Standard Solar Models in the Light of New Helioseismic Constraints
I The Solar Core

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ABSTRACT

In this paper, we examine a new updated solar model taking advantage of the recent reexamination of the nuclear reaction rates and of the microscopic diffusion of helium and heavy elements. Our best model fits the helioseismic data reasonably well: the base of the convective zone is at \( R_{bcz} = 0.715 \), the photospheric helium in mass fraction is 0.243, the sound speed square difference between the Sun and the model, \( \delta c^2/c^2 < 1\% \). This model leads to a reestimate of neutrino fluxes: 7.18 SNU for the chlorine experiment, 127.2 SNU for the gallium detector and \( 4.82 \times 10^6 \) cm\(^{-2}\)s\(^{-1}\) for the \( ^8\text{B} \) neutrino flux. Acoustic mode predictions are also estimated.

We then consider the radiative zone and discuss what we learn from such a model confronted with the present helioseismic constraints from space experiments aboard SOHO. We present three models which respect these constraints and fit the seismic observations better by taking advantage of the known physical uncertainties: nuclear reaction rates, CNO abundances and microscopic diffusion. We also study some current questions as the possibility of mixing in the nuclear core, the revision of the solar radius, or the influence of the solar age.

We conclude that the standard model, inside its inherent uncertainties, is robust in the light of the present acoustic mode detection and that mixing in the core is not really favoured, even though a proper understanding of the angular momentum evolution with time has not yet been obtained.

The initial solar helium abundance seems more and more constrained; this study supports an initial abundance between 0.273 and 0.277 in mass fraction.
This analysis allows us to define minimal values for neutrino predictions, compatible with present seismic results. We notice that a reduction of about 30% on chlorine and water detectors, which is more than half of the discrepancy with the experimental results, is still supported by the present study. This work emphasizes also the fact that acoustic mode determination does not put strong constraints on the nuclear plasma characteristics.

We finally estimate g-mode frequencies in the range which may be accessible to the satellite SOHO, these results emphasize the substantially improved sensitivity of these modes on the details of the nuclear solar core, and show the frequency dependence of these modes for the different models previously discussed.

1. INTRODUCTION

Standard solar models, and stellar evolution in general, have been substantially improved in recent years, taking advantage of the impressive helioseismic tool. The quality and accuracy of this technique has revealed that the Sun may be considered a real laboratory to check the physical processes we introduce in calculations: equation of state, radiation transport and gravitational settling of elements.

Effectively the extraction of the internal radial sound speed from a great number of acoustic modes determined by ground networks GONG (Hill et al. 1994, 1996), IRIS (Grec et al. 1991, Gelly et al. 1997), BiSON (Elsworth et al. 1994, Chaplin et al. 1996a, b) and LOWL (Tomczyk et al. 1995), has allowed the determination of the base of the convection zone \( r_{bcz} = 0.713 \pm 0.003 R_\odot \) (Christensen-Dalsgaard, Gough and Thompson 1991, Basu and Antia 1997), the verification of the equation of state for light elements, with the precise determination of photospheric helium content \( Y_{ph} = 0.249 \pm 0.003 \) (Vorontsov, Baturin and Pamyatnykh 1992, Basu and Antia 1995), thus compelling the introduction of the slow microscopic diffusion process.

Three additional space helioseismic experiments aboard SOHO (Domingo et al. 1995), GOLF (Gabriel et al. 1995, 1997), VIRGO (Fröhlich et al. 1995, 1997) and MDI (Scherrer et al. 1995, Kosovichev et al. 1997) give us new constraints on the transition region between radiation and convection zones and on the nuclear core, through a detailed analysis of the rotation and sound speed profiles. In particular, the GOLF experiment, accessing more than 100 modes penetrating the solar core, will hopefully constrain the nuclear energy production region down to 0.05 \( R_\odot \) and its evolution with time, if any (Lazrek et al. 1997, Turck-Chièze et al. 1997).
In parallel, solar neutrino detection has been substantially improved and enlarged with GALLEX (Hampel et al. 1996 and references therein) and SAGE (Abdurashitov et al. 1994, 1996) results, sensitive to pp neutrinos. Additionally, the very recent SuperKamiokande (Suzuki 1994, Totsuka et al. 1996) experiment has detected, for the first time, 4500 solar neutrinos in less than 1 year. The next years in this field are also extremely promising with the beginning of SNO (McDonald 1995) and BOREXINO (Raghavan 1995).

This exceptional circumstance opens new debates on stellar modeling aspects and on neutrino predictions. All the measurements together, for at least half of a solar cycle, provide a unique occasion to investigate models beyond the standard stellar framework and to bring some quantitative answers to dynamical aspects which may induce turbulence and the mixing of elements in different parts of the Sun or more generally local instabilities. Another important issue could be a determination of specific properties of neutrinos which are not accessible in the laboratory.

In this paper, we first discuss the improved microscopic physics of our standard model, including the recent updated nuclear reaction rates which are fundamental in the core description and the microscopic diffusion (section 2). We describe the reference model obtained with the stellar evolution code, CESAM, developed by P. Morel (1997) which is numerically sufficiently accurate to discuss the new generation of experimental results (better than $10^{-4}$). We compare our predictions on neutrino fluxes, sound speed profile and low degree acoustic modes (using the pulsation code of J. Christensen Dalsgaard, 1982), to measurements or observations (section 3). In section 4, the differences are examined and a new step consists of discussing some uncertainties of the physical processes already included in order to better match the helioseismic constraints. Then, we question some other processes as mixing processes, radius determination or solar age. Section 5 is devoted to a general discussion and to gravity mode predictions supported by the present acoustic mode observations. Finally section 6 summarizes our results.

2. PHYSICAL INPUTS

2.1. Composition and Opacity Coefficients

Chemical composition plays a crucial role in solar modeling, mainly via the opacity coefficients and the nuclear reaction rates. Improvements on photospheric abundances have been made since the review of Anders and Grevesse (1989), suppressing the disagreement between photospheric and meteoritic iron and improving the knowledge of carbon, nitrogen and oxygen photospheric abundances (Grevesse and Noels 1993). The present solar
photospheric metallicity agrees very well now with the meteoritic composition but the present accuracy does not exclude a small effect of diffusion between the initial composition and the present photospheric observation. Effectively, this study has led to a ratio \( Z/X = 0.0245 \) but with an uncertainty of about 10-15 % which seems largely dominated by CNO determination. The first impact of these improvements on the solar model has been studied by Turck-Chièze and Lopes (1993).

Other crucial progress has been achieved by the helioseismic community in the determination of the photospheric helium content. Until recently, the solar initial helium content, thought to be equal to the photospheric abundance, was deduced from solar evolution in the absence of photospheric line measurements. The possibility of reaching a precise photospheric value from helioseismic determination of the adiabatic exponent, has been useful complementary information. This value of \( 0.25 \pm 0.01 \) (Vorontsov, Baturin and Pamyatnykh 1992) or \( 0.249 \pm 0.003 \) (Basu and Antia 1995) is not very far from cosmological and certainly smaller than the initial solar value deduced from solar modeling. This result has largely confirmed the need to introduce microscopic diffusion in stellar modeling (see section 2.4).

Despite all these improvements, there are still some elements such as \( ^7\text{Li} \) and \( ^9\text{Be} \), for which the observed surface abundance depletion is difficult to reproduce, without invoking other physical processes than the ones introduced in the classical picture such as mass loss by winds or turbulent mixing.

The CESAM code used in this study, follows the time evolution from the zero-age main sequence (ZAMS) or the pre-main sequence (PMS), of 12 chemical elements, namely: \( ^1\text{H} \), \( ^2\text{H} \), \( ^3\text{He} \), \( ^4\text{He} \), \( ^7\text{Li} \), \( ^7\text{Be} \), \( ^{12}\text{C} \), \( ^{13}\text{C} \), \( ^{14}\text{N} \), \( ^{15}\text{N} \), \( ^{16}\text{O} \), \( ^{17}\text{O} \). Apart for the study of \( ^7\text{Li} \) depletion, the impact of the PMS on the solar structure or the neutrinos fluxes, is negligible.

