Comparison of structure functions measured in neutrino and charged lepton deep inelastic scattering can be used to test basic properties of parton distribution functions such as the validity of charge symmetry. Recent experiments indicate a substantial discrepancy between $F_2^\nu$ and $F_2^\mu$ in the region of small Bjorken-$x$. We discuss nuclear corrections and strange and anti strange quark effects and show that none of them can account for the observed discrepancy. These results suggest surprisingly large CSV effects in nucleon sea distributions.

1 Introduction

Comparison of structure functions measured in neutrino and charged lepton deep inelastic scattering can be used to test basic symmetry properties of parton distribution functions such as the validity of charge symmetry. Such comparisons are based on the interpretation of structure functions in terms of parton distribution functions measured in neutrino, anti neutrino and charged lepton deep inelastic scattering.

A very useful quantity in the comparison of the structure functions is the “charge ratio”

$$R_c(x) = \frac{F_2^{\nu N_0}(x)}{\frac{1}{18} F_2^{\mu N_0}(x) - x[s(x) + \bar{s}(x)]/6}.$$ 

A deviation $R_c(x) \neq 1$, at any value of $x$, must arise either from CSV effects or from $s(x) \neq \bar{s}(x)$.

Recent experimental measurements allow a precise comparison between $F_2^\nu(x, Q^2)$ and $F_2^\mu(x, Q^2)$. In Fig. 1 we show the “charge ratio” calculated by using the CCFR

and NMC

structure functions for $F_2^\nu$ and $F_2^\mu$, respectively. The structure functions are integrated over $Q^2 = 3.2$ GeV$^2$ in the region of overlap of the two experiments. Since the CCFR structure function has been measured on an iron target nuclear corrections (nuclear EMC effect, shadowing and anti shadowing) have to be applied to the data. In Fig. 1, the open circles represent the result without nuclear corrections. The open circles are calculated by using nuclear corrections obtained from charged lepton DIS. While, in the region of intermediate values of Bjorken $x (0.1 \leq x \leq 0.4)$, the two structure functions are in very good agreement, in the small $x$-region ($x < 0.1$), they differ by as much as 10-15% when nuclear corrections are taken into account. This discrepancy could be interpreted as evidence for charge symmetry violation. However, several corrections have to be applied to the data before any conclusions may be drawn. We see especially that the result is very sensitive to nuclear corrections. The CCFR Collaboration made a careful study of overall normalization, charm threshold and iso-scalar correction effects. Here, we re-examine nuclear corrections for neutrinos and discuss $s(x) \neq \bar{s}(x)$ effects before we turn to possible charge symmetry violation.

2 Nuclear corrections

Nuclear corrections for neutrinos are generally calculated using correction factors from charged lepton reactions at the same kinematic values. A priori, there is no reason that neutrino and charged lepton heavy target correc-
structure function is larger by a factor of $\sim$ as the coupling to the electro-magnetic current but the are related to each other by the "Weinberg sum rule" and that of the electro-magnetic current to vector mesons the weak current to the vector and axial vector mesons become important. The coupling of determined by the absorption of pions on the target. $Q^2$-values, shadowing due to Pomeron exchange (which is of leading twist) becomes dominant, leading to identical (relative) shadowing in neutrino and charged lepton DIS.

We calculated shadowing corrections to the CCFR $\nu$ structure function in the framework of this “two phase” model and used these corrections in calculating the charge ratio $R_c$ of Eq. 3. There are also nuclear effects in the Deuteron. However, because of the low density of the Deuteron, these are (relatively speaking) very small and have a negligible effect on the charge ratio. We integrated the structure functions above $Q^2 = 3.2$ GeV$^2$ in the overlapping kinematic region of the two experiments and used a parametrization of the nuclear corrections and used a parametrization of the nuclear corrections in charged lepton DIS to correct the data in the non-shadowing region. The result is shown as solid circles in Fig.1. At small $x$, careful consideration of neutrino shadowing corrections decreases, but does not resolve, the low-$x$ discrepancy between the CCFR and NMC data.

