Experimental Realization of Teleporting an Unknown
Pure Quantum State via Dual Classical and Einstein-Podolski-Rosen Channels

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We report on a quantum optical experimental implementation of teleportation of unknown pure quantum states. This realizes all the nonlocal aspects of the original scheme proposed by Bennett et al. and is equivalent to it up to a local operation. We exhibit results for the teleportation of a linearly polarised state and of an elliptically polarised state. We show that the experimental results cannot be explained in terms of a classical channel alone.

PACS 03.65Bz, 42.50.-p

In [1], Bennett et al showed that an unknown quantum state can be "dis-

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assembled into, then later reconstructed from, purely classical information and purely nonclassical Einstein-Podolsky-Rosen (EPR) correlations." They called this process *teleportation*. In their scheme, a sender, traditionally called Alice, is given a state unknown to her. She also has one of two particles prepared in an EPR state (such as a singlet state). She performs a Bell measurement on the combined system of the unknown state and her EPR particle and transmits the result of this measurement by a classical channel to Bob who has the second of the EPR particles. Depending on the result of the measurement, Bob performs one of four possible unitary transformations on his particle and it will now be in the unknown state.

In the experiment reported in this paper we take an approach first suggested in [2] in which a total of two photons, rather than three, are used. The EPR state is realized by **k**-vector (or path) entanglement and the polarisation degree of freedom of one of the photons is employed for preparing the unknown state. This avoids the difficulties associated with having three photons and, as will be seen below, makes the Bell measurement much more straight forward (indeed, in three photon schemes, where the Bell measurement is made on two photons, feasible schemes can have only a 50% rate of success even in the ideal case [3]). However, this approach does place a restriction on us in that the preparer must prepare his state on one of the EPR photons, and so the unknown state cannot come from outside. Nevertheless, the scheme to be described here realizes all the nonlocal aspects of the original teleportation scheme, and is equivalent to the original scheme upto a local operation (since, in principle, the unknown state of a particle from outside could be swapped onto the polarisation degree of freedom of

Alice's EPR particle by a local unitary operation). In particular, as in the original scheme, we emphasize that if the preparer does not tell Alice what state he has prepared then there is no way Alice can find out what the state is.

No experiment is perfect, so we need to know how good the experiment has to be before we can say we have quantum teleportation. We will take as our objective to show that the experimental results cannot be explained by a classical channel alone (that is without an EPR pair). Thus, consider the following scenario. With probability of $\frac{1}{3}$ the preparer prepares one of the states $|\phi_a\rangle$ ($a = 1, 2, 3$) which correspond to equally spaced linearly polarised states at $0°$, $120°$, $-120°$. He then gives this state to Alice (without telling her which one it is). Alice makes a measurement on it in an attempt to gain some information about the state. The most general measurement she can make is a positive operator valued measure. She will never obtain more information if some of the positive operators are not of first rank, and thus we can take them all to be of first rank, that is proportional to projection operators. Let Alice's measurement have L outcomes labeled $l = 1, 2, \ldots L$ and let the positive operator associated with outcome l be $|\varepsilon_l\rangle\langle\varepsilon_l|$. We require that

$$\sum_{l=1}^{L} |\varepsilon_l\rangle\langle\varepsilon_l| = I \tag{1}$$

Note that in general the states $|\varepsilon_l\rangle$ are neither orthogonal to each other or normalized but rather form an overcomplete basis set. The probability of getting outcome l given that the state prepared is $|\phi_a\rangle$ is $|\langle\phi_a|\varepsilon_l\rangle|^2$ Alice sends the information l to Bob over the classical channel and Bob prepares a state $|\phi_l^c\rangle$ (the c denotes that the state has been "classically teleported").

This state is chosen so as to give the best chance of passing a test for the original state. Bob now passes this state onto a verifier. We suppose that the preparer has told the verifier which state he prepared and the verifier sets his apparatus to measure the projection operator, $|\phi_a\rangle\langle\phi_a|$, onto this state. The probability that the classically teleported state will pass the test in this case is $|\langle\phi_a|\phi_l^c\rangle|^2$ The average probability, S, of passing the test is given by

$$S = \sum_{a,l} \frac{1}{3}|\langle\phi_a|\phi_l^c\rangle|^2|\langle\phi_a|\varepsilon_l\rangle|^2 \tag{2}$$

In the appendix we show this classical teleportation protocol must satisfy

$$S \leq \frac{3}{4} \tag{3}$$

To show that we have quantum teleportation we must show that the experimental results violate this inequality [4].

