. We propagate $\sim 10^6$ light
rays through these planes by com-
puting the deflections due to the
matter at every plane. Fast Fourier
Transforms are used to compute gra-
dients of the potential that provide
the shear tensor at each plane. The
outcome of the simulation is a map of
the shear and convergence on square
patches of side length $1 - 5'$. Sev-
eral realizations for each model are
needed to compute reliable statistics
on scales ranging from $1'$ to $10'$. In the weak lensing regime, the
magnification and induced ellipticity
are given by linear combinations of
the Jacobian matrix of the mapping
from the source to the image plane.
The Jacobian matrix is defined by

\[ \Phi_{ij} = \frac{\partial \delta \theta_i}{\partial \theta_j} \] (1)

where $\delta \theta_i$ is the $i$–th component of
the perturbation due to lensing of
the angular position on the source
plane, and $\theta_j$ is the $j$–th compo-
nent of the position on the image

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Figure 1. The dimensionless power spectrum of $\kappa$. For the cosmological model indicated by $\Omega_m$ and $\Gamma$ in the panel, the power spectrum from ray tracing shown by the solid curves is compared with the linear (long-dashed) and non-linear analytical (short-dashed) predictions. The angular wavenumber $l$ is given in inverse radians – the smallest $l$ plotted corresponds to modes with wavelength of order $L \simeq 3^\circ$, where $L$ is the side-length of the field.

The convergence is defined as $\kappa = -(\Phi_{11} + \Phi_{22})/2$, while the two components of the shear are $\gamma_1 = -(\Phi_{11} - \Phi_{22})/2$ and $\gamma_2 = -\Phi_{12}$. The convergence $\kappa$ can be reconstructed from the measured shear $\gamma_1, \gamma_2$, up to a constant which depends on the mean density in the survey area. If the survey is sufficiently large and there is little power on scales larger than the survey, this error can be neglected.

2 Results of Simulations

The N-body simulations used for the ray tracing are taken from four different cosmological simulations, the parameters of which are summarized in Table 1. The N-body simulations use an adaptive $P^3M$ method with $256^3$ particles, and were carried out using codes kindly made available by the Virgo consortium (e.g. Jenkins et al. 1997). Coupled with ray tracing on a $2048^2$ grid these simulations provide us with a small scale resolution down to $\lesssim 0.5^\circ$, well into the nonlinear regime for weak lens-
Table 1. Summary of the parameters used for the N-body simulations. $h$ is the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$, $\Gamma$ is the shape parameter of the power spectrum, and the other parameters have their conventional meaning.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\Omega_m$</th>
<th>$\Omega_{\Lambda}$</th>
<th>$h$</th>
<th>$\sigma_8$</th>
<th>$\Gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCDM</td>
<td>1.0</td>
<td>0.0</td>
<td>0.5</td>
<td>0.60</td>
<td>0.50</td>
</tr>
<tr>
<td>$\tau$CDM</td>
<td>1.0</td>
<td>0.0</td>
<td>0.5</td>
<td>0.60</td>
<td>0.21</td>
</tr>
<tr>
<td>ACDM</td>
<td>0.3</td>
<td>0.7</td>
<td>0.7</td>
<td>0.90</td>
<td>0.21</td>
</tr>
<tr>
<td>OCDM</td>
<td>0.3</td>
<td>0.0</td>
<td>0.7</td>
<td>0.85</td>
<td>0.21</td>
</tr>
</tbody>
</table>

The power spectrum of $\kappa$ measured from the simulations is compared with the analytical predictions of Jain & Seljak (1997) in figure 1. We will focus in this contribution on the extraction of parameters from simulated noisy data. Figure 2 shows the reconstructed convergence field from noisy ellipticity data on a single 3° field. The galaxies were randomly distributed and assigned intrinsic ellipticities with each component drawn randomly from a Gaussian with rms=0.4. The reconstructed $\kappa$ is compared with the field without noise, as well as with a pure noise field, to show the significance of reconstructed features. The field is smoothed on the scale at which the noise and signal should be comparable. The figure shows that with observational parameters feasible on large telescopes, a field a few degrees on a side can allow one to reconstruct the large-scale features dominated by groups and clusters of galaxies. Statistically one can do much better.

