Behaviour of copper matrix in quench process calculated by 2-strand 4-wire model

S. W. Kim and D. Leroy
CERN, CH-1211,Geneva 23, Switzerland

Abstract — **In a simulation of a superconducting strand, it is usual to treat the strand as superconducting wire and copper wire in a parallel connection. When simulating a multi-stranded cable, strands are treated as a mixture of superconductor and copper, usually. All parameters are calculated from combination of those for superconductor and copper. This means the role of copper was just changing properties of the strand. However in real current transition process, copper matrix may play a role of a path of current and heat. In this paper, one strand is considered as two wires; superconductor and copper, and calculations were done for a cable with two strands. The simulation model and some results which show behavior of the copper matrix in quench process are presented in this paper. The difference between 'matrix heating' and 'filament heating' is discussed, also.**

I. INTRODUCTION

Many analytic and numerical calculations have been done for simulating the quench development in a superconducting wire or cable [1]-[5]. Usually a strand was treated as a parallel circuit of superconductor and copper having a very good electrical and thermal contact [6].

When a multi-stranded cable is calculated, one strand is considered as one wire having an averaged property of the superconductor and the matrix [3]-[5]. With this assumption, when a normal zone is nucleated in one of the strands, the current moves directly from one superconductor to another superconductor.

However in a real superconducting cable, there is no direct contact between superconductors, and all the transition current should pass more than two copper matrices. This paper uses a more realistic circuit model for a two-strand cable as shown in Fig. 1.

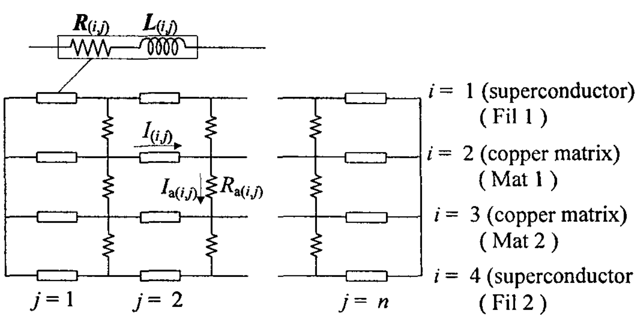

Fig. 1. Circuit diagram of the 2-strand 4-wire model. The same elements are used for the thermal calculation also.

Manuscript received September 14, 1998.

II. EQUATIONS

The circuit equation for the one loop of elements (i,j) and $(i+1,j)$ is given as follows;

$$R_{(i,j)}I_{(i,j)} - R_{(i+1,j)}I_{(i+1,j)} + R_{a(i,j)}I_{a(i,j)} - R_{a(i,j-1)}I_{a(i,j-1)} + \sum_{k,l=1,1}^{4,n}\left\{\left(L_{(i,j)(k,l)} - L_{(i+1,j)(k,l)}\right)\frac{\partial I_{(k,l)}}{\partial t}\right\} = 0 \quad (1)$$

where, $R_{(i,j)}$ is the resistance of the element (i,j) and $R_{a(i,j)}$ is adjacent resistance between elements (i,j) and $(i+1,j)$. $L_{(i,j)(k,l)}$ is the mutual inductance between the elements (i,j) and (k,l), and $L_{(i,j)(i,j)}$ is the self inductance of the element (i,j). $I_{(i,j)}$ and $I_{a(i,j)}$ are the currents in the element and the contact, respectively. The current in the contact can be represented by element currents as follows,

$$I_{a(i,j)} = \sum_{k=1}^{i} I_{(k,j)} - \sum_{k=1}^{i} I_{(k,j+1)} = \sum_{k=1}^{i}\left(I_{(k,j)} - I_{(k,j+1)}\right) \quad (2)$$

The total current of the two-strand cable (I_{total}) is a constant value during the calculation. The total current is the sum of the currents in the four wires, and this condition is presented by the following equation.

$$I_{total} = \sum_{i=1}^{4} I_{(i,j)} \quad \text{(for j=1,n)} \quad (3)$$

Substituting (2) into (1) yields the equation for one loop. Combining all the $(3\times n)$ loop equations and the n equations for current condition shown in (3) produces the following matrix equation,

$$[L]\frac{\partial}{\partial t}[I] + [R][I] = [G] \quad (4)$$
$$[I] = \left[I_{(1,1)} \quad I_{(1,2)} \quad \cdots \quad I_{(4,n)}\right]^T$$

$[L]$ is a matrix of size $(3n\times 3n)$. This matrix is calculated from the self and mutual inductances of all elements. $[R]$ is calculated from the resistances, and the size is $(3n\times 3n)$. [G] is a column matrix with $3n$ elements including zeros and I_{total}.