The experiment is shown in fig. 1. Pairs of polarisation entangled photons are created directly using type II degenerate parametric downconversion by the method described in [5,6]. The crystal is pumped by a 200mW UV cw argon laser with wavelength 351.1nm. The downconverted photons have wavelength 702.2nm. The state of the photons at this stage is $\frac{1}{\sqrt{2}}(|v\rangle_1|h\rangle_2 + |h\rangle_1|v\rangle_2)$ However, we want a **k**-vector entangled state so next we let each photon pass through a calcite crystal after which the state becomes

$$\frac{1}{\sqrt{2}}(|a_1\rangle|a_2\rangle + |b_1\rangle|b_2\rangle)|v\rangle_1|h\rangle_2 \tag{4}$$

By this method a polarisation entangled state has been converted into a **k**-vector entangled state. Here, $|a_1\rangle|v\rangle_1$, for example, represents the state of photon 1 in path a_1 and having vertical polarisation. Since each photon has

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the same polarisation in each of the two paths it can take, the polarisation part of the state factors out of the $\mathbf{k}$-vector entanglement. The EPR pair for the teleportation procedure is provided by this $\mathbf{k}$-vector entanglement. By means of quarter wave plates orientated at some angle γ to the vertical and Fresnel romb polarisation rotators acting in the same way on paths a_1 and b_1 as shown in Fig. 1 the polarisation degree of freedom of photon 1 is used by the preparer to prepare the state $|\phi\rangle = \alpha|v\rangle_1 + \beta|h\rangle_1$. This is the state to be teleported. The state of the whole system is now

$$\frac{1}{\sqrt{2}}(|a_1\rangle|a_2\rangle + |b_1\rangle|b_2\rangle)(\alpha|v\rangle_1 + \beta|h\rangle_1)|h\rangle_2 \tag{5}$$

We now introduce four orthonormal states which are directly analogous to the Bell states considered in [1]:

$$|c_\pm\rangle = \frac{1}{\sqrt{2}}(|a_1\rangle|v\rangle_1 \pm |b_1\rangle|h\rangle_1) \tag{6}$$

$$|d_\pm\rangle = \frac{1}{\sqrt{2}}(|a_1\rangle|h\rangle_1 \pm |b_1\rangle|v\rangle_1) \tag{7}$$

We can rewrite (5) using these states as a basis:

$$\frac{1}{2}|c_+\rangle(\alpha|a_2\rangle + \beta|b_2\rangle)|h\rangle_2 + \frac{1}{2}|c_-\rangle(\alpha|a_2\rangle - \beta|b_2\rangle)|h\rangle_2$$
$$+ \frac{1}{2}|d_+\rangle(\beta|a_2\rangle + \alpha|b_2\rangle)|h\rangle_2 + \frac{1}{2}|d_-\rangle(\beta|a_2\rangle - \alpha|b_2\rangle)|h\rangle_2 \tag{8}$$

For Alice, it is simply a question of measuring on the basis $|c_\pm\rangle$, $|d_\pm\rangle$. To do this we first rotate the polarisation of path b_1 by a further $90°$ (in the actual experiment this was done by setting the angle of the Fresnel romb in path b_1 at $\theta + 90°$ rather than by using a separate plate as shown in the figure) so that the state $|b_1\rangle|v\rangle_1$ becomes $-|b_1\rangle|h\rangle_1$ and the state $|b_1\rangle|h\rangle_1$ becomes

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$|b_1\rangle|v\rangle_1$. Thus,

$$|c_\pm\rangle \rightarrow \frac{1}{\sqrt{2}}(|a_1\rangle \pm |b_1\rangle)|v\rangle_1) \qquad (9)$$

$$|d_\pm\rangle \rightarrow \frac{1}{\sqrt{2}}(|a_1\rangle \mp |b_1\rangle)|h\rangle_1) \qquad (10)$$

Paths a_1 and b_1 now impinge on the two input ports of an ordinary 50:50 beamsplitter (BS). At this beamsplitter each of the two polarisations h, and v interfere independently. After the beamsplitter there are two polarisers which are set either to transmit h or to transmit v polarisation. When the path lengths have been set appropriately, the state $\frac{1}{\sqrt{2}}(|a_1\rangle \pm |b_1\rangle)$ gives rise to a click at detector $D_{A_\pm}$. If the polarisers are set to v then a click at $D_{A_\pm}$ corresponds to measurement onto $|c_\pm\rangle$ and if the polarisers are set to h then a click at $D_{A_\pm}$ corresponds to a measurement onto $|d_\mp\rangle$. In this way each of the four Bell states can be measured.