Figure 3 shows the power spectrum of $\kappa$ measured from simulated noisy data. On scales larger than 10' the signal dominates the noise. The scales on which the power can be measured are dominated by the density power spectrum on scales of about $1 - 10$ h$^{-1}$Mpc at $z \sim 0.3$. Thus weak lensing surveys a few degrees on a side will be sensitive to the dark matter power spectrum on these scales.

Figure 4 shows one attempt at estimating the density parameter $\Omega_m$ from simulated data. The simplest way to estimate $\Omega_m$ is to measure the skewness $S_3$ of the convergence (Bernardea et al. 1997; Jain & Seljak 1997; Schneider et al. 1998). We found that $S_3$ was sensitive to the tails of the probability distribution function (pdf) and therefore required large sample sizes. Instead, we fit the pdf to an Edgeworth expansion and estimated $S_3$ as a parameter. The result is shown in figure 4 and shows that cosmological models with different values of $\Omega_m$ can be distinguished at a high level of significance (the error bars are 1-$\sigma$). The dashed curves show the predictions of perturbation theory.

Finally, figure 5 shows the convergence field centered on typical rich clusters in the Einstein-de Sitter model, $\tau$CDM, and the open CDM model. The fields are 10' arcminutes on a side and are chosen because current observational data is already capable of detecting signal in the outer parts of clusters. Comparison of the EdS and open models shows that the convergence fields are morphologically different: clusters in the open model appear more compact, regular and isolated. The clusters in the EdS model are not fully relaxed and appear linked to the surrounding large-scale structure through filaments or more irregular structures. These differences can be quantified by using topological measures and by measuring moments such as the quadrupole or higher moments. In figure 6 we simply show the mean profile of the convergence around clusters, which also shows differences between the two models. The source galaxies have been taken to be at $z = 1$ and the 10 richest clusters in a field 3 degrees on a side have been used to obtain the mean profiles.
Figure 2. The reconstructed convergence field from noisy ellipticity data on a single 3° field. The upper left panel shows the field without noise or sparse sampling. The upper right panel shows the reconstructed field using $2 \times 10^5$ galaxies per square degree at a mean $z = 1$. The lower left panel used $1/4$th as many galaxies. The lower right panel shows a map of the “curl” field generated from the same ellipticity data as in the lower left panel. Fluctuations in this field are solely due to noise, therefore comparison of the two panels shows the significance of the reconstructed features in the lower left panel.

3 Conclusion

We have shown results on estimating the power spectrum and $\Omega_m$ from simulated, noisy weak lensing data. With several fields a degree on a side, the dark matter power spectrum on scales of $1 - 10 \ h^{-3}\text{Mpc}$ can be probed and $\Omega_m$ estimated to within about 0.1-0.2. We have also shown preliminary results on the morphological differences between clusters in different cosmologies. Further work is needed to quantify the differences and explore the effects of noise and of varying the redshift distribution of source galaxies.

Acknowledgments

We are grateful to Matthias Bartelmann and Peter Schneider for helpful discussions. It is a pleasure to
Figure 3. *Convergence power spectrum estimated from simulated noisy data.* The shear field on a single field $3^\circ$ on a side is sampled by randomly distributed galaxies with intrinsic ellipticities assigned as in figure 2. The power spectrum of the reconstructed $\kappa$ from the ellipticity data is shown with error bars obtained from 10 independent realizations of the noisy data.

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References


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Figure 4. The density parameter $\Omega_m$ estimated from the pdf of simulated noisy data. The skewness parameter $S_3$ is estimated by minimizing the $\chi^2$ with respect to an Edgeworth expansion of the pdf. The four curves with decreasing peak heights are for the open, cosmological constant, Einstein-de Sitter models, all with $\Gamma = 0.2$ CDM power spectra, and an Einstein-de Sitter model with $\Gamma = 0.5$ CDM power spectrum.
Figure 5. Clusters in open and Einstein-de Sitter cosmologies. The convergence in fields 10' on a side centered on a rich cluster is shown for the EdS model in the upper panels and for the open model in the lower panels. The values of the convergence range from over 10% in the center to below 1% in the outermost regions. Figure 6 gives the mean profiles of the convergence.
Figure 6. The profile of the convergence in clusters in open and EdS cosmologies. The dashed curve shows the average convergence profile measured from 10 clusters in the EdS model while the solid curve shows the corresponding profile for the open model.