The most recent observed solar abundances (Grevesse and Noels 1993) are used in the models described below, after normalization of the metals to 1 (see Table 1). For the \( ^3\text{He} \) case, starting on the zero-age main sequence, we use the value of \( ^3\text{He}/^{1}\text{H} = 4.4 \pm 1.5 \times 10^{-5} \) including deuterium burning, discussed in Turck-Chièze et al. (1993). This value is not ruled out by the recent reestimate of \( ^2\text{H}/^{1}\text{H} \) abundance of Gautier and Morel (1997).

The Livermore opacity tables OPAL96 (Iglesias and Rogers 1996) are also based on Grevesse and Noels (1993) composition and now include 19 heavy elements to properly introduce the respective composition of the heavy elements in these calculations. The tables have been introduced in the CESAM code and the mean Rosseland opacity, \( \kappa_R \) is calculated for each mass shell, interpolating across the tables for \( X, T, \rho \) and \( Z \). One of the difficulties
in using such tables is to extend them outside their original range. We accomplished this by using Kurucz’s low temperature opacities below $T_{\text{inf}} = 5700K$. Specific studies have been performed with higher matching temperatures (up to $T= 10^4 K$) to check the atmospheric $T(\tau)$ laws (Brun et al. 1997). The smoothest transition was obtained for a temperature of $T= 9300K$ (the discrepancy between these two calculations is smaller than 2%), even though the very small molecular components encourage the use of the Livermore tables down to the photospheric temperature (Sharp and Turck-Chiëze, 1998). The recent updated OPAL opacities may introduce up to a 10% change in the opacity coefficients due to the improvement of both the physical description, the numerical procedures and the inclusion of 19 instead of 12 heavy elements. A comparison between OPAL93 (Iglesias et al. 1992) and OPAL96 opacity tables indicates variations up to 5% in the intermediate region of the solar models ($0.3 \, R_\odot \, - \, 0.7 \, R_\odot$). Such modifications have a clear effect on the sound speed profile at the level of accuracy we reach today (i.e few % in opacity $\rightarrow$ few $10^{-3}$ in sound speed) and leads to a slight degradation when comparing with the helioseismic data. These opacity modifications explain partly why solar models using older opacities, like model S of Christensen-Dalsgaard et al. (1996), have slightly different sound speed profiles. We have verified that our result is independent of the opacity interpolation subroutine (the subroutine delivered by the Livermore group leads to a radial sound speed profile in agreement within 0.001 with the one obtained in using the opacity interpolation subroutine provided by G. Houdek 1996). The other sources of difference are attributed to the used nuclear reaction rates (see 2.3) and the solar age used for the computation (see 4.4.2).

Table 1: Metal Abundances

<table>
<thead>
<tr>
<th>Element</th>
<th>Relative Number Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.24551</td>
</tr>
<tr>
<td>N</td>
<td>0.06458</td>
</tr>
<tr>
<td>O</td>
<td>0.51295</td>
</tr>
<tr>
<td>Others</td>
<td>0.17696</td>
</tr>
</tbody>
</table>
2.2. Equation of State

We have introduced the Livermore EoS in the CESAM code. This equation of state considers the correct ionization treatment of the chemical species present in the solar plasma, pressure ionization and quantum corrections other than electronic degeneracy (Rogers, Swenson and Iglesias 1996). Moreover it is consistent with the opacity coefficients.

Fig 1: Comparison of the main thermodynamical quantities: the pressure $P$, the density $\rho$, and the adiabatic exponent $\Gamma_1$ for the two equations of state: VDC and OPAL96 in the sense $(VDC-OPAL)/OPAL$. $\delta P/P$, $\delta \rho/\rho$ and $\delta \Gamma_1/\Gamma_1$.

In figure 1 we compare this equation of state with the one used in our previous solar models (Turck-Chièze et al. 1988, Turck-Chièze and Lopes 1993, Dzitko et al. 1995). It was based on a treatment of a perfect gas for ions and an estimate of the partial ionization of light elements using the Saha equation (Vardya 1960), a correct estimate of the electron degeneracy and a coulomb term recently reestimated beyond the Debye approximation (Dzitko et al. 1995). We call it V (Vardya) D (degeneracy) C (coulomb correction) equation of state in the following. This equation had the advantage of being analytical, which allows an estimate of the different terms and an easy derivation of each of them, but the partial ionization of the heavy elements was not considered and it was not accurate enough to estimate the partial ionization of the light elements, even if molecular components were included. The oscillatory behaviour of $\frac{\delta \Gamma_1}{\Gamma_1}$ is due to the different locations of the partial
ionization zones of hydrogen and helium. In fact the improvements of the Livermore equation of state are effectively mainly in this zone corresponding to $T < 5 \times 10^5 \text{K}$. The small difference of 0.5\% in the central region could be due to the complete ionization of all the species assumed in VDC EoS but may be also due to a slight difference in the coulomb correction for intermediate plasma coupling.

Due to the small role of the heavy elements in the equation of state, we have only calculated models with OPAL EoS at fixed $Z$ element composition. Most of the models used $Z = 0.0195$. Guzik et al. (1995) show that the effect of a fixed $Z$ in the EoS was observable ($< 1 \mu\text{Hz}$) in the acoustic mode frequencies. On the contrary, the opacity $\kappa_R$ has been evaluated at each point with a precise $Z$ composition (see section 2.4), as the heavy elements, and more specifically $^{16}\text{O}$ and $^{56}\text{Fe}$ are strong contributors to the mean Rosseland opacities (Courtaud et al. 1990).

Due to its lower temperature boundary $T_{\text{inf}} = 5000 \text{K}$, (the minimal temperature value in our models is $\sim 4200 \text{K}$), Livermore EoS could not be used along the whole structure. So VDC EoS was used for the very upper layers. This absence of consistency creates a small discontinuity in pressure, and a difference of about $1 \mu\text{Hz}$ on acoustic-mode frequencies at high frequency.

### 2.3. Nuclear reaction rates

The nuclear reaction rates are the most important ingredients in the description of the solar nuclear core and for the prediction of the neutrino fluxes, mainly for secondary neutrinos as those of the ppII, ppIII chains and CNO cycle (Turck-Chièze and Brun 1997). This is the reason why considerable effort is still devoted to the best determination of the different reaction interactions, in improving the theoretical determination of the electronic capture on $^7\text{Be}$ (Gruzinov and Bahcall 1997), the experimental conditions for $(^3\text{He},^3\text{He})$ (Arpesella et al. 1996) and $(^7\text{Be},p)$ (Hammache et al. 1998), the extrapolation of measurements to the solar range of energy, the effect of solar plasma screening on the reaction rates (Turck-Chièze and Lopes 1993, Dzitko et al. 1995).

The importance of these ingredients in the solar model calculation has led some of us to properly examine all the progress obtained and determine a new update of all the reaction rates and screening effects, including recent works and commenting the ones which have been excluded in order to get updated nuclear S factors and corresponding uncertainties (Adelberger et al. 1998).
This updated compilation, including Mitler screening prescription (Dzitko et al. 1995), has been introduced in the CESAM code in place of Caughlan and Fowler (1988) rates and weak screening (Salpeter 1954). The new values for the cross section factor $S(0)$ and its derivatives are listed in Table 2, for the main nuclear reaction rates of the pp chain.

