On Breaking Cosmic Degeneracy

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Abstract

It has been argued that the power spectrum of the anisotropies in the Cosmic Microwave Background (CMB) may be effectively degenerate, namely that the observable spectrum does not determine a unique set of cosmological parameters. We describe the physical origin of this degeneracy and show that at small angular scales it is broken by gravitational lensing: effectively degenerate spectra become distinguishable at $\ell \sim 3000$ because lensing causes their damping tails to fall at different rates with increasing ℓ. This effect also helps in distinguishing nearly degenerate power spectra such as those of mixed dark matter models. Forthcoming interferometer experiments should provide the means of measuring otherwise degenerate parameters at the $5 - 25\%$ level.

It has recently been pointed out [21] that in a parameter space including open models the contours of the estimated likelihood function for planned experiments are highly elongated in certain directions. This indicates that there is a near degeneracy between cosmological parameters which will limit the accuracy with which the individual parameters can be measured using only CMB observations. Note that with sufficiently good data as expected from planned satellite experiments, no such degeneracy arises in the more limited subset of flat models, contrary to an earlier claim [4]. Our intent with this letter is to first elucidate the physical origin of this effective degeneracy and then show how it is broken at small angular scales. We will not attempt to make detailed estimates of the precision with which particular future experiments may determine cosmological parameters.

The cosmic degeneracy has a simple physical explanation which can be used to easily identify models that have near degenerate CMB power spectra. We restrict our discussion to homogeneous, isotropic cold dark matter models with arbitrary spatial curvature, adiabatic scalar perturbations and no tensor perturbations. A curve that produces degenerate spectra can be found by first specifying $h^2\Omega_m$, $h^2\Omega_b$ and the primordial spectrum. This fixes the comoving scale of the acoustic peaks. One then varies Ω_m (or h) and Ω_Λ in such a way that the angular size distance to the surface of last scattering remains constant. In doing so, the change in the redshift of the surface of last scattering must be taken into account,

$$z_* \cong 10^3 \Omega_b^{-0.027/(1+0.11 \ln \Omega_b)} \qquad (1)$$

[9]. The Hubble parameter is $H_o = 100h$ km/s/Mpc and Ω_m, Ω_b and Ω_Λ are, respectively, the density in matter, baryons and vacuum energy measured in units of the critical density. In contrast to the apparently similar but non-degenerate behavior found for a grid of models with $\Omega_\Lambda + \Omega_m = 1$ [4], this prescription gives models whose spectra are completely degenerate at small angles, or large multipole number ℓ. This is what we mean by the description "effectively degenerate" as opposed to a case of "near" degeneracy which may arise due to experimental limitations.

However, as can be seen in Figure (1) these spectra are not degenerate at large angular scales. This difference is due to the decay in the potential fluctuations in $\Omega_m < 1$ models,

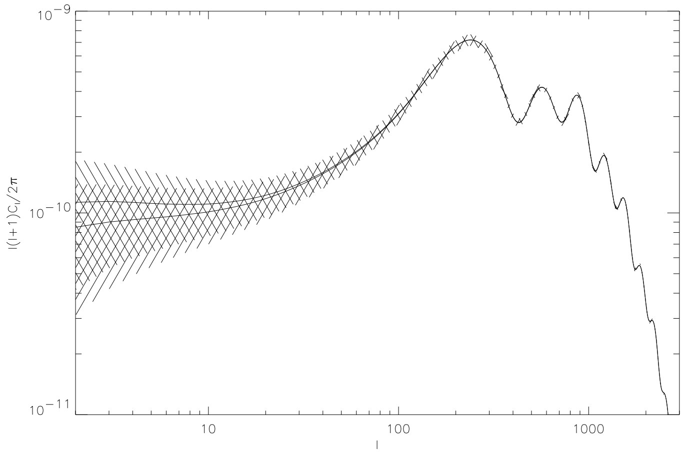

Figure 1: An example of effectively degenerate spectra. For these models $h^2\Omega_b = 0.015$ and $h^2\Omega_m = 0.15$. For the top model $\Omega_m = 0.31$ and $\Omega_\Lambda = 0.61$. For the bottom one $\Omega_m = 0.60$ and $\Omega_\Lambda = 0.34$. The 1σ cosmic variance for each of the models is shown with the hash lines assuming full-sky coverage.