All the self and mutual inductances are computed from the geometry of the elements in the circuit (Fig. 1), and are used in the simulation. For example, The computed self inductance of the element (1,1) is 2.11×10^{-9} H. The self inductance of the element (2,1) is 2.79×10^{-9} H, and all the mutual inductances have smaller values.

Equation (4) is the governing equation, and each element of the matrix $[R]$ is presented as a function of temperature.

The heat balance equation is used to calculate temperature of each element [5],[6]. For each time step in calculation,

$$C_p^{\prime}\Delta\theta = (heat\,generation) + (heat\,input) - (heat\,output) \qquad (5)$$

where C_p is the volumetric specific heat of the element and $\Delta\theta$ is the temperature increment in the element during one time step.

In the above equation, "heat generation" includes the Joule heatings in the element and in the contact resistances. "Heat input" is the conduction heat from neighboring elements of higher temperature. "Heat output" includes the conduction to the cold neighbours and cooling helium.

Temperature of each element can be calculated from (5) knowing the temperatures of the previous step. All the other parameters - resistivity, heat conductivity, specific heat of superconductor and copper - are then calculated at the values of temperature, current density and magnetic field.

All the necessary equations were combined together and translated into a program in C language, and as many as possible cases were simulated. The parameters and physical values used in this paper are shown in Table I.

Unfortunately, many parameters are unknown, and the values in Table I are taken arbitrary within reasonable ranges of the values. Because of this, many test calculations were done with parameters in a wide range to confirm validity of this simulation. Usually, one hundredth to one hundred times of values given in Table I were used in the test runs.

The result of this test was good enough, the cable showed the same tendency in the wide range of parameters, and we could catch the behavior of filaments and matrices in strands.

III. Results of Simulation

A. High current quench and low current quench

Fig. 2 shows the currents in the filaments and the matrices after the superconducting filamentary zone of one strand ('Fil 1') is quenched by energy deposition. The transport current is 80% of the cable critical current. This means that before heating, each strand is carrying 40 % of the cable critical current or 80 % of its own critical current. When the 'Fil 1' quenched, the current moves to the matrix of the strand ('Mat 1') and 'Fil 2.' When the current in 'Fil 2' reaches its critical value, the cable quenches.

TABLE I

Parameters and Physical Values Used in Calculation

Parameter	Value
Contact resistance between filament and matrix	$10^{-10}\ \Omega m^{-1}$
Contact resistance between strands	$10^{-8}\ \Omega m^{-1}$
Coefficient of heat transfer between filament and matrix	$10^{3}\ Wm^{-2}K^{-1}$
Coefficient of heat transfer between strands	$10^{2}\ Wm^{-2}K^{-1}$
Cooling coefficient of copper matrix to helium	$10^{2}\ Wm^{-2}K^{-1}$
Sample length	0.5 m
Time step for calculation	2×10^{-8} sec

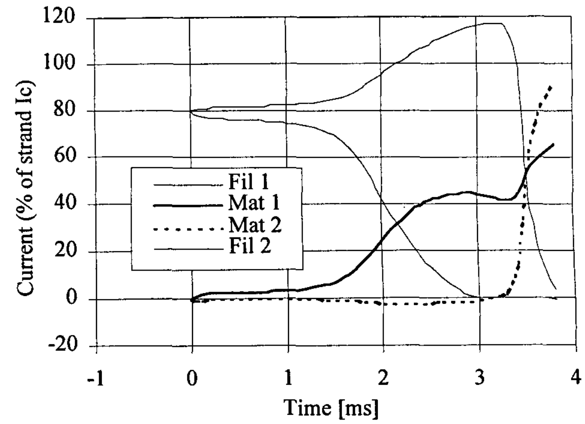

Fig. 2. Current at the heating point when the filamentary zone of strand 1 (Fil 1) is heated and the cable quenches. Cable current is 80% of the cable critical current. Time scale is set from –1 for a better comparison with Fig. 4.

This quench process occurs only when the total cable current is more than the 'quench current' of one strand. If the total current is less than the 'quench current' of one strand, a current transition cannot make a quench.

Fig. 3 shows the case of low current. Almost all of the current moved to 'Fil 2' and went into a steady state after about 15 ms, but heat energy is still moving from 'Fil 1' to 'Fil 2,' and the temperature of 'Fil 2' is increasing. If this heat energy is enough to make a quench in 'Fil 2' this cable will be quenched.

In both cases of high current and low current, 'Mat 1' plays a role of a current buffer. This means that the matrix of the heated strand prevents a direct current transition from a superconductor to a superconductor. However, the matrix of the second strand does not participate in this current re-sharing.