### 3 Strange and anti strange quark effects

Since the CCFR-Collaboration uses both neutrino and anti neutrino events in the structure function analysis the extracted structure function $F_2^{CCFR}$ is a flux weighted average between $\nu$ and $\bar{\nu}$ structure functions $^3$

$$F_2^{CCFR}(x, Q^2) = \alpha F_2^\nu(x, Q^2) + (1 - \alpha) F_2^{\bar{\nu}}(x, Q^2).$$

Here, we define the relative neutrino flux as $\alpha = \Phi_\nu/(\Phi_\nu + \Phi_{\bar{\nu}})$, where $\Phi_\nu$ and $\Phi_{\bar{\nu}}$ are the $\nu$ and $\bar{\nu}$ fluxes, respectively. $F_2^{CCFR}$ is equal to $\frac{1}{2}[F_2^\nu(x, Q^2) + F_2^{\bar{\nu}}(x, Q^2)]$ if $\alpha = \frac{1}{2}$ or if the two structure functions are equal, i.e. charge symmetry is valid and $s(x) = \bar{s}(x)$. The value of $\alpha$ depends on the energy of the incident neutrinos and anti neutrinos in the CCFR-experiment. In the relevant kinematic region, at small $x$ which corresponds to large incident energies, it is $\approx 0.83$ so to a good approximation $F_2^{CCFR}(x, Q^2)$ can be regarded as a neutrino structure function.

The remaining small-$x$ discrepancy in the charge ratio is either from different strange quark distributions $s(x) \neq \bar{s}(x)$ or from charge symmetry violation. First, we examine the role played by the strange quark distributions. Assuming charge symmetry, $s(x)$ and $\bar{s}(x)$ are given by a linear combination of neutrino and muon structure functions,

$$\frac{5}{6} F_2^{CCFR}(x, Q^2) - 3 F_2^{NMC}(x, Q^2) = \frac{1}{2} x [s(x) + \bar{s}(x)] + \frac{5}{6} (2\alpha - 1) x [s(x) - \bar{s}(x)].$$

For larger $Q^2$-values the contributions of vector and axial vector mesons become important. The coupling of the weak current to the vector and axial vector mesons and that of the electro-magnetic current to vector mesons are related to each other by the "Weinberg sum rule" $f^2_{\rho^0} = f^2_{\rho^1} = 2 f^2_{\rho^0}$. Since the coupling of the vector (axial vector) mesons to the weak current is twice as large as the coupling to the electro-magnetic current but the structure function is larger by a factor of $\sim 18/5$ in the neutrino case, we expect that shadowing due to VMD in neutrino reactions is roughly half of that in charged lepton scattering. For large $Q^2$-values, shadowing due to Pomeron exchange (which is of leading twist) becomes dominant, leading to identical (relative) shadowing in neutrino and charged lepton DIS.

Figure 1: The “charge ratio” $R_c$ vs. $x$ calculated using CCFR data for neutrino and NMC data for muon structure functions. Open triangles: no heavy target corrections; open circles: $\nu$ data corrected for heavy target effects using corrections from charged lepton scattering; solid circles: $\nu$ shadowing corrections calculated in the “two phase” model. Both statistical and systematic errors are shown.
and \(s\) can be solved for Eqs. (5) and (6) form a pair of linear equations which require charge symmetry, but allowing \(s(x) \neq \bar{s}(x)\), cannot resolve the discrepancy between \(F_2^{CCFR}(x, Q^2)\) and \(F_2^{NMC}(x, Q^2)\). The experimental results are incompatible, even if \(\bar{s}(x)\) is completely unconstrained.