Table 2: Nuclear S factors and derivatives for the main reactions of the hydrogen burning (see Aldelberger et al. 1998 for the others) and screening factors for weak or intermediate screening.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$S(0)$ (MeV barn)</th>
<th>$S'(0)$ (barn)</th>
<th>$S''(0)$ (b MeV$^{-1}$)</th>
<th>$f_{WS}$</th>
<th>$f_{Mitler}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1$H(p,e$^+$ $\nu$)$^2$H</td>
<td>4.00 $10^{-25}$</td>
<td>4.48 $10^{-24}$</td>
<td></td>
<td>1.05</td>
<td>1.045</td>
</tr>
<tr>
<td>$^3$He($^3$He,2p)$^4$He</td>
<td>5.4</td>
<td>-4.1</td>
<td>4.6</td>
<td>1.21</td>
<td>1.18</td>
</tr>
<tr>
<td>$^3$He($\alpha,\gamma$)$^7$Be</td>
<td>5.3 $10^{-4}$</td>
<td>-3.0 $10^{-4}$</td>
<td></td>
<td>1.21</td>
<td>1.18</td>
</tr>
<tr>
<td>$^7$Be(p,\gamma)$^8$B</td>
<td>1.9 $10^{-5}$</td>
<td>-1.35 $10^{-5}$</td>
<td>7.33 $10^{-5}$</td>
<td>1.21</td>
<td>1.17</td>
</tr>
<tr>
<td>$^{14}$N(p,\gamma)$^{15}$O</td>
<td>3.5 $10^{-3}$</td>
<td>-0.0128</td>
<td></td>
<td>1.40</td>
<td>1.29</td>
</tr>
</tbody>
</table>

If this table is compared to the previous recommendations, three main changes are apparent. The recommended $S_{pp}(0)$ value is between that suggested by Kamionkowski and Bahcall (1994) and that used by Turck-Chièze and Lopes (1993). It includes a reestimate of the neutron lifetime of 888 ± 3 s (Barnett et al. 1996). The present uncertainty of 2.2 %, is dominated by the meson current effect. The ($^3$He,$^3$He) reaction rate has been slightly increased due to the recent remeasurement of this cross section in the laboratory down to 12 keV without revealing any resonance. The error bar of ±8% is mainly due to the systematic error and the uncertainty on the effect of screening in the laboratory. The situation of ($^3$He,$^4$He) is still confusing as the adopted reaction rate is the mean value of those of two types of experiments leading to two different results. So this reaction rate may be reduced in the future as the direct measurements lead to a smaller value than the $^7$Be activity measurements. Finally, the reestimate of the ($^7$Be,p) reaction rate agrees with the analysis of Turck-Chièze et al. (1993) and reduces the $S(0)$ recommended by Bahcall and Pinsonneault (1995) by 15%. The difficulty of this experiment due to the radioactive target leads to an error bar of at least 10%. The measurement of Filippone et al. (1983) is now confirmed by the very recent experiment of Hammache et al. (1998). This experiment will be continued at low energy. Effectively, a second problem for this reaction rate is the extrapolation at the solar energy of 20 keV. The calculation is still extremely uncertain, depending on the way $^7$Be and $^8$B nuclei are described. A “prudent conservative range” for this astrophysical factor of $19^{+8}_{-4}$ eVb is consequently recommended.
Several papers have been published on the stellar screening effect. Discussions during the preparation of the compilation have led to a recommended extension of the weak screening to treat intermediate plasma for reaction between two reactants of Z greater than 1. Following our previous study (Turck-Chièze and Lopes 1993; Dzitko et al. 1995), we use in the present paper the formalism for weak screening for the pp interaction and intermediate screening based on Mitler’s work (Mitler 1977).

We recall that this intermediate screening (see table 2) reduces the neutrino fluxes in comparison with weak screening (except for \( ^1\text{H}(\text{p}, e^+ \nu)^2\text{H} \)) but leads to greater values than those of Graboske et al. (1973). This last formalism is not adequately accurate for solar neutrino predictions and does not treat the electron degeneracy. Recent work (Gruzinov et Bahcall 1998) confirms this tendency and could even support a value nearly compatible with the weak screening for heavy elements of Z \( \geq 8 \). The difference between these two recent works on screening is smaller than the uncertainty of the corresponding nuclear reaction rates.

2.4. Microscopic Diffusion

This slow process, which requires an atom needing more than \( 10^{10} \) years to move along the solar radius, has been neglected in solar modeling in the past. But now, due to the evidence of surface element depletion by helioseismic analysis of photospheric helium, this process must be introduced in the standard treatment of the solar model.

Consequently, the time evolution of the mass fraction abundances \( X_i \) of each chemical species must be expressed as follows:

\[
\frac{dX_i}{dt} = \frac{dX_i}{dt}_{\text{nucl}} + \frac{dX_i}{dt}_{\text{diff}} = \frac{dX_i}{dt}_{\text{nucl}} - \frac{1}{\rho r^2} \frac{d}{dr} \left( \rho r^2 V_i X_i \right)
\]

In this expression a mass loss term and a turbulent term are still missing. For the diffusive velocity \( V_i \), the CESAM code uses the approximate formulae proposed by Michaud and Proffit (1993, eq. 17, 18, 19) for a \(^1\text{H}-^4\text{He} \) mixture and several trace elements.

Formula (19) of Michaud and Proffit (1993), describing the thermal part of the diffusive velocity of a trace element i in a \(^1\text{H}-^4\text{He} \) background, is in agreement with a more detailed calculation based on Burgers’ equations to within 20% (Burgers 1969).

In the CESAM code, these formulae are applied to diffuse the following elements, \(^1\text{H}, ^2\text{H}, ^3\text{He}, ^4\text{He}, ^7\text{Li}, ^7\text{Be}, ^{12}\text{C}, ^{13}\text{C}, ^{14}\text{N}, ^{15}\text{N}, ^{16}\text{O}, ^{17}\text{O} \) and of course an extra element “Ex” in
order to diffuse the remaining chemical composition (i.e. \( X_{Ex} = 1 - \sum_{i=H}^{O17} X_i \)), treated as silicium (Morel et al. 1997).

Figure 2 shows the diffusive velocities for the different elements in our reference model along with the contribution of the thermal velocity (up to 40% of the total velocity). It is interesting to note, that the bumps close to \( T=10^7K \) on \( ^{12}C \) and \( ^{14}N \) microscopic velocities, arise from the composition gradient due to the nuclear processes occurring in this region.

Figures 3 a) and b) clearly show the impact of the variation of the composition along the solar radius on the opacity coefficient. The large bump around the base of the convective zone shows the crucial role of the heavy elements in this range, mainly iron and oxygen. As is well known, this has a direct consequence on the sound speed profile. Due to the approximation done in this calculation, we have compared our results with other more recent detailed calculations of microscopic diffusion processes. We notice that the diffusion prescription we use in the present work, matches a more complete description based on Burgers full equations within 5-10% (Thoul, Bahcall and Loeb 1993, Turcotte et al. 1998), and has the advantage of a dramatic saving in CPU time.

Turcotte et al. (1998) have performed a very complete microscopic calculation taking into account the radiative acceleration using monochromatic opacity tables and the average atomic number to treat the fact that the chemical species are not completely ionized. For example, the photospheric helium variation, between a model without or with diffusion, is about 11.1% for Turcotte et al. (1998), when partial ionization is included (model C), and 10.4% for the full calculation (model H). In our model, where complete ionization has been assumed for all elements, we find a 10.6% helium variation. The radiative acceleration reduces the settling mainly for the elements of the iron peak but does not act strongly on He, C, N, O (less than 2% of the gravitational acceleration except at the base of the convective zone where an effect of 4-6% could be reached) (Turcotte et al. 1998).

For heavy elements the difference is slightly greater mainly due to the expression of \( v_{\text{therm}} \) which underestimates this effect by about 20% (Michaud and Proffitt 1993). Effectively the Z variation is about 10.3% for Turcotte’s model C but 8.5% for model H (one notes also that \( \text{grad/g} \) may reaches 0.4 for the elements of iron peak), and only 8.1% for our diffusive model. Another part of the discrepancy should come from the neglect of the partial ionization of heavy elements such as O, Fe, near the convective zone, while an additional small contribution to the difference may arise from the different ages used in the calculations (4.52 Gyr compared with 4.57 Gyr).