the Integrated Sachs-Wolfe (ISW) effect. Unfortunately this difference is of limited use for discriminating between models because of the magnitude of the cosmic variance on these scales. The cosmic variance is the unavoidable uncertainty that results from comparing a cosmological model which predicts only the statistical distribution of observables with a finite sampling of that distribution as represented by the data: there are many possible skies but we observe only one. If the entire sky could be used for measuring the CMB anisotropies, the variance in the power spectrum would be $\sigma^2_{C_\ell} = 2\langle C_\ell\rangle^2/(2\ell+1)$ where gaussianity has been assumed. In addition, cutting out parts of the sky, such as the galactic plane, will increase this by a factor that is approximately inversely proportional to the fraction of the sky sampled. The cosmic variance is indicated in Figure (1) by the hashed lines. Note that if the parameter space is increased to include models with tensor perturbations, a slope nearly equivalent to the ISW effect can be generated, making the spectra even more degenerate. It is evident that cosmic variance prevents us from differentiating between models whose C_ℓ spectra differ only on large angular scales.

In summary, if one set of parameters is found to fit the observed CMB power spectrum, a whole family of models can be found with the above prescription which fits the spectrum equally well except in the small ℓ region where the cosmic variance is largest. One could always hope to use other observables such as supernova surveys and more direct measures of the Hubble parameter to break the degeneracy in the CMB power spectrum, as suggested by Zaldarriaga, Spergel & Seljak [21]. This is a promising approach, but is unlikely to provide the accuracy of parameter estimation that has been claimed for the CMB fluctuations. In this Letter we show that in standard models the observable CMB power spectrum is in fact not degenerate on small scales due to the processing of the background light after it leaves the surface of last scattering. Gravitational lensing has the effect of both smoothing out the acoustic peaks and troughs in the power spectrum ([16] and references therein) and of making the damping tail a less steep function of ℓ [13, 3]. It is the latter effect that we concentrate on here.

Gravitational lensing by large-scale structure (LSS) magnifies and demagnifies patches of the sky with very nearly the same probability. However, the steep damping of structure with decreasing scale on the surface of last scattering results in a bias in the lensing which transfers power from lower to higher ℓ in the damping tail of the power spectrum [13]. This results in the power spectrum falling less rapidly with increasing ℓ. Since the lensing contribution is different for otherwise degenerate spectra, the degeneracy will be broken and the fractional difference between two such models will increase with increasing ℓ. The strength of this lensing effect increases with increasing Ω_m and is generally greater the more steeply the damping tail drops.

The transformation of the C_ℓ's by lensing is given by

$$\begin{aligned} C_\ell^{ob} \;\simeq\; & \sum_{\ell'=0}^{\infty} C_{\ell'} \frac{2\ell'+1}{2} \int_0^\pi ds \sin(s) P_\ell[\cos(s)] \left\{ e^{-\ell'^2 \langle \beta_\parallel(s)^2 \rangle /2} P_{\ell'}[x] \right. \\ & \left. + \frac{1}{2} \left[\langle \beta_\parallel(s)^2 \rangle - \langle \beta_\perp(s)^2 \rangle \right] P'_{\ell'}[x] \right\}_{x=\cos(s)} \end{aligned} \tag{2}$$

where C_ℓ^{ob} is the observed power spectrum and the second moments of the components of the relative deflection of two light paths are given by

$$\left\langle \beta_{\parallel,\perp}(s)^2 \right\rangle \simeq \left(\frac{2}{g(r)} \right)^2 \int_0^r d\overline{r} \int \frac{dk}{2\pi} k^3 P_\phi(k,\overline{r}) g(r-\overline{r})^2 \left\{ 1 - J_o[ksg(\overline{r})] \pm J_1[ksg(\overline{r})]/ksg(\overline{r}) \right\} . \tag{3}$$

where $g(r) = \{R\sinh(r/R), r, R\sin(r/R)\}$ for the open, flat and closed models respectively. The curvature scale is $R = |H_o\sqrt{1-\Omega-\Omega_\Lambda}|^{-1}$, $P_\phi(k,\overline{r})$ is the power spectrum of potential fluctuations and r is the coordinate distance to the surface of last scattering. This result is derived elsewhere [13]. An effectively numerically equivalent expression was obtained by Seljak [16].