4 Charge symmetry violation

We have seen that neither neutrino shadowing corrections nor allowing \(s(x) \neq \bar{s}(x)\) removes the low-x discrepancy. There remain the two possible explanations. Either one of the experimental structure functions (or \(s(x)\)) is incorrect at low x, or parton charge symmetry is violated in this region. Assuming the possibility of parton CSV, we can combine the dimuon data for \(s(x)\), (Eq. 6), with Eq. 5 to obtain

\[
\frac{5}{6} F_2^{CCFR}(x, Q^2) - 3 F_2^{NMC}(x, Q^2) = x s^{\mu\bar{\nu}}(x)
\]

The left hand side of Eq. 7 and the coefficient in front of \(s-\bar{s}\) are positive. Consequently, the smallest CSV effects will be obtained when \(\bar{s}(x) = 0\). Note that this violates the requirement that the net strangeness of the nucleons be zero. Thus, setting \(\bar{s}(x) = 0\) gives the absolute lower limit on charge symmetry violation. In Fig. 2 we show the results obtained for \(x s(x)\) (open circles) and \(x \bar{s}(x)\) (solid circles) by solving the resulting linear equations, Eqs. 5 and 6, with \(\alpha \approx \alpha' \approx 0.83\). Both the structure functions and dimuon data have been integrated over \(Q^2 > 3.2\ \text{GeV}^2\) in the overlapping kinematical regions. In averaging the dimuon data we used the CTEQ4L parametrization for \(s^{\mu\bar{\nu}}(x)\). We see that the results are completely unphysical, since the equations require \(\bar{s}(x) < 0\). Since \(\alpha \approx 0.83\) is closer to the critical value for \(\alpha' = 1\) than for \(\alpha' = 0.83\) the results are even more unphysical if we use only the neutrino dimuon data. Similarly, if we decrease \(\alpha\) \(\bar{s}(x)\) becomes more negative.

In conclusion, our analysis strongly suggests that requiring charge symmetry, but allowing \(s(x) \neq \bar{s}(x)\), cannot resolve the discrepancy between \(F_2^{CCFR}(x, Q^2)\) and \(F_2^{NMC}(x, Q^2)\). The experimental results are incompatible, even if \(\bar{s}(x)\) is completely unconstrained.

Under the assumption \(s(x) = \bar{s}(x)\), this relation could be used to extract the strange quark distribution. However, as is well known, \(s(x)\) obtained in this way is inconsistent with results extracted from independent experiments.

Opposite sign dimuon production in deep inelastic \(\nu\) and \(\bar{\nu}\) scattering provides a direct determination of both \(s(x)\) and \(\bar{s}(x)\). The CCFR Collaboration extracted \(s(x)\) and \(\bar{s}(x)\) from a leading order (LO) next to leading order (NLO) analysis of their dimuon data. While the strange and anti strange distributions were different in the LO analysis they were equal within experimental errors in the NLO analysis.

To test the hypothesis that the low-x discrepancy in the charge ratio of Eq. 3 could be accounted for by allowing \(s(x) \neq \bar{s}(x)\) we combined the dimuon production data, averaged over \(\nu\) and \(\bar{\nu}\) events, with the structure functions from neutrino and charged lepton scattering (Eq. 5). Defining \(\alpha' = N_{\nu}/(N_{\nu} + N_{\bar{\nu}})\), where \(N_{\nu} = 5,030, N_{\bar{\nu}} = 1,060\) (\(\alpha' \approx 0.83\)) are respectively the \(\nu\) and \(\bar{\nu}\) events from the dimuon production experiment, the flux-weighted experimental distribution \(x s(x)^{\mu\bar{\nu}}\) from dimuon production is

\[
x s^{\mu\bar{\nu}}(x) = \frac{1}{2} x \left[ s(x) + \bar{s}(x) \right] + \frac{1}{2} (2\alpha' - 1) x \left[ s(x) - \bar{s}(x) \right].
\]

Eqs. (5) and (6) form a pair of linear equations which can be solved for \(s(x)\) and \(\bar{s}(x)\). We can simultaneously test the compatibility of the various experiments.

Compatibility of the two experiments requires that physically acceptable solutions for \(\frac{1}{2} x [s(x) + \bar{s}(x)]\) and \(\frac{1}{2} x [s(x) - \bar{s}(x)]\), satisfying both Eq. 5 and Eq. 6, can be found. Clearly, solutions do not exist if \(\frac{1}{2} (2\alpha - 1) = (2\alpha' - 1)\). (Note that the left-hand sides of Eq. 5 and Eq. 6 are different.) This gives the critical values \(\alpha = \frac{1}{2} (3\alpha' + 1)\) or \(\alpha \approx 0.7\) for \(\alpha' \approx 0.83\) and \(\alpha = 0.8\) for \(\alpha' = 1\).