In conclusion, the equations we use lead to a in reasonably good agreement with more
Fig 2: Diffusive velocities and thermal contribution to these velocities for the different element considered.
Fig 3: a) Opacity difference between diffusive and non-diffusive models. b) Corresponding relative variation of helium and heavy elements.
precise and extremely time consuming calculations in the case of the Sun. For greater mass or more evolved stars, this treatment may appear too simple. Nevertheless, this process is probably partly inhibited by turbulence at the basis of the convection zone. The discrepancy between the solar sound speed and that deduced from a model including turbulence, will give part of the answer (see our next paper: “II The radiation-convection transition”).

As far as the neutrino predictions are concerned, there is an underestimation of about 5% compared to models of Turcotte et al. (1998) or Bahcall and Pinsonneault (1995) (see Turck-Chièze 1996, Turck-Chièze and Brun 1997 for a comparison).

2.5. Convection

The standard mixing length theory (MLT) (Böhm-Vitense 1958) is used to describe the convective transport of the energy, taking into account the optical width of the eddies (Henyey et al. 1965, see also Berthomieu et al. 1993). This concept is insufficient to compare with present helioseismic data (mainly for the absolute acoustic modes frequencies) and needs to be improved.

2.6. Atmospheric Treatment

The treatment of the outer layers is important to determine the absolute values of the acoustic modes. One major difference between our previous models (Turck-Chièze et al. 1988, Turck-Chièze and Lopes 1993 and Dzitko et al. 1995), based on the code of Paczynski (1969) and the present ones based on the CESAM code is the atmosphere treatment. The Paczynski code uses the Henyey method to solve the structure equations. The solution of these equations is done from the central part to some region located in the convective zone and from $R=1.003 R_{\odot}$ down to this same point. In this scheme, one then minimizes the discrepancies at the adjusting point and iterates the calculation until it matches the structure. In contrast, CESAM code uses a reconstructed atmosphere based on a $T(\tau,T_{\text{eff}})$ law (Morel et al. 1994), deduced from the ATLAS9 atmosphere code (Kurucz 1991), and adds it to the main structure. The matching point could be at different optical depth $\tau_b$. Normally, the atmosphere is better described, but at the transition a temperature gradient exists. It leads to a discontinuity which deteriorates the quality of the mode frequency predictions (see Brun et al. 1997 and Turck-Chièze et al. 1997). Pending improvements in the CESAM code will address this problem.
3. STANDARD MODEL RESULTS

In this section, we present the results of our standard solar model, calculated with the CESAM code (Morel 1997), including all the physical quantities described above. In this calculation, we also use the physical parameters included in table 3.

Table 3: Solar Observations: physical parameters, helioseismic observations, solar neutrino detections

<table>
<thead>
<tr>
<th>Physical parameters</th>
<th>Helioseismic observations</th>
<th>Solar neutrino detections</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_\odot = (1.9891 \pm 0.0004) \times 10^{33}$ g</td>
<td>$Y_{surf} = 0.249 \pm 0.003$</td>
<td>$^{71}$Ga = 76 $\pm$ 8 SNU (cal = 0.91 $\pm$ 0.08) for GALLEX</td>
</tr>
<tr>
<td>$R_\odot = (6.9599 \pm 0.0002) \times 10^{10}$ cm</td>
<td>$R_{bcz}/R_\odot = 0.713 \pm 0.003$</td>
<td>$^{71}$Ga = 70 $\pm$ 8 SNU (cal = 0.95 $\pm$ 0.12) for SAGE</td>
</tr>
<tr>
<td>$L_\odot = (3.846 \pm 0.004) \times 10^{33}$ ergs.s$^{-1}$</td>
<td></td>
<td>$^{37}$Cl = 2.55 $\pm$ 0.25 SNU for Homestake</td>
</tr>
<tr>
<td>Age = 4.52 $\pm$ 0.04 Gyr</td>
<td></td>
<td>$^8$B = 2.7 $\pm$ 0.1 $\times 10^6$ cm$^2$ s$^{-1}$ for Kamiokande</td>
</tr>
<tr>
<td>$(Z/X)_\odot = 0.0245 \times (1 \pm 0.1)$</td>
<td></td>
<td>$^8$B = 2.44 $\pm$ 0.26 $\times 10^6$ cm$^2$ s$^{-1}$ for Super Kamiokande</td>
</tr>
</tbody>
</table>

The mass, radius and luminosity are the usual values. The present age is deduced from a study of the age of the earth and of the solar system formation (Guenther 1989, 1992). This value supposes that we begin our calculation on the main sequence and is not in contradiction with the compilation of Wasserburg in Bahcall, Pinsonneault and Wasserburg (1995). The heavy element composition is deduced from the compilation of Grevesse and Noels (1993) and we have chosen a $^3$He mass composition of $\sim 9.10^{-5}$ which takes into account the best determination of the isotopic ratio $^3$He/$^4$He and the deuterium burning during the premain sequence.

We iterate our calculation to reach the solar luminosity, radius and Z/X ratio estimated
by Grevesse (1996) within $10^{-5}$ by varying the solar initial helium, the ratio $Z/X$ of the
initial composition and the mixing length parameter $\alpha$. In doing so, we assume the relative
composition of the heavy elements remains constant with time, which is not totally true, as
seen in the previous section where we discuss microscopic diffusion in detail.

We generally consider about 400-600 mesh points (this number may be increased if
needed) and 40 time steps with an adaptable step varying from 10 Myr to 200 Myr. The
numerical integration of the structure equations is based on the splines collocation method
(de Boor 1978, Morel 1997).

We summary also in table 3, the results from helioseismology: the determination of the
photospheric helium ($Y_{surf}$) (Vorontsov, Baturin and Pamyatnykh 1992, Basu and Antia
1995) and the position of the base of the convection zone ($R_{bcz}/R_\odot$)(Christensen-Dalsgaard
et al. 1991). The mean values (on the period of observations) of the three types of
neutrino detections are also listed: gallium from GALLEX (Hampel et al. 1996) and SAGE
(Abdurashitov et al. 1996) (the calibration value is not introduced in the experimental
mean value), chlorine from Homestake (Davis 1994) and water for Kamiokande (Fukuda
et al. 1996) and SuperKamiokande (Totsuka 1996). They are directly compared with the
results of our present solar model in table 4.

The diffusion of helium and heavy elements is clearly visible in the sound speed
difference between the Sun and a model including this physical process, even though it
increases the discrepancy on neutrino fluxes ($\sim +27\%$ for $^{37}\text{Cl}$ detector). This point has
already been mentioned by several groups (Proffitt and Michaud (1991), Bahcall and
Pinsoneault (1992), Christensen-Dalsgaard et al. (1993), Berthomieu et al. (1993) for
helium diffusion; Michaud and Proffit (1994), Bahcall and Pinsonneault (1995), Morel,
Provost and Berthomieu (1997) for diffusion of helium + heavy elements. Figure 4 shows
a comparison of the square of the sound speed deduced by S. Basu and J. Christensen
Dalsgaard (Turck-Chi`eze et al. 1997), from the GOLF (Lazrek et al. 1997, ) and LOWL
(Tomczyk et al. 1995) experiments compared with our standard models with and without
microscopic diffusion.