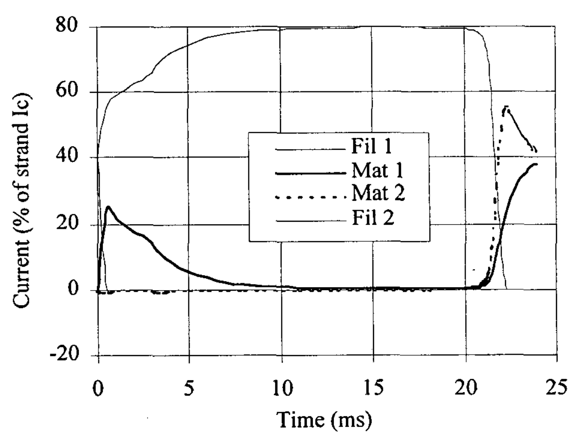

Fig. 3. Current at the heating point when the filamentary zone of strand 1 (Fil 1) is heated and the cable quenches. Cable current is 40% of the cable critical current.

The negative current in the 'Mat 2' is caused by inductive voltage. In a numerical calculation, filaments and matrices are treated as elements of a given length, in series connection. All of the elements have self and mutual inductance, and the mutual inductance can induce negative voltage and hence negative current. A real cable is not divided by elements, and the negative current should be very small.

B. Filament heating and matrix heating

The role of these matrices does not change when the matrix of a strand is heated and then the cable quenches, as shown in Fig. 4. This figure shows almost the same curves as Fig. 2, and this means that the quench process in the matrix heating case is the same as the filament heating case.

The most visible difference is in the behavior of Fil 1 and Mat 1 at the beginning. 'Current decreasing in Fil 1' starts just after heating in the case of the filament heating. On the contrary in the matrix heating case, there is a time delay and the "decreasing" itself is slow. However, after some time (about 2 ms in Fig. 4) the two cases show almost identical curves.

Though, the curve shapes are very similar, the minimum quench energy (MQE) values of these two cases are very different. The MQE values for the filament heating and matrix heating are 39.9 μJ and 160 μJ, respectively. This means that making a quench by heating the matrix needs 4 times more energy.

C. Classical model and 2-strand 4-wire model

The results obtained by the "2-strand 4-wire" model are now compared with the classical "mixed model" calculation. For the comparison, the strand current is obtained by addition of the filament current and the matrix current, as shown in Fig. 5. This figure also shows the strand currents calculated from the mixed model.

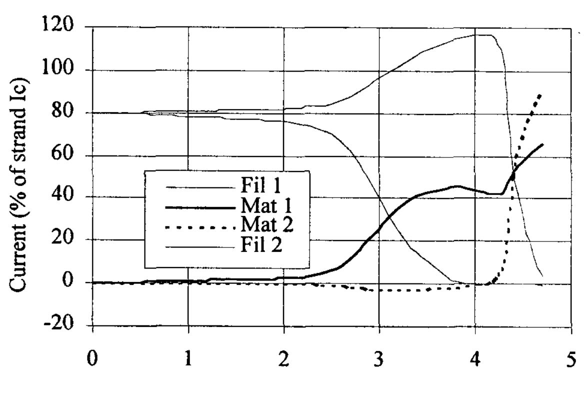

Fig. 4. Current at the heating point when the matrix of strand 1 (Mat 1) is heated by the MQE. Cable current is 80% of the cable critical current.

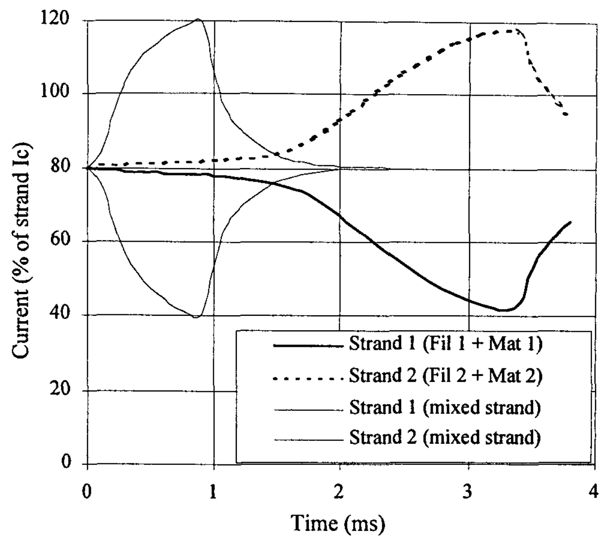

Fig. 5. Current at the heating point calculated by the 2-strand 4-wire model (filament heating) and the mixed strand model. In the mixed strand calculation, the contact resistance is 1.02×10^{-8} Ωm^{-1} and the coefficient of heat transfer between strand is 0.83×10^{2} $Wm^{-2}K^{-1}$, respectively. These values are equivalent to the values from filament to filament values in the 2-strand 4-wire model.

However, the peak value of current is at 3.3 ms in the 2-strand 4-wire calculation and 0.8 ms in the mixed strand calculation. This suggests that the actual quench process may take a much longer time than estimated by a "classical" calculation.