In Fig. 2 we show the results obtained for \(x s(x)\) (open circles) and \(x \bar{s}(x)\) (solid circles) by solving the resulting linear equations, Eqs. 5 and 6, with \(\alpha \approx \alpha' \approx 0.83\). Both the structure functions and dimuon data have been integrated over \(Q^2 > 3.2\ \text{GeV}^2\) in the overlapping kinematical regions. In averaging the dimuon data we used the CTEQ4L parametrization for \(s^{\mu\bar{\nu}}(x)\). We see that the results are completely unphysical, since the equations require \(\bar{s}(x) < 0\). Since \(\alpha \approx 0.83\) is closer to the critical value for \(\alpha' = 1\) than for \(\alpha' = 0.83\) the results are even more unphysical if we use only the neutrino dimuon data. Similarly, if we decrease \(\alpha\) \(\bar{s}(x)\) becomes more negative.

In conclusion, our analysis strongly suggests that requiring charge symmetry, but allowing \(s(x) \neq \bar{s}(x)\), cannot resolve the discrepancy between \(F_2^{CCFR}(x, Q^2)\) and \(F_2^{NMC}(x, Q^2)\). The experimental results are incompatible, even if \(\bar{s}(x)\) is completely unconstrained.
setting \( \bar{s}(x) = s(x) \). We also show the results we obtain by setting \( \alpha' = 1 \) which corresponds to using only a subsample of the di-muon data containing only neutrino events. For \( \bar{s}(x) = s(x) \), the result is the same. However, setting \( \bar{s}(x) = 0 \) gives us a higher lower limit on CSV than that for \( \alpha' \). The result is shown as open triangles in Fig. 3.

The magnitude of the extracted CSV is also sensitive to the strange quark distribution used in the analysis. In Fig. 4 we show the CSV obtained by using the LO (open circles) and NLO (solid triangles) CCFR distributions and the CTEQ4L (solid circles) and CTEQ4D (solid rectangles) parametrizations. The uncertainty due to different parametrizations has been partly taken into account since the calculated errors already include the uncertainty of the dimuon measurement and most of the parametrizations lie within the experimental errors of the dimuon data (except of LO-CCFR \( s(x) \)).

In conclusion, the CSV effect required to account for the low-\( x \) NMC-CCFR discrepancy is extraordinarily large. It is roughly the same size as the strange quark distribution at small \( x \). This CSV term is roughly 25\% of the light sea quark distributions for \( x < 0.1 \), and the sign gives \( \bar{dd}(x) > \bar{uu}(x) \) and \( \bar{dd}(x) > \bar{uu}(x) \).

5 Influence of CSV on other observables

Clearly, CSV effects of this magnitude need further experimental verification. It is hard to imagine how such large CSV effects are compatible with the high precision of charge symmetry measured at low energies. The level of CSV required is surprising, as it is at least two orders of magnitude larger than theoretical CSV estimates\(^{11,12} \). Theoretical considerations suggest that \( \delta\bar{d}(x) \approx -\delta\bar{u}(x) \), with this sign CSV effects also require large flavor symmetry violation. Since most of the observables are proportional to the sum of \( \delta\bar{d}(x) \) and \( \delta\bar{u}(x) \) rather than to their difference this large CSV could remain unobserved in many experiments. Here, we briefly discuss the effects of the extracted CSV on FSV.