Including the effects of diffusion, we observe (figure 4, table 4 and table 5):

a) a reduction of the discrepancy between the observed sound speed deduced from GOLF +
LOWL data, and the calculated one by a factor greater than 2 when microscopic diffusion
is incorporated.
b) a 10% increase of the heavy element composition, at the center leading to a $Y_{surf} \sim 0.243$,
in reasonable agreement with the value inferred by helioseismology (Basu et Antia 1995).
c) a much better agreement with the observed base of the convection zone for models with
Table 4: Thermodynamical Quantities and Neutrino Predictions of the Standard Solar Model: \( \Delta R/R_\odot \) and \( \Delta L/L_\odot \): accuracy on \( R_\odot \) and \( L_\odot \), \( \alpha \): mixing length parameter, \( Y_0, Z_0, (Z/X)_0 \): initial helium, initial heavy element and initial ratio heavy element on hydrogen, \( Y_s, Z_s, (Z/X)_s \): idem for photospheric compositions, \( \tau_b \) is the optical depth of the bottom of the atmosphere, \( R_{bcz}, T_{bcz} \) are the radius and temperature at the base of the convective zone, \( Y_c, Z_c, T_c, \rho_c \): central helium, heavy element contents, central temperature and density; \( P_0 \): gravity mode period; \(^{37}\)Cl, \(^{71}\)Ga, \(^{8}\)B respective neutrino predictions for the chlorine, gallium and water detectors. OPAL/K 5800K means that we use OPAL96 opacities above 5800K and Kurucz ones below.

<table>
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<tr>
<th>Parameters</th>
<th>model without diffusion</th>
<th>with diffusion</th>
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<tr>
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<td>OPAL/K 5800K</td>
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<td>Diffusion</td>
<td>no</td>
<td>yes</td>
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<tr>
<td>( \Delta R/R_\odot )</td>
<td>( 2.26 \times 10^{-5} )</td>
<td>( 1.57 \times 10^{-5} )</td>
</tr>
<tr>
<td>( \Delta L/L_\odot )</td>
<td>( 1.79 \times 10^{-5} )</td>
<td>( 3.67 \times 10^{-5} )</td>
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<tr>
<td>( \alpha )</td>
<td>1.713</td>
<td>1.840</td>
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<tr>
<td>( Y_0 )</td>
<td>0.265</td>
<td>0.273</td>
</tr>
<tr>
<td>( Z_0 )</td>
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<td>( 1.964 \times 10^{-2} )</td>
</tr>
<tr>
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<td>0.0277</td>
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<tr>
<td>( Y_s )</td>
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<td>0.243</td>
</tr>
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<td>( 1.810 \times 10^{-2} )</td>
</tr>
<tr>
<td>( (Z/X)_s )</td>
<td>0.0245</td>
<td>0.0245</td>
</tr>
<tr>
<td>( \tau_b )</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( R_{bcz}/R_\odot )</td>
<td>0.729</td>
<td>0.715</td>
</tr>
<tr>
<td>( T_{bcz} \times 10^6 ) (K)</td>
<td>2.055</td>
<td>2.172</td>
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<tr>
<td>( Y_c )</td>
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<td>0.635</td>
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<tr>
<td>( Z_c )</td>
<td>( 1.807 \times 10^{-2} )</td>
<td>( 2.084 \times 10^{-2} )</td>
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<tr>
<td>( T_c \times 10^6 ) (K)</td>
<td>15.44</td>
<td>15.67</td>
</tr>
<tr>
<td>( \rho_c ) (g/cm(^3))</td>
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<td>151.85</td>
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<tr>
<td>( P_0 ) (mm)</td>
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<td>35.75</td>
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<tr>
<td>(^{37})Cl (SNU)</td>
<td>5.65</td>
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</tr>
<tr>
<td>(^{71})Ga (SNU)</td>
<td>119.4</td>
<td>127.2</td>
</tr>
<tr>
<td>(^{8})B (10(^6)/cm(^2)/s)</td>
<td>3.66</td>
<td>4.82</td>
</tr>
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Fig 4: Sound speed square difference between the Sun measured by GOLF+LOWL experiments (Lazrek et al. 1997, Tomczyk et al. 1995) and our models without (—) and with diffusion (full line with experimental error bars).
Table 5: Global acoustic mode frequencies ($\mu$Hz) obtained with our standard model for $l=0,1,2,3$.

<table>
<thead>
<tr>
<th>n</th>
<th>$l = 0$</th>
<th>$l = 1$</th>
<th>$l = 2$</th>
<th>$l = 3$</th>
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<td>0</td>
<td>0.000</td>
<td>0.000</td>
<td>352.584</td>
<td>392.639</td>
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<tr>
<td>1</td>
<td>258.179</td>
<td>284.435</td>
<td>382.864</td>
<td>415.854</td>
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<tr>
<td>2</td>
<td>404.229</td>
<td>448.288</td>
<td>514.278</td>
<td>564.492</td>
</tr>
<tr>
<td>3</td>
<td>535.571</td>
<td>596.675</td>
<td>664.025</td>
<td>718.155</td>
</tr>
<tr>
<td>4</td>
<td>679.996</td>
<td>746.366</td>
<td>811.419</td>
<td>866.505</td>
</tr>
<tr>
<td>5</td>
<td>824.820</td>
<td>893.258</td>
<td>959.419</td>
<td>1014.559</td>
</tr>
<tr>
<td>6</td>
<td>972.161</td>
<td>1039.122</td>
<td>1104.663</td>
<td>1161.104</td>
</tr>
<tr>
<td>7</td>
<td>1117.509</td>
<td>1185.051</td>
<td>1250.198</td>
<td>1306.229</td>
</tr>
<tr>
<td>8</td>
<td>1262.906</td>
<td>1329.136</td>
<td>1394.069</td>
<td>1450.387</td>
</tr>
<tr>
<td>9</td>
<td>1406.906</td>
<td>1472.349</td>
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<td>1686.125</td>
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<tr>
<td>12</td>
<td>1821.537</td>
<td>1884.412</td>
<td>1945.228</td>
<td>2000.485</td>
</tr>
<tr>
<td>13</td>
<td>1956.901</td>
<td>2020.064</td>
<td>2081.446</td>
<td>2137.350</td>
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<td>14</td>
<td>2092.957</td>
<td>2156.401</td>
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<td>2273.280</td>
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<td>15</td>
<td>2228.494</td>
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<td>2408.334</td>
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<tr>
<td>16</td>
<td>2363.384</td>
<td>2426.353</td>
<td>2487.029</td>
<td>2543.238</td>
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<tr>
<td>17</td>
<td>2497.456</td>
<td>2560.885</td>
<td>2621.774</td>
<td>2678.527</td>
</tr>
</tbody>
</table>
elemental diffusion: \( R_{bcz} = 0.715 R_\odot \) vs \( R_{bcz} = 0.729 R_\odot \) for non-diffusive models, which is due to the mean Rosseland opacity increase over the whole structure influenced by C, N, O and Fe composition modification, with a dominant peak close to the base of the convection zone. This has the effect to increase \( \nabla_{\text{rad}} \) and thus to favour convective energy transport and a broader convective zone.

d) a very reasonable agreement between our acoustic mode predictions and the most recent observations from GOLF (Lazrek et al. 1997, Rhodes et al. 1997) for the low frequency range. The agreement is between 0.5 – 2\( \mu \) Hz. At high frequencies, the agreement is less, due to the description of the outer layers. See Turck-Chièze et al. (1997) for a discussion on absolute frequency values for different solar models. If one compares our calculations with and without diffusion, we note an improvement of 4-5\( \mu \)Hz at low order for models incorporating microscopic diffusion, due to a higher helium and Z element composition in the region of the sun where radiation dominates, and also a slight improvement for the fine spacing value \( \tilde{\delta}_{0,2} \).