Figure (2) shows four sets of effectively degenerate spectra and two mixed dark matter spectra which will be discussed later. For all of the models, $h^2\Omega_b = 0.015$ and each set has a different value of $h^2\Omega_m$. The matter power spectrum used in these calculations is of the form $P(k) = A^2 k^n T^2(k/h\Gamma)$. We use the standard CDM transfer function of [2] with $\Gamma = \Omega_m h \exp(-\Omega_b - \Omega_b/\Omega_m)$ [19]. For each set of equal $h^2\Omega_m$, the matter power spectrum in the model with the lowest h is COBE-normalized [5]. The conjugate model of each set is normalized so that its CMB power spectrum is degenerate with the COBE-normalized spectrum. This requires changing the normalization by a factor of $\sim 1.14 - 1.02$ with the lower range being for the higher $h^2\Omega_m$. The nonlinear evolution of the matter power spectrum is followed using the method of Peacock & Dodds [14]. The nonlinear evolution of the power spectrum contributes significantly to lensing at these angular scales. All of the intrinsic, pre-lensed CMB power spectra are calculated using the Seljak & Zaldarriaga [18] computer code up to $\ell = 3000$. Beyond this point we use an extrapolation of the spectrum.

The bottom panel of figure (2) gives the fractional difference between the spectra that would be degenerate were there no lensing. The splitting of the spectra is more pronounced as $h^2\Omega_m$ gets larger. The normalizations of the spectra in the top panel have been changed

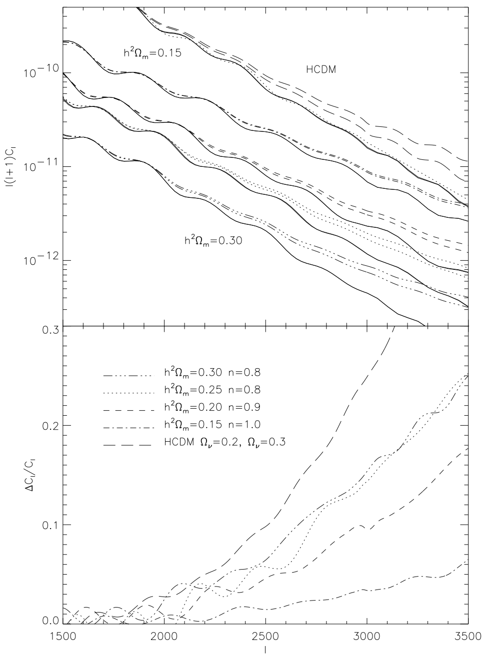

Figure 2: The damping tails of CMB power spectra. For the lower four sets of spectra in the top panel the unlensed spectra (all of them) are shown in solid lines and the lensed spectra are dotted, dashed etc. The upper spectra of each of these sets have $h = 0.7$ and the lower have $h = 0.5$ except in the case of $h^2\Omega_m = 0.30$ which has $h = 0.55$ and 0.7. The top set of spectra labeled HCDM are for mixed dark matter models except the dotted spectrum which is unlensed CDM with $\Omega_m = 1$, $h = 0.6$. The normalizations have been changed for display purposes. The bottom panel is the fractional difference between the lensed spectra.

Table 1: Model Parameters

$h^2\Omega_m$	$h^2\Omega_b$	h	Ω_Λ	n	σ_8
0.30	0.015	0.55	0.003	0.8	0.85
0.30	0.015	0.70	0.34	0.8	0.88
0.25	0.015	0.50	0.00	0.8	0.78
0.25	0.015	0.70	0.43	0.8	0.81
0.20	0.015	0.50	0.17	0.9	0.84
0.20	0.015	0.70	0.52	0.9	0.90
0.15	0.015	0.50	0.17	1.0	0.84
0.15	0.015	0.70	0.52	1.0	0.90

for display purposes (otherwise the lensed spectra for the $h^2\Omega_m = 0.15 - 0.30$ models would overlap). All matter power spectra are COBE-normalized and satisfy the σ_8 constraint on the variance in the average overdensity in a $8h^{-1}$Mpc sphere as determined from the abundance of galaxy clusters [20]. The low h model of the $h^2\Omega_m = 0.30$ models has $h = 0.55$ instead of $h = 0.5$ so that $\Omega_m \leq 1$ for all the models. The $h^2\Omega_m = 0.20$, 0.25 and 0.30 models have tilted primordial spectra so that their σ_8's satisfy the cluster abundance constraints. The reduction in the strength of the lensing in the tilted models is partially made up for by the steepening of the CMB power spectrum which acts to increase the enhancement. Most of the difference in the lensed models is due to the difference in the implied COBE normalization. The differences in the global geometry and the shape and evolution of the power spectrum play a smaller role. The parameters for each of the models are given in Table 1.