The Drell-Yan experiment measures the ratio of the dimuon cross sections of proton-Deuteron and proton-proton scattering. Since CSV is significant in the small \( x \) region, it is a good approximation to keep only the contributions to the Drell-Yan cross sections which come from the annihilation of quarks of the projectile and anti quarks of the target. In this approximation, the ratio \( R \equiv \sigma_{pp}^D/(2\sigma_{pp}^p) \) is given by

\[
\frac{\sigma_{pp}^D}{2\sigma_{pp}^p} \approx \frac{1}{2} \left[ 1 + \frac{d_2}{u_2} - \frac{\delta d_2}{u_2} \right] + \frac{R_1}{4} \left[ 1 + \frac{d_2}{u_2} - \frac{\delta d_2}{u_2} \right].
\]

Here, we introduced the notation \( R_1 \equiv \frac{d_1}{u_1} \) and \( q_{1,2} \equiv q(x_{1,2}) \) for the quark distributions. Neglecting \( R_1 \) for large \( x_F \), which corresponds to large \( x_1 \), we have

\[
R = \frac{\sigma_{pp}^D}{2\sigma_{pp}^p} \approx \frac{1}{2} \left[ 1 + \frac{(d_2 - \delta d_2)}{u_2} \right].
\]
and obtain
\[ \delta \]
We can make the approximations on the right hand side of Eq. 11 by a parametrization of \( \bar{s} \). We see that, in a first approximation, the quantities extracted from the two experiments are the same even if CSV is present, the term \( \Delta Q \) distribution are integrated for each data point over the regions as in the analysis of the charge ratio.

The difference, \( \bar{d} - \bar{u} \), can also be extracted from the difference between the proton and neutron structure functions measured by the the NMC Collaboration\(^4\). In this case we have

\[ \frac{1}{2}(u_0 - d_0) = \frac{3}{2x}(F_2^p - F_2^n) = (\bar{d} - \bar{u}) - \frac{2}{3}(\delta \bar{d} + \delta \bar{u}) - \frac{1}{6}(\delta \bar{u} + \delta \bar{u}). \]  

We can make the approximations \( \delta q \approx \delta \bar{q} \) and \( \delta \bar{d} \approx -\delta \bar{u} \), and obtain

\[ \frac{1}{2}(u_0 - d_0) = \frac{3}{2x}(F_2^p - F_2^n) \approx [(\bar{d} - \delta \bar{d}) - \bar{u}]. \]  

We see that, in a first approximation, the quantities extracted from the two experiments are the same even if both CSV and FSV are present. However, if CSV is present, the term \( \delta \bar{d} \) has to be subtracted from the measured quantity to obtain the difference \( \bar{d} - \bar{u} \).

We inverted Eq. 11 by dividing both sides by \( \bar{d} - \delta \bar{d} + \bar{u} \equiv \bar{u}(r_2 + 1) \), approximating \( \bar{d} - \delta \bar{d} + \bar{u} \) on the left hand side of Eq. 11 by a parametrization of \( \bar{d} + \bar{u} \) and solving for \( r_2 \). The structure functions and the parton distribution are integrated for each data point over the same \( Q^2 \) regions as in the analysis of the charge ratio. The result is shown in Fig. 5 as solid triangles. If we subtract the contribution of CSV from the ratio \( r_2 \) we obtain the result shown as open triangles in Fig. 5.

\[ \text{Figure 5: The ratio } \bar{d}/\bar{u} \text{ extracted from the Drell-Yan data assuming the validity of charge symmetry (solid circles). If CS is violated this ratio corresponds to } (\bar{d} - \delta \bar{d})/\bar{u}. \text{ The result obtained by correcting for CSV is shown as open triangles. The same for the ratio extracted from the difference of the proton and Deuteron structure functions is shown as solid and open triangles, with and without CSV, respectively.} \]

6 Conclusions

In conclusion, we have examined in detail the discrepancy at small \( x \) between the CCFR neutrino and NMC muon structure functions. The only way we can make these data compatible is by assuming charge symmetry violation in the sea quark distributions. The CSV amplitudes necessary to obtain agreement with experiment are extremely large – at least two orders of magnitude greater than theoretical predictions of charge symmetry violation. If CSV effects of this magnitude are really present, then one must include charge symmetry violating quark distributions in phenomenological models from the outset, and re-analyze the extraction of parton distributions.

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References