First of all we can see these two methods are in a good agreement. These two figures show almost the same curve shapes and the peak current is the same.

D. Minimum Quench Energy (MQE) of cable

Fig. 6 shows MQE values of a single strand and a 2-strand cable calculated by the 2-strand 4-wire model. The MQE curve of a strand obtained by mixed strand model is also shown in this figure. The results of the calculations show that the MQE of a strand for the filament heating is a little bit lower than that of mixed strand model, and MQE for the matrix heating is much higher than that of mixed strand model.

In the case of 2-strand cable, when the current is over 0.6 I_c, the MQE curves are in a parallel position with single strand curves. In low current region 2-strand MQE curves are apart from the single strand MQE curves. This is because of the different quenching processes for high current and low current, as explained with Fig. 2 and Fig. 3. Quench current of one strand is about 0.6 Ic of cable.

In Fig. 6 the two 2-strand MQE curves cross at 0.4Ic. It means that the matrix heating MQE is less than the filament heating MQE when current is very low. When matrix is heated, the hottest part in the cable is the matrix, and the main heat energy flow is from Mat 1 to Mat 2 and to Fil 2. In the case of the filament heating there is one more step in the heat transfer, which is Fil 1 to Mat 1. This additional step makes the heat transfer less efficient. This lower efficiency explains the crossing of two MQE curves for 2-stranded cables.

The most important conclusion we can retrive from Fig. 6 is the relation between experimental MQE values and the real quench energy. In experiments, we heat the surface of a strand by small spot heaters, and this means heating in experiments is more close to matrix heating.

Fig. 6 shows that MQE of filament heating is much less than that of matrix heating. Hence, it can be said that the cable can be quenched by a lower energy, if something happens in filaments.

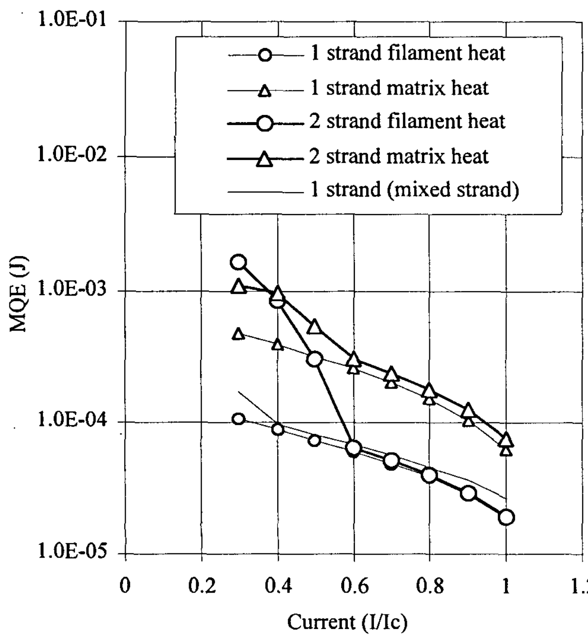

Fig. 6. Calculated MQE values. Calculations were done for a single strand and a 2-stranded cable, and these results are compared with that of the mixed strand calculation.

IV. Conclusions

We could see the differences between a high current quench process and a low current quench process. A high current quench is triggered by a current transition and a low current quench is triggered by a heat transfer.

Filament heating and matrix heating showed very similar quench process, but the MQE values of the matrix heating are much bigger than those of the filament heating. This is because only a part of the matrix energy moves into the filament.

The copper matrix in current decreasing strand plays a role of a current buffer, but the matrix in current increasing strand does not participate at the current transition.

Real quench process may take longer time than that expected by the classical mixed strand calculation.

References

[1] B. Turk, "Influence of a transverse conductance on current sharing in a two-layer superconducting cable," Cryogenics, August 1974.

[2] G. Ries, "Stability in superconducting multistrand cables," Cryogenics, September 1974.

[3] G. B. J. Mulder, L. J. M. van de Klundert and V. S. Vysotski, "Quench Development in Superconducting Cable Having Insulated Strands With High Resistive Matrix (Part 2, Analysis)," IEEE Transaction on Magnetics, vol. 28, No. 1, January 1992.

[4] N. A. Buznikov, A. A. Pukhov, A. L. Rakhmanov and V. S. Vysotsky, "Current redistribution between strands and quench process in a superconducting cable," Cryogenics, vol. 36, No. 4, 1996.

[5] M. N. Wilson and R. Wolf, "Calculation of Minimum Quench Energies in Rutherford Cables," IEEE Transaction on Applied Superconductivity, vol. 7, No. 2, June 1997.

[6] Pierre Bauer, "Stability of Superconducting Strands for Accelerator Magnets, " Doctoral Thesis, University of Vienna, 1998.