On the other hand, the higher abundances of He and metals in the solar core, increase the discrepancy with neutrino experiments (\(^{71}\text{Ga} +7\%, \(^{37}\text{Cl} +27\%, \(^{8}\text{B} +32\%\)).

e) a different shape of figure 4 in comparison with previous studies (Basu et al. 1997, Turck-Chièze et al. 1997), due to the improvement in modeling the physical processes, specifically the introduction of the OPAL96 opacity coefficients, including 19 elements and the updated reaction rates. This shows the importance of a proper treatment of the known physical processes at the level of accuracy we can reach today to be able to interpret the residual difference.

f) For the neutrino predictions, we use the recent work of Bahcall et al. (1996) for the updated absorption cross sections. Compared to the previous predictions, our diffusive model (see table 4) does not reach such large neutrino flux predictions due to the reestimate of the nuclear reaction rates which leads to a reduction of the neutrino fluxes and practically a compensation of the microscopic diffusion effect.

4. NEW CONSTRAINTS ON SOLAR MODELS FROM RECENT SEISMIC RESULTS

The recent sound speed profile, deduced from the helioseismic experiments leads to a characteristic deviation with standard solar models. It shows a bump near the base of the convection zone and a slower sound speed in the nuclear core with the transition close to the \(^{3}\text{He} \) production zone (i.e. \( \sim 0.28 R_\odot \)) (see Turck-Chièze and Brun 1997) or near the
edge of the nuclear core. The physical processes which are at the origin of this behaviour, must be investigated in order to improve the description of the solar interior and also on the prediction of the observables. In a previous paper (Turck-Chièze et al. 1997), we have shown the sensitivity of this shape to some ingredients of the calculation.

In this section, we check first the consequence of the known uncertainties of the physical processes already included in our reference solar model on the sound speed (see section 2). Then we discuss the possibility of some mixing in the core (sometimes evoked to decrease the central temperature), and some specific points as the actual knowledge of the solar radius.

The main bump, close to the convective-radiative transition zone is of great interest, as it could certainly be the first manifestation of hydrodynamical phenomena. Coupled with the rotation profile in this range it is a strong constraint on tachocline instabilities inside the Sun (Zahn 1992, Spiegel and Zahn 1992, Kosovichev et al. 1997, Corbard et al. 1998). Nevertheless, the detailed interpretation of this peak needs to review also the influence of the quality of some opacity coefficients and the knowledge of the abundances. This concerns mainly the oxygen and iron elements. A preliminary study has been done by Turck-Chièze et al. (1997), which show that the bump may be reduced by a localized 5% variation in the opacity coefficients, which is certainly the order of uncertainty we may have on this quantity at the radiation-convection edge. Moreover, such a study shows that the effect is local and has no consequence on the central part of the Sun. This region will be extensively studied in our next paper "II The radiation convection transition".

As the knowledge of the nuclear part of the Sun is the main objective of this study, we shall concentrate on the radiative zone in this paper.

4.1. The role of the uncertainties on opacity coefficients and heavy element compositions

Considering the broad deviation in the intermediate region between the nuclear region and the convection zone, and noting the improvement done by the microscopic diffusion, which modifies the composition across the sun, we first investigate a possible solution of the discrepancy based on an opacity increase compatible with the abundance uncertainties on the C, N, O, Fe elements. In fact, their ionization states play a crucial role in the opacity coefficient in the radiative zone. Therefore, in order to simulate an opacity correction connected to the treatment of the heavy elements in the absorption opacity coefficients or to the knowledge of their abundances, we fit a correction to $\kappa(T, \rho, X, Y, Z)$ compatible with
Figure 1 of Courtaud et al. (1990), which represented the proportion of the heavy element opacity to the total opacity. This curve has been established for an iron composition about 30% greater than we believe today. In fact, in current tables the C, N, O, Fe elements contribute about 70% of the opacity near the base of the convective zone (Richer 1997, Richer et al. 1998). The correction we applied is of 1.5% in the center and 5% at the base of the convective zone. The results appear in table 6 for the structural quantities and figure 5a) for the sound speed. We note that the sound speed matches the helioseismic data better, \( \delta c^2/c^2 < 0.6\% \), \( ^4\text{He}_{\text{surf}}=0.248 \) and \( \text{R}_{bcz}=0.712 \), without changing greatly the neutrino flux predictions. Also, as expected, the predicted acoustic mode frequencies match closer the GOLF frequencies (see figure 8).

This model (Opac Inc) shows the sensitivity of the Sun structure to localised opacity changes (see also Gabriel 1997) and the necessity to improve our knowledge on the radiation-matter interaction as well as on chemical element abundances.

### 4.2. The role of the uncertainties on the nuclear reaction rates

The great sensitivity of the neutrino fluxes produced by the interaction of electron and proton with the \(^7\text{Be}\) to the different nuclear reaction rates has been largely discussed in the past (Turck-Chi`eze and Lopes 1993, Castellani, Degl’Innocenti and Fiorentini 1993) and has stimulated new measurements and theoretical efforts (see section 2). On the other hand the sensitivity of the sound speed to these important ingredients of the solar modeling is not so large, except for the pp reaction rate, as it modifies the energy production directly (Dzitko et al. 1995, Turck-Chi`eze et al. 1997).

Here we discuss a “minimal nuclear solar model” (Min Nuc), where we modify the nuclear cross sections within the experimental error bars of the new compilation in order to get the minimal neutrino fluxes (Dzitko et al., 1995). The modifications are: +2.2% on \(^1\text{H}(p,e^+\nu)^2\text{H}, +8\%\) on \(^3\text{He}(^3\text{He},2p)^4\text{He}, -10\%\) on \(^3\text{He}(\alpha,\gamma)^7\text{Be}\) and -20% on \(^7\text{Be}(p,\gamma)^8\text{B}\). The results are summarized in table 6 and figure 5b. They show a slightly better agreement with the present helioseismic results in the solar core and a reduction of the neutrino fluxes by -6.5% for \(^{71}\text{Ga}\), -28% for \(^{37}\text{Cl}\), -33% for \(^8\text{B}\) at 1σ level, which are, except for gallium case, half the discrepancies between neutrino detections and predictions. The corresponding acoustic modes frequencies are modified by less than 0.2\(\mu\)Hz (see figure 8).

One may add some comments on the CNO uncertainties which are rather large: (p, \(^{12}\text{C}\)): 15%, (p, \(^{13}\text{C}\)): 13%, (p, \(^{14}\text{N}\)): 11%, (p, \(^{16}\text{O}\)): 18%. As the CNO contribution to the total luminosity is rather small, 1.26% in our standard model, an error in these cross
Fig 5: Sound speed square difference between the Sun seen by GOLF +LOWL (see Turck-Chièze et al. 97) and models. Full line standard model with experimental seismic error bars. Dashed line: a) a model with an increase of opacity between 1% in the central region up to 5% at the bottom of the convective zone, b) a model taking into account the uncertainty on the nuclear reaction rates to define a “minimal nuclear” model, c) a model which takes into account the uncertainty on the microscopic diffusion, here the velocities are increased by 15% in comparison with our standard model.
Table 6: Solar models including a modification of the basic physical ingredients inside the present uncertainties. Same variables than in table 5.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Opac Inc</th>
<th>Min Nuc</th>
<th>Microsc. Inc</th>
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<td>OPAL/K 5800</td>
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<td>yes</td>
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<td>0.274</td>
</tr>
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<td>(1.96 \times 10^{-2})</td>
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<td>0.0277</td>
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</tr>
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<td>0.240</td>
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<tr>
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<td>0.0245</td>
<td>0.0245</td>
</tr>
<tr>
<td>( \tau_b )</td>
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<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( \text{R}<em>{\text{bcz}}/R</em>\odot )</td>
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<td>0.715</td>
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<td>15.70</td>
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<tr>
<td>( \rho_c \ (g/cm^3) )</td>
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<td>150.64</td>
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<td>119.0</td>
<td>128.4</td>
</tr>
<tr>
<td>( ^{8}\text{B} \ (10^6 \text{cm}^2/\text{s}) )</td>
<td>5.10</td>
<td>3.21</td>
<td>5.00</td>
</tr>
</tbody>
</table>
sections has a small impact on the neutrino predictions: about 11% luminosity modification of this small part has an effect of 0.7 SNU on gallium prediction, 0.11 SNU on chlorine and practically no effect on the boron flux. On the other hand, if we try to minimize the neutrino fluxes on gallium and chlorine detectors by reducing the CNO contributors, this can lead up to an added -2 SNU on gallium and -0.5 SNU on chlorine at the 1 σ level.