At Bob's end, path b_2 is rotated through 90° by a half wave plate. Then paths a_2 and b_2 are combined at a polarising beamsplitter orientated to transmit vertical and reflect horizontal polarisation. The state in (8) becomes

$$\frac{1}{2}|c_+\rangle(\beta|v\rangle_2 + \alpha|h\rangle_2) + \frac{1}{2}|c_-\rangle(-\beta|v\rangle_2 + \alpha|h\rangle_2)$$
$$+\frac{1}{2}|d_+\rangle(\alpha|v\rangle_2 + \beta|h\rangle_2) + \frac{1}{2}|d_-\rangle(-\alpha|v\rangle_2 + \beta|h\rangle_2) \qquad (11)$$

Bob's photon can be transformed back to the original state $\alpha|v\rangle + \beta|h\rangle$ by applying an appropriate unitary transformation depending on the outcome of Alice's measurement. However, we are simply interested in verifying that the appropriate state has been produced at Bob's end so rather than performing these unitary transformations we will simply orientate the measuring apparatus at end 2 appropriately for each of Alice's outcomes (the transformations

can either be seen as active transformations or as a passive re-orientation of our reference system with respect to which verification measurements are made). In the case of linear polarisation the verification measurements can be accomplished by rotating the polarisation of the state through an an angle θ_B by means of Fresnel romb devices, then letting it impinge on a polarising beamsplitter followed by two detectors $D_B(\theta_B)$ (a photon originally incident with polarisation θ_B would certainly be detected at this detector), and $D_B(\theta_B^\perp)$. In the more general case of elliptical polarisation we can add a quarter wave plate orientated at an angle γ_B to the vertical before the polarisation rotator.

Wide filters ($\Delta\lambda$=20nm) were placed just before each detector. Wide, rather than narrow, filters were used so that the count rate was high enough to allow measurements to be made in a few seconds in order that problems associated with phase drift were minimize.

The beamsplitter BS and the trombone arrangement in path a_2 were each mounted on a computer controlled micrometrical stage which could be incremented in 0.1μm steps. To align the system a half wave plate orientated at 45° to the vertical was placed before one of the calcite crystals to rotate the polarisation by 90°. This had the effect of sending both photons to Alice's end or Bob's end. The correct values of Δz and Δy were found by looking for an interference dip in the coincidence count rates between A_+ and A_- at Alice's end, and between $D_B(45°)$ and $D_B(-45°)$ at Bob's end. After alignment, the half wave plate was rotated to 0° to the vertical so that it had no effect on the polarisation of photons passing through it.

We will report separately two aspects of the experiment. First using three

equally spaced linear polarisation settings $(0°, +120°, -120°)$ we will see that the classical teleportation limit in equation (3) is surpassed. Second we will see that, for some arbitrary states (linear and elliptically polarised), we see all the expected features.

Let $I(\theta_B)$ be the coincidence count between $D_B(\theta)$ and one of Alice's detectors. Let $I_\parallel$ be the coincidence rate when θ_B is orientated so as to measure the projection operator onto the corresponding term in (11). For example, if $\theta = 120°$ then Alice's output $|c_+\rangle$ corresponds requires $\theta_B = -30°$. Let $I_\perp$ be the count rate when θ_B is rotated through $90°$ from the value used to measure $I_\parallel$. We will have $I_\parallel = k|\langle\phi|\phi_{\text{tele}}\rangle|^2$ and $I_\perp = k|\langle\phi|\phi_{\text{tele}}^\perp\rangle|^2$ where $|\phi\rangle$ is the prepared state and $|\phi_{\text{tele}}\rangle$ is the state that is actually produced at Bob's side by the teleportation process. If the state produced at Bob's end is not pure then this analysis is easily adapted by summing over a particular decomposition of the impure state. The normalization constant k depends on the detector efficiencies. Since $I_\parallel + I_\perp = k$, we have that

$$|\langle\phi|\phi_{\text{tele}}\rangle|^2 = \frac{I_\parallel}{I_\parallel + I_\perp} \tag{12}$$

