Comment on “Searching for Evolutions of Pure States into Mixed States in the Two-State System $K^0-\bar{K}^0$”

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It has been recently proposed to study generic dynamical evolutions of the neutral kaon system that go beyond quantum mechanics. We explicitly show that, unless the condition of complete positivity is enforced, those dynamics are physically inconsistent and should be rejected.

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In a recent paper, H.-J. Gerber discusses possible extensions of the time-evolution of the neutral kaon system, beyond standard quantum mechanics. The system can be modeled by a two-dimensional Hilbert space and its states can be described by a $2 \times 2$ density matrix $\rho$. The time-evolution equation considered in [1] is:

$$\frac{\partial}{\partial t} \rho(t) = L[\rho(t)] ,$$

where $L$ is a generic linear map. In this note, we would like to point out that time-evolutions generated by equations of the form (1), without further restrictions, are in general physically inconsistent.

In going beyond conventional quantum mechanics, one has to be careful not to destroy the probabilistic interpretation of $\rho$, on which all physical considerations are based. To be specific, the finite time evolution map $\gamma_t : \rho(0) \mapsto \rho(t)$ must at least preserve the positivity of the eigenvalues of $\rho(t)$ for all times $t \geq 0$. However, this condition alone does not guarantee the positivity of the eigenvalues of density matrices of correlated kaons evolving with $\gamma_t \otimes \gamma_t$: in fact, $\gamma_t$ has to be not only positive, but also completely positive. [2,3,5] This requirement is physically unavoidable, since correlated kaons are indeed produced in the decay of $\phi$ mesons, and are the focus of experimental investigations in the so called $\phi$-factories. Without the complete positivity of $\gamma_t$, the time evolution $\gamma_t \otimes \gamma_t$ of correlated kaons would be beset by negative eigenvalues of evolving density matrices. [2,3]

In general, since the neutral kaon system is unstable, the evolution $\gamma_t$ must satisfy a forward in time composition law, decrease the trace of the kaon state and increase its von Neumann entropy. Together with complete positivity, this fixes the generator $L$ in (1) to be $[2,6]$

$$L[\rho] = -i [M, \rho(t)] - \frac{1}{2} (\Gamma \rho + \rho \Gamma) + L_D[\rho] .$$

The first two pieces in the r.h.s. of (2) correspond to the Wéisskopf-Wigner standard evolution with effective hamiltonian $H = M - (i/2) \Gamma$. The term $L_D$ is a dissipative contribution [6]. By writing $\rho$ as a 4-vector with components $(\rho_0, \rho_1, \rho_2, \rho_3)$ along the identity matrix $\sigma_0$ and the Pauli matrices $\sigma_i, i = 1, 2, 3$, $[L_D]$ acts as:

$$[L_D] = -2 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & c \\ 0 & b & a & \beta \\ 0 & c & \beta & \gamma \end{pmatrix} ,$$

and the six real parameters must satisfy

$$2R \equiv a + \gamma - a \geq 0 , \quad RS \geq b^2 ,$$
$$2S \equiv a + \gamma - a \geq 0 , \quad RT \geq c^2 ,$$
$$2T \equiv a + \gamma - a \geq 0 , \quad ST \geq \beta^2 ,$$
$$RST \geq 2bc\beta + R\beta^2 + Sc^2 + Tb^2 .$$

Concretely, let $\rho_+ = (\sigma_0 \pm \sigma_3)/2, \rho_\pm = (\sigma_1 \pm i\sigma_2)/2$ and set for simplicity $M = \Gamma = 0, a = b = c = 0, \alpha = \gamma = 1, \beta = 1/2$: since $0 = ST < \beta^2 = 1/4$, $\gamma_t$ is positive but not completely positive. The initial projections $\rho_\pm$ evolve
into $\rho_\pm(t) = 1/2(\sigma_0 \mp s\sigma_2 \pm r\sigma_3)$, where $r = e^{-2t}\cosh t$
and $s = e^{-2t}\sinh t$, and keep positive eigenvalues, while $\sigma_\pm(t) = 1/2(\sigma_1 \mp ir\sigma_2 \pm is\sigma_3)$. Therefore, the initial state
$\rho_S = 1/2(\rho_+ \otimes \rho_- + \rho_- \otimes \rho_+ - \sigma_+ \otimes \sigma_- - \sigma_- \otimes \sigma_+)$, describing two correlated kaons, evolves with $\gamma_t \otimes \gamma_t$ into

$$
\rho_S(t) = \frac{1}{4} \begin{pmatrix}
1 - x & -iy & -iy & -1 + x \\
    iy &  1 + x & -1 - x &   iy \\
    iy &  -1 - x &  1 + x &   iy \\
-1 + x &  -iy &  -iy &  1 - x
\end{pmatrix},
$$

where $x = e^{-4t}\cosh(2t)$, $y = e^{-4t}\sinh(2t)$, and develops a negative eigenvalue as soon as $t > 0$. [3,5]

Concluding, physical consistency demands that it is the time evolution generated by the equation in (2), and not the generic one given by (1) that should be compared with the experimental data. [4,5]