We notice that the sound speed in the nuclear range may be a good constraint on the pp reaction rate which is not measurable in the laboratory due to its very small cross section. But one must not forget that helioseismology has at present difficulty in discriminating between different sources of uncertainties. In the next section, we discuss the effect of these modifications on the g-mode predictions.

One finally observes that the $^8B$ neutrino flux and the $^7Be$ neutrino flux, respectively peaked at 0.05 and 0.1 R$_\odot$ are not well constrained due to the present seismic uncertainties (see figure 5). A slightly smaller temperature in the very central region is not excluded.

### 4.3. The role of the uncertainties on the effect of the microscopic diffusion

As it has been mentioned previously, one may question the way we introduce the microscopic diffusion velocities for two reasons: first because we use approximate formulae at about 5-10 % accuracy, secondly because the settling of the chemical species is inhibited by turbulent diffusion (Proffitt and Michaud 1991, Richard et al. 1996, Gabriel 1997). Therefore, in this section we look to the sensitivity of the results to a variation of the microscopic diffusion velocity. We have first considered a model where we have increased the helium diffusion velocity by 8% and the heavy elements diffusion velocity by 15% in order to simulate a more complete microscopic calculation. A improvement of the sound speed by no more than 0.1 % occurs with a slight increase of the neutrino fluxes and a decrease of the surface helium composition to 0.241, which seems to be a rather small value. Then to increase the effect on the sound speed, we have considered a model where both the helium and the heavy element microscopic diffusion are increased by 15% (Microsc Inc). In these conditions, the effect on the sound speed is noticeable (see figure 5c), but, as seen in table 6, the reduction of the photospheric helium is important and difficult to reconcile with the helioseismic photospheric value except if some turbulent mixing at the base of the convective zone is evoked to reduce the element settling.

Finally, we notice a slight improvement in the sound speed when we increase the microscopic diffusion, but it seems that such increase is not really supported by the global observations.
4.4. Other questions?

4.4.1. Are the helioseismic results compatible with $^3$He turbulent diffusion?

In order to try to solve the neutrino problem and to understand the weakest points of classical stellar evolution in solar-like stars as the Li and Be abundances or the evolution of the rotation along the stellar life, much theoretical works have been devoted to the study of possible instabilities inside the stars (Schatzman and Maeder 1981, Lebreton and Maeder 1987, Zahn 1992, Morel and Schatzman 1996, for a review and further references: Turck-Chièze et al. 1993). Some of them have been rejected by physical considerations, such as the partial inhibition of turbulent mixing by the gradient of composition. Some others have proposed very recently such as mixing due to gravity waves (Schatzman 1993, Kumar et al. 1997, Zahn, Talon and Matias 1997). An additional question is the reaction of the $^3$He peak located at the boundary of the nuclear core at $0.3 \, R_\odot$ (see figure 5 of Turck-Chièze and Brun 1997).

The solar angular velocity profile would help in this discussion but this quantity is difficult to extract below $0.4 R_\odot$ due to the small value of the splitting of the acoustic modes. It is typically 4 times the estimated error bar and largely influenced by the inherent effect of the stochastic excitation. Some tentative estimates which appear partly contradictory, exist (Elsworth et al. 1995, Lazrek et al. 1996). With very continuous data produced by SOHO helioseismic experiments (GOLF, VIRGO and MDI), we begin to make progress on the accuracy of this rotational splitting (Lazrek et al. 1997). We hope to have a second constraint from the results of the COROT mission (Catala et al. 1995), an asteroseismic project which will be launched in 2002, through measurements of the rotational splitting of young clusters.

In the absence of definitive constraint on the internal rotation of the Sun, we limit our investigation to the profile of the well known sound speed, in calculating “ad hoc” models which may simulate any reasonable mixing. Following the work of Lebreton and Maeder (1987), we have computed several models which include a turbulent term $D_T = \nu_{\text{rad}} \ast Re^*$. We have used different critical Reynold numbers, $Re^* = \text{cte} = 20$ or $Re^* = 20 \exp^{-1/2(r-a/\sigma)^2}$ with $a = 0.3$ or 0.71. We have also tried an other expression for the turbulent term, proposed by Morel and Schatzman (1996) to describe the effect of internal waves:

$$D_T = D_0 \ast \exp^{-1/2(r-0.22/\sigma)^2}$$ (2)

with $D_0$ values varying from $10^2$ to $10^4$ cm$^2$ s$^{-1}$ and $\sigma$ one from 0.02 to 0.04. In the different cases, the photospheric helium is slightly increased, the neutrino flux on chlorine detector
or water never reduced by more than 15%, the sound speed profile never improved and is even worst (e.g. $\delta c^2/c^2 > 2\%$) (see also Richard and Vauclair 1997). It seems to show that the present results do not support evidence for mixing today (see figure 6 for a typical example).

![Fig 6: Sound speed square difference between the Sun seen by GOLF +LOWL (see Turck-Chièze et al. 97) and a model where we have introduced a small amount of mixing of the elements following expression (2) with $D_0 = 1000 \text{ cm}^2 \text{s}^{-1}$.](image)

Of course this is not a definitive conclusion as we cannot exclude a temporal dependence of this kind of coefficient. A more general study must be supported by theoretical investigation of the evolution of the angular momentum with time.

4.4.2. What is the effect of the solar age?

The present accuracy of the determination of the sound speed is so impressive that we have noticed different results for very similar physics included in standard solar models. This must be solved in the near future if one hopes to progress on the interpretation of the seismic tool. Here we question the problem of the age of the Sun where we adjust the present luminosity and radius. Figure 7 illustrates this point in showing two models converged at two different ages: 4.52 and 4.6 Gyr (also currently used), starting at the beginning of hydrogen burning. This increase leads to an increase of the neutrino predictions by less than 2.5% but the effect is not negligible on the sound speed. This point must be taken into account in a comparison between several models which include premainsequence
or not and use slightly different solar ages. Is it a clear signature that the Sun could be a little older than one generally thinks?

Fig 7: Sound speed square difference between the Sun seen by GOLF +LOWL (see Turck-Chièze et al. 97) and two models converged at different solar age: our standard model at 4.52 Gyr (full line) and a model converged at 4.6 Gyr (dashed line).

4.4.3. Is the solar radius well determined?

Due to the quality of the helioseismic measurements, one generally adjusts the solar radius at the estimated value with an accuracy of some $10^{-5}$. This is useful for the determination of the absolute value of the acoustic wave frequencies which vary as $R_{\odot}^{-3/2}$. If we believe the present inversions, we may question the slope observed just above the base of the convective zone (see figures 5a, b and c), where the temperature gradient is purely adiabatic. So to verify this point, we have modified the radius by -300 km to see how the agreement will be modified following the recent possible reestimates of the photospheric solar radius (Schou et al. 1997, Antia 1998, Brown and Christensen-Dalsgaard 1998). In this case we have found a more constant agreement just above the edge of the convective zone even if the absolute difference remains 0.1%. The effect on the frequencies is a reduction of 2 $\mu Hz$ at 3 mHz and -0.64 $\mu Hz$ at 1 mHz. This point could contribute to the general trend observed between measurements and absolute predictions (Turck-Chièze et al. 1997).
5. DISCUSSION

In the previous section, we have discussed several modifications of our reference model to see which may better match the present acoustic mode observations, in varying the main physical processes inside their uncertainties, but also evoking some other processes, such as mixing in the core. We consider that the two modified models: changing the opacities in the radiative region or the nuclear reaction rates, inside the known error bars, match better the present helioseismic data, one in the intermediate region between the nuclear one and the convection zone, the other in the solar core. Consequently, we discuss these two models in details.