To beat the classical teleportation limit the average value of this quantity must exceed $3/4$. This average is taken over the three equally spaced linear polarisations (each weighted with probability $\frac{1}{3}$) and over each of the four possible outcomes of Alice's Bell state measurement (each weighted by $\frac{1}{4}$). With the average understood to be in this sense we can write

$$S = \Big[\frac{I_\parallel}{I_\parallel + I_\perp}\Big]_{\text{av}} \tag{13}$$

This quantity was measured and we found that

$$S = 0.831 \pm 0.009$$

This represents a violation of the classical teleportation limit by 9 standard deviations. Note, if $I_\parallel = I_{\max}$ and $I_\perp = I_{\min}$ (as we would expect, and as is indeed true for the data to be discussed below), then if the quantity in (12) greater than 3/4 then the visibility is greater than 50% (and vice versa).

Now consider in more detail one linear polarisation case. All the quarter wave plates were removed in this case. In Fig. 2, we show coincidence count rates between each outcome of Alice's Bell state measurement ($c_\pm$ and $d_\pm$) and Bob's detector $D_B(\theta_B)$ as a function of θ_B for the particular case where the preparer prepared linear polarisation with $\theta = 22.5°$ with respect to the horizontal direction. Note that the displacements of the maxima are 22.5°, 67.5°, 112.5° and -22.5°. These are consistent with equation (11) where $\alpha = \sin(\theta)$ and $\beta = \cos(\theta)$.

To prepare an elliptical polarised state we set $\theta = 0$ and inserted the quarter wave plates at angle γ equal to 20° to the vertical. This produced the elliptically polarised state

$$\frac{1}{\sqrt{2}}[(1 + i\cos(2\gamma))|\updownarrow\rangle + \sin(2\gamma)|\leftrightarrow\rangle]$$

To verify that the state had been teleported according to equation (11) quarter wave plate was used at Bob's end orientated at angle γ_B. A different setting of this was used corresponding to each of Alice's outcomes. For outcomes $|d_\pm\rangle$ we set $\gamma_B = \pm\gamma + 90°$. This converts the corresponding state at Bob's side to $|v\rangle$. For outcomes $|c_\pm\rangle$ we set $\gamma_B = \pm\gamma$. This converts the corresponding state at Bob's side to $|h\rangle$. Then the state was analysed in linear polarisation over a range of values of θ_B. The results, shown in fig. 3, demonstrate that the state after the quarter wave plate is vertically or horizontally polarised as required.

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In this paper we have seen how a state which is unknown to Alice can be disassembled into purely classical and purely nonlocal EPR correlations, and then it can be reconstructed at a distant location. In the reconstruction procedure we took an essentially passive view of the unitary transformations. That is to say the verifier took them into account in his verification process by orientating his apparatus appropriately. Work is currently under progress to implement the transformations in an active way by using fast Pockel cells fired by Alice's detectors. Also, work is under way to use polarising beamsplitters rather than the polarisers at Alice's end, followed by four detectors. This would allow an 'in principle' teleportation efficiency of 100%.

Appendix

In this appendix we prove inequality (3) for the classical teleportation protocol. Define the normalized state $|\Omega_l\rangle$ by $|\Omega_l\rangle = \sqrt{\mu_l}|\varepsilon_l\rangle$ where $\mu_l = \langle\varepsilon_l|\varepsilon_l\rangle$. By taking the trace of (1) we obtain

$$\sum_l \mu_l = 2 \tag{14}$$

The 2 here corresponds to the dimension of the Hilbert space. Define

$$T_l = \sum_a |\langle\phi_a|\phi_l^c\rangle|^2 |\langle\phi_a|\Omega_l\rangle|^2 \tag{15}$$

Then

$$S = \sum_l \frac{1}{3}\mu_l T_l \tag{16}$$

By varying with respect to the vectors $|\phi_l^c\rangle$ and $|\Omega_l\rangle$ we obtain $T_l^{\max} = \frac{9}{8}$. Hence,

$$S \leq \sum_l \frac{1}{3}\mu_l \frac{9}{8}$$

By using (14) we obtain (3) as required.

Acknowledgements

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