Adding more parameters of course complicates things, but lensing of the damping tail is likely to remain a strong discriminator between models. Tensor modes contribute only to the large scale anisotropies where lensing has no affect so they cannot produce an additional set of effectively degenerate models, but only add extra dimensions to the degenerate parameter subspace. If the tensor mode contribution is large enough, it can be seen over cosmic variance ($T/S \gtrsim 0.14$ [11]), otherwise it might be detectable in the polarization anisotropies. Adding hot dark matter changes the heights and positions of the acoustic peaks at intermediate scales, but if there is only one species of neutrino with a cosmologically interesting mass ($0.4 \gtrsim \Omega_\nu \gtrsim 0.2, m_\nu = 46.6\Omega_\nu h^2$eV) these changes are quite small and will be difficult to detect even with future satellite missions [8]. There are two mixed dark matter (HCDM) spectra in Figure (2), both with $\Omega_m + \Omega_\nu = 1$ and $h = 0.6$. The lensing for these models was calculated with only the linear evolution of the matter power spectra given by [15]. Nonlinear evolution would probably increase the difference between the lensed spectra. It is already evident that lensing of the damping tail would be a useful probe of hot dark matter if most of the other cosmological parameters were to some degree constrained by other observations. There remains the possibility that additional effective or near degeneracies exist in this or other expanded parameter spaces. This will always be a possibility because of the unlimited number of additional parameters that could be added.

Radio interferometry presently holds the most promise for observing the CMB at the small angular scales that are necessary to observe these lensing effects. The Cosmic Background

Interferometer (CBI) is now being built [6]. This is an array of 13 one meter dishes that is expected to measure the CMB power spectrum in the range $690 < \ell < 3800$ at $27-36$GHz. The width of the ℓ-space window function, $\Delta\ell$, for an interferometer is $\sim D$, the diameter of the dishes in units of wavelength. The width may be further reduced by mosaicing, and $\Delta\ell \sim 50-100$ is expected. Widths of this order do not significantly change the predicted difference between lensed spectra. The sample variance is given by $\sigma^2_{sam} \simeq (4\pi/A)\sigma^2_{cos}\Delta\ell$ where A is the solid angle covered. For an instrument such as CBI, reducing the sample variance to 10% at $\ell = 3000$ will require covering an area of $13-27$ deg^2 or for 20% accuracy $3-7$ deg^2, depending on $\Delta\ell$.

A possible complication to these lensing predictions is a wide class of foregrounds that may be present at these angular scales. Among these are radio galaxies, the kinematic and thermal Sunyaev-Zel'dovich effect from galaxy clusters and perhaps the most worrying, the kinematic Sunyaev-Zel'dovich effect from either bubbles of ionized gas surrounding quasars [1] or Lyα absorption systems [12]. These effects adds a component to the power spectrum at small angular scales without distorting the thermal spectrum. This decreases the steepness of the damping tail which reduces the lensing effect. On the other hand if reionization is uniform it will damp the spectrum and increase the effect of lensing. The majority of the lensing takes place after reionization affects the CMB. The Rees-Sciama effect is not likely to change our predictions significantly because it is expected to contribute only $\sim 1\%$ or smaller to the power spectrum at the scales of interest [17]. The extent to which the Vishniac effect contributes to the spectrum is strongly dependent on the ionization history of the universe, but is probably small [10, 7]. In any case, the contribution to the CMB power spectrum from these two effects will generally increase with increasing small-scale structure just as lensing does, and will then complement lensing in splitting the degenerate spectra. Foregrounds may lift the degeneracy of spectra by themselves, but to determine in what way this might happen it is necessary to relate the ionization history of the universe to the underlying cosmological model. At present this connection is very uncertain.

In summary, we have shown that if sufficient sensitivity and low enough sample variance can be obtained with future radio interferometers, the problem of degeneracy in the CMB power spectrum can be reduced and effectively removed in many models. Lensing can separate degenerate spectra on the $\sim 5-25\%$ level over a wide range of ℓ space. Because the range is wide, distinguishing between models would require significantly less precision in individual C_ℓ's. It is the rate at which the damping tail falls that distinguishes models. Removing the degeneracy would allow for more accurate measurements of the cosmological parameters based on CMB measurements alone and would enable us to differentiate more definitively between the pre- and post-recombination states of the universe.

We would like to thank the referee for helpful comments and suggestions. This research has been supported in part by grants from NASA and DOE.

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