5.1. Influence of the physical processes on the acoustic mode predictions

In figure 8, we present the acoustic mode frequency differences between these two models and our reference model for l= 0 and 2. We note that the GOLF results of the first year agree rather well with the model.

![Figure 8: Difference between theoretical acoustic frequencies of various models (the reference being the standard diffusive model), full symbol for the model with a modification of opacity, empty symbol for the model with a modification of nuclear reaction rates. Superimposed GOLF data set taken between 11 April to 25 August 1996 (Lazrek et al. 1997).](image)
where the opacity coefficients have been slightly modified. In the case of the modification of the nuclear reaction rates the best agreement in the solar core, observed on the sound speed, is not visible in the absolute values of the acoustic modes as they are influenced very little by the solar core.

5.2. The corresponding gravity mode predictions

It is clearly established that the gravity modes are a better probe of the nuclear core. Table 7 shows some theoretical predictions of these modes, obtained with the pulsation code of Christensen-Dalsgaard (1982) for our reference model. In this table, we concentrate on the most observable gravity modes (e.g. l= 1 and l= 2) which may be accessible to the satellite SOHO. Following the first asymptotic approximation, the gravity frequency is proportional to \( I = \int_0^r N/r \, dr \), where \( N \) is the Brunt Väisälä frequency (\( N^2 = g ( \Gamma_1 \frac{dp}{dr} - \frac{dlnP}{dr}) \)). 60% of the frequency of the gravity modes is built on the value of the Brunt Väisälä frequency in the inner 0.2 \( R_\odot \).

Therefore in figure 9, we show the comparison of the convergence of this integral for our two modified models in comparison with our reference model, to better understand how a physical modification of the solar model influences the gravity modes.
Table 7: Gravity mode frequencies ($\mu$Hz) obtained from our standard model, with the pulsation code of Christensen-Dalsgaard (1982).

<table>
<thead>
<tr>
<th>n</th>
<th>$l = 1$</th>
<th>$l = 2$</th>
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<tbody>
<tr>
<td>1</td>
<td>260.3</td>
<td>294.5</td>
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<tr>
<td>2</td>
<td>189.3</td>
<td>254.0</td>
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<td>3</td>
<td>151.6</td>
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<tr>
<td>4</td>
<td>126.3</td>
<td>192.2</td>
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<tr>
<td>5</td>
<td>107.9</td>
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<td>6</td>
<td>94.31</td>
<td>149.5</td>
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<td>7</td>
<td>83.68</td>
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<td>68.23</td>
<td>110.4</td>
</tr>
<tr>
<td>10</td>
<td>62.45</td>
<td>101.4</td>
</tr>
</tbody>
</table>

Fig 10: Difference between the gravity modes of the "Opac Inc" model full symbol or of the "Min Nuc" model empty symbol and those of our standard solar model.
We then calculate the g-mode frequencies for these different models with the pulsation code. The modification of the nuclear reaction rates introduced in the model "Min Nuc" does induce a decrease of frequency which may reach $2\mu Hz$ in the range between 50-300 $\mu Hz$ (figure 10). This illustrates that the gravity modes are four to ten times more sensitive to the solar core modifications (see also Brun and Turck-Chièze 1997) than acoustic modes. We also notice that a slight opacity increase and the modified nuclear reaction rates, change the gravity mode frequencies in the opposite direction. These variations and their dependence in frequency, are substantially above the experimental resolution and could be measured if the gravity modes maybe identified at low order. One may also remark that the frequencies dependence of the gravity modes on physical processes is not easy to anticipate and contribute to the difficulty of the identification of such modes. As the opacity change acts in the opposite way compared to the nuclear modified model for the central thermodynamic quantities, but not for $\delta c^2/c^2$, or $^4\text{He}_{\text{surf}}$ and $R_{bcz}$, this emphasizes the fact that a coupled analysis of acoustic modes and gravity modes could be useful to differentiate different physical processes.

6. CONCLUSION

In this study of the radiative region of the Sun, we have observed that the standard picture of the stellar evolution is very robust when challenged by the present seismic constraints, if one accepts the modification of some physical processes such as opacity or nuclear reaction rates inside their inherent error bars. This is a very important issue. We have proposed two models which may better match the information extracted from acoustic mode measurements, in order to improve the standard observable predictions: acoustic modes, gravity modes and neutrino fluxes.

The present seismic data do not support clearly any mixing in the core: effectively, any tentative model of mixing does not improve the sound speed comparison. This fact does not allow us to say that there is no mixing in the solar core as one can imagine compensation effects or mixing in the past, all of them cannot be detected on the present profile. We consider only that the present helioseismic sound speed profile does not allow us to deduce any mixing characteristics. A detailed analysis of the radial rotation profile coupled to a theoretical time dependent sound speed profile is necessary before conclusion on this specific point.

Even if the present solar structure seems rather well under control, we have noticed that our solar neutrino predictions are significantly smaller than recently published ones. After a comparison of the used ingredients, we can say that this discrepancy is mainly due
to the reestimate of the nuclear reaction rates. Moreover, a persistent large uncertainty in chlorine and water detector neutrino predictions exists. A 30% reduction of the predicted values is not ruled out and even favoured by the present study.

It seems that the present seismic results contrain more and more the pp reaction rate and could favour a slightly higher value compatible with the estimated error bar. It is possible to believe that one begins to be sensitive to the effect of the \((^3He, ^3He)\) interaction (shape of the sound speed at low radius). But we also note that the solar structure in the inner 0.1 R\(_\odot\) is not yet constrained by the present seismic results. This confirms that the sensitivity of the present helioseismology (sound speed and acoustic mode analysis) to the nuclear reaction rates stays rather low. Moreover the sensitivity to all the assumptions on screening or reaction rates of the pp chain II or III or CNO cycle is still rather poor (see Turck-Chièze et al. 1997). We can then conclude that the properties of the internal plasma is not checked by present seismology results \((r < 0.2 R\odot)\). This point can only be checked by improved nuclear experiments at low energy or maybe by high energy laser experiments which are presently under investigation in our laboratory.

In parallel, the knowledge of the internal rotation and of the g-mode frequencies will contribute toward the knowledge of the nuclear core and to solve some issues that the classical stellar evolution has not totally answered such as the photospheric lithium or the history of the angular momentum of the Sun.

We have tried to determine some standard models (inside the known error bars of the ingredients), in better agreement with the seismic constraints. In doing such models, we constrain, better than previously, the initial helium content. This study supports a value between 0.273-0.277 in mass fraction.

As our accuracy progresses, we must also be more careful with the available information. We have observed different shapes of the sound speed in the core over the last 2 years, supported by different observations or different physical processes introduced in the calculation. We notice that the solar age plays a non negligible effect. The next years are absolutely crucial with all experiences of solar neutrino detection or seismology in operation. Moreover, due to the extreme accuracy we reach now, the comparison between models of different groups will contribute to definitively fix what we know on the present solar core.

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$\nu_{\text{model}} - \nu_{\text{std}} \ (\mu\text{Hz})$

Order $n$

$0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$
\[ \frac{C_{\text{exp}}^2 - C_{\text{th}}^2}{C_{\text{th}}^2} \]
\[
\frac{(C_{\text{exp}}^2 - C_{\text{th}}^2)}{C_{\text{th}}^2}
\]
\[
\frac{(C_{\text{exp}}^2 - C_{\text{th}}^2)}{C_{\text{th}}^2}
\]
\[ \frac{(C^2_{\text{exp}} - C^2_{\text{th}})}{C^2_{\text{th}}} \]
\[
\frac{(C_{\text{exp}}^2 - C_{\text{th}}^2)}{C_{\text{th}}^2}
\]
$\nu_{\text{model}} - \nu_{\text{std}} \ (\mu\text